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Scientific Thought

By

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NEW YORK
HARCOURT, BRACE & COMPANY, INC.
LONDON: KEGAN PAUL, TRENCH, TRUBNER & CO., LTD.
1923

PRINTED IN GREAT BRITAIN BY
THE EDINBURGH PRESS, 9 AND 11 YOUNG STREET, EDINBURGH.

IN PIAM MEMORIAM

IACOBI MUDIE

DUCIS ILLIUS DORSETIAE

APUD CONDISCIPULOS IN UNIVERSITATE ANDREANA

QUI A.D. MDCCCXCV TAODUNI SCOTORUM NATUS

A.D. MDCCCCXVI IN PUGNA AD THESSALONICAM OCCUBUIT

. . . Manibus date lilia plenis :
Purpureos spargam flores, animamque nepotis
His saltem accumulem donis, et fungar inani
Munere.—VIRGIL, *Aeneid,* VI

At ego tibi sermone isto . . . varias fabulas conseram, auresque tuas benevolas lepido susurro permulceam, modo si papyram Ægyptiam argutia Nilotici calamo inscriptam non spreveris inspicere . . .—APULEIUS, *The Golden Ass*

PREFACE

The present book is ultimately based on a course of lectures delivered to the third year students of science at the University of Bristol in the session 1920-21. It is an admirable custom, which, like many other benefits, that University owes to my distinguished predecessor, Professor Lloyd Morgan, that all students of science are expected to attend such a course before completing their career. It seemed worth while to elaborate the lectures, to remove their more obvious blemishes, and to present them to a wider public.

In the *First Part* I have started with the highly sophisticated concepts of the classical mathematical physics, have tried to express them clearly, and have then discussed the modifications which recent advances in scientific knowledge have necessitated in these concepts. I have carried this account to the end of the Second Theory of Relativity. I have not penetrated into the still more revolutionary speculations of Weyl, because I do not feel that I yet understand them well enough myself to venture to explain them to others. A philosopher who regards ignorance of a scientific theory as a sufficient reason for not writing about it cannot be accused of complete lack of originality, as a study of recent philosophical literature will amply prove.

I begin with an *Introduction*, which states what I think Philosophy to be about, and how I think it to

be connected with the special sciences. I then try to explain in simple terms the nature and objects of Whitehead's *Principle of Extensive Abstraction.* This seems to me to be the "Prolegomena to every future Philosophy of Nature." It is quite possible to explain its motives and general character without entering deeply into those logico-mathematical complications which are inevitable when it is applied in detail. Next, greatly daring, I have discussed the difficult problems which centre upon the general notion of Time and Change. Here I have tried to make some answer to the very disturbing arguments by which Dr M'Taggart has claimed to disprove the reality of these apparently fundamental features of the Universe. After this the rest of the *First Part* should be fairly plain sailing to anyone of decent general education, though I do not pretend that it can be understood without effort by persons who are unfamiliar with the subjects which it treats.

In some of these later chapters the reader will find a number of mathematical formulæ. He must not be frightened of them, for I can assure him that they involve no algebraical processes more advanced than the simple equations which he learnt to solve at his mother's knee. I myself can make no claims to be a mathematician: the most I can say is that I can generally follow a mathematical argument if I take enough time over it. I like to believe that, in expounding the Theory of Relativity, a clumsy mathematician has some of the qualities of his defects. His own former difficulties will at least suggest to him the places where others are likely to have trouble.

In *Part II* we start afresh at a quite different level. Here I try to point out the sensible and perceptible facts which underlie the highly abstract concepts of

science, and the cruder, but still highly sophisticated, concepts of common-sense. Beside the intrinsic interest and importance of this topic it has a direct bearing on *Part I.* A great deal of the difficulty which many people have in accepting the newer views of Space, Time and Motion, arises from the fact that they regard the traditional concepts as perfectly plain and obvious, whilst they feel that the later modifications are paradoxes, forced on them *vi et armis* by a few inconvenient and relatively trivial facts. The moment we recognise how extraordinarily remote the classical concepts are from the crude facts of sense-experience, from which they must have been gradually elaborated, this source of incredulity vanishes. The hold of the tradition is loosened; and we are prepared to consider alternative, and possibly more satisfactory, conceptual syntheses of sensible facts.

I have tried in *Part II* to focus before my mind what seems to me to be the most important work that has been done on these subjects since 1914, when the publication of my *Perception, Physics and Reality* unhappily precipitated a European war. If I have learnt nothing else since then, I have at least come to see the extreme complexity of the problem of the external world and of our supposed knowledge of it. My obligations to Moore, Russell, Whitehead and Stout are continual, and will be perfectly obvious to anyone acquainted with the literature of the subject. I here make my grateful acknowledgments to them, once for all. To a less extent I have been influenced by Alexander and Dawes Hicks. I have merely mentioned Dawes Hicks's theory of appearance and then left it. This is not because I think it either impossible or unimportant, but because I am here deliberately trying to work out a different view, which I also think to be possible and important.

I cannot claim to have put forward any new and startling theory of the universe. If I have any kind of philosophical merit, it is neither the constructive fertility of an Alexander, nor the penetrating critical acumen of a Moore; still less is it that extraordinary combination of both with technical mathematical skill which characterises Whitehead and Russell. I can at most claim the humbler (yet useful) power of stating difficult things clearly and not too superficially.

> "Excudent alii spirantia mollius aera,
> Credo equidem; vivos ducent de marmore vultus;"

but I hope that I may at least have smolten some of the metal and hewn some of the stone which others will use in their constructions.

I must end by thanking Dr R. S. Paton of Perth for kindly reading the proofs and helping me with the index; Mr E. Harrison, of Trinity College, Cambridge, for his gallant efforts to involve my dedication in "the decent obscurity of a learned language"; and the printers for the care which they have taken in printing what must have been a rather troublesome piece of work.

C. D. BROAD.

LONDON, *Sept.* 1922.

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INTRODUCTION

"Noli, Lector, expectare hoc loco, contra Philosophiam aut Philosophos orationem invectivam. . . . Distinguo inter Philosophos et non Philosophos, et inter Philosophiam veram, vitae humanae Magistram sapientissimam, humanae naturae decus singulare, et illam, quae jam diu pro Philosophia habita est, fucatam et garrulam meretriculam."

(HOBBES, *Leviathan*, Part IV. cap. xlvi.)

The Subject-matter of Philosophy, and its Relations to the special Sciences

I SHALL devote this introductory chapter to stating what I think Philosophy is about, and why the other sciences are important to it and it is important to the other sciences. A very large number of scientists will begin such a book as this with the strong conviction that Philosophy is mainly moonshine, and with the gravest doubts as to whether it has anything of the slightest importance to tell them. I do not think that this view of Philosophy is true, or I should not waste my time and cheat my students by trying to teach it. But I do think that such a view is highly plausible, and that the proceedings of many philosophers have given the general public some excuse for its unfavourable opinion of Philosophy. I shall therefore begin by stating the case against Philosophy as strongly as I can, and shall then try to show that, in spite of all objections, it really is a definite science with a distinct subject-matter. I shall try to show that it really does advance and that it is related to the special sciences in such a way that

the co-operation of philosophers and scientists is of the utmost benefit to the studies of both.

I think that an intelligent scientist would put his case against Philosophy somewhat as follows. He would say: "Philosophers discuss such subjects as the existence of God, the immortality of the soul, and the freedom of the will. They spin out of their minds fanciful theories, which can neither be supported nor refuted by experiment. No two philosophers agree, and no progress is made. Philosophers are still discussing with great heat the same questions that they discussed in Greece thousands of years ago. What a poor show does this make when compared with mathematics or any of the natural sciences! Here there is continual steady progress; the discoveries of one age are accepted by the next, and become the basis for further advances in knowledge. There is controversy indeed, but it is fruitful controversy which advances the science and ends in definite agreement; it is not the aimless wandering in a circle to which Philosophy is condemned. Does this not very strongly suggest that Philosophy is either a mere playing with words, or that, if it has a genuine subject-matter, this is beyond the reach of human intelligence?"

Our scientist might still further strengthen his case by reflecting on the past history of Philosophy and on the method by which it is commonly taught to students. He will remind us that most of the present sciences started by being mixed up with Philosophy, that so long as they kept this connexion they remained misty and vague, and that as soon as their fundamental principles began to be discovered they cut their disreputable associate, wedded the experimental method, and settled down to the steady production of a strapping family of established truths. Mechanics is a case in point. So long as it was mixed up with Philosophy it made no progress; when the true laws of motion were discovered by the experiments and reasoning of Galileo

it ceased to be part of Philosophy and began to develop into a separate science. Does this not suggest that the subject-matter of Philosophy is just that ever-diminishing fragment of the universe in which the scientist has not yet discovered laws, and where we have therefore to put up with guesses? Are not such guesses the best that Philosophy has to offer; and will they not be swept aside as soon as some man of genius, like Galileo or Dalton or Faraday, sets the subject on the sure path of science?

Should our scientist talk to students of Philosophy and ask what happens at their lectures, his objections will most likely be strengthened. The answer may take the classical form: "He tells us what everyone knows in language that no one can understand." But, even if the answer be not so unfavourable as this, it is not unlikely to take the form: "We hear about the views of Plato and Kant and Berkeley on such subjects as the reality of the external world and the immortality of the soul." Now the scientist will at once contrast this with the method of teaching in his own subject, and will be inclined to say, if *e.g.* he be a chemist: "We learn what *are* the laws of chemical combination and the structure of the Benzene nucleus, we do not worry our heads as to what exactly Dalton thought or Kekule said. If philosophers really know anything about the reality of the external world why do they not say straightforwardly that it is real or unreal, and prove it? The fact that they apparently prefer to discuss the divergent views of a collection of eminent 'back-numbers' on the question strongly suggests that they know that there is no means of answering it, and that nothing better than groundless personal opinions can be offered."

I have put these objections as strongly as I can, and I now propose to see just how much there is in them. First, as to the alleged unprogressive character of Philosophy. This is, I think, an illusion; but it is a very natural one. Let us take the question of the

reality of the external world as an example. Common-sense says that chairs and tables exist independently of whether anyone happens to perceive them or not. We study Berkeley and find him claiming to prove that such things can only exist so long as they are perceived by someone. Later on we read some modern realist, like Alexander, and we are told that Berkeley was wrong, and that chairs and tables can and do exist unperceived. We seem merely to have got back to where we started from, and to have wasted our time. But this is not really so, for two reasons. (i) What we believe at the end of the process and what we believed at the beginning are by no means the same, although we express the two beliefs by the same form of words. The original belief of common-sense was vague, crude and unanalysed. Berkeley's arguments have forced us to recognise a number of distinctions and to define much more clearly what we mean by the statement that chairs and tables exist unperceived. What we find is that the original crude belief of common-sense consisted of a number of different beliefs, mixed up with each other. Some of these may be true and others false. Berkeley's arguments really do refute or throw grave doubt on some of them, but they leave others standing. Now it may be that those which are left are enough to constitute a belief in the independent reality of external objects. If so this final belief in the reality of the external world is much clearer and subtler than the *verbally* similar belief with which we began. It has been purified of irrelevant factors, and is no longer a vague mass of different beliefs mixed up with each other.

(ii) Not only will our final belief differ in content from our original one, it will also differ in certainty. Our original belief was merely instinctive, and was at the mercy of any sceptical critic who chose to cast doubts on it. Berkeley has played this part. Our final belief is that part or that modification of our original one that has managed to survive his criticisms. This

does not of course *prove* that it is true; there may be other objections to it. But, at any rate, a belief that has stood the criticisms of an acute and subtle thinker, like Berkeley, is much more likely to be true than a merely instinctive belief which has never been criticised by ourselves or anyone else. Thus the process which at first sight seemed to be merely circular has not really been so. And it has certainly not been useless; for it has enabled us to replace a vague belief by a clear and analysed one, and a merely instinctive belief by one that has passed through the fire of criticism.

The above example will suggest to us a part at least of what Philosophy is really about. Common-sense constantly makes use of a number of concepts, in terms of which it interprets its experience. It talks of *things* of various kinds; it says that they have *places* and *dates*, that they *change*, and that changes in one *cause* changes in others, and so on. Thus it makes constant use of such concepts or categories as thinghood, space, time, change, cause, etc. Science takes over these concepts from common-sense with but slight modification, and uses them in its work. Now we can and do *use* concepts without having any very clear idea of their meaning or their mutual relations. I do not of course suggest that to the ordinary man the words *substance*, *cause*, *change*, etc., are mere meaningless noises, like *Jabberwock* or *Snark*. It is clear that we mean something, and something different in each case, by such words. If we did not we could not use them consistently, and it is obvious that on the whole we do consistently apply and withhold such names. But it is possible to apply concepts more or less successfully when one has only a very confused idea as to their meaning. No man confuses place with date, and for practical purposes any two men agree as a rule in the places that they assign to a given object. Nevertheless, if you ask them what exactly they mean by *place* and *date*, they will be puzzled to tell you.

Now the most fundamental task of Philosophy is to take the concepts that we daily use in common life and science, to analyse them, and thus to determine their precise meanings and their mutual relations. Evidently this is an important duty. In the first place, clear and accurate knowledge of anything is an advance on a mere hazy general familiarity with it. Moreover, in the absence of clear knowledge of the meanings and relations of the concepts that we use, we are certain sooner or later to apply them wrongly or to meet with exceptional cases where we are puzzled as to how to apply them at all. For instance, we all agree pretty well as to the place of a certain pin which we are looking at. But suppose we go on to ask: "Where is the image of that pin in a certain mirror; and is it in this place (whatever it may be) in precisely the sense in which the pin itself is in *its* place?" We shall find the question a very puzzling one, and there will be no hope of answering it until we have carefully analysed what we mean by *being in a place*.

Again, this task of clearing up the meanings and determining the relations of fundamental concepts is not performed to any extent by any other science. Chemistry *uses* the notion of substance, geometry that of space, and mechanics that of motion. But they assume that you already know what is meant by *substance* and *space* and *motion*. So you do in a vague way; and it is not their business to enter, more than is necessary for their own special purposes, into the meaning and relations of these concepts as such. Of course the special sciences do in some measure clear up the meanings of the concepts that they use. A chemist, with his distinction between elements and compounds and his laws of combination, has a clearer idea of substance than an ordinary layman. But the special sciences only discuss the meanings of their concepts so far as this is needful for their own special purposes. Such discussion is incidental to them, whilst

it is of the essence of Philosophy, which deals with such questions for their own sake. Whenever a scientist begins to discuss the concepts of his science in this thorough and disinterested way we begin to say that he is studying, not so much Chemistry or Physics, as the *Philosophy* of Chemistry or Physics. It will therefore perhaps be agreed that, in the above sense of Philosophy, there is both room and need for such a study, and that there is no special reason to fear that it will be beyond the compass of human faculties.

At this point a criticism may be made which had better be met at once. It may be said: "By your own admission the task of Philosophy is purely verbal; it consists entirely of discussions about the meanings of words." This criticism is of course absolutely wide of the mark. When we say that Philosophy tries to clear up the meanings of concepts we do not mean that it is simply concerned to substitute some long phrase for some familiar word. Any analysis, when once it has been made, is naturally *expressed* in words; but so too is any other discovery. When Cantor gave his definition of Continuity, the final result of his work was expressed by saying that you can substitute for the word "continuous" such and such a verbal phrase. But the essential part of the work was to find out exactly what properties are present in objects when we predicate continuity of them, and what properties are absent when we refuse to predicate continuity. This was evidently not a question of words but of things and their properties.

Philosophy has another and closely connected task. We not only make continual use of vague and unanalysed concepts. We have also a number of uncriticised beliefs, which we constantly assume in ordinary life and in the sciences. We constantly assume, *e.g.* that every event has a cause, that nature obeys uniform laws, that we live in a world of objects whose existence and behaviour are independent of our

knowledge of them, and so on. Now science takes over these beliefs without criticism from common-sense, and simply works with them. We know by experience, however, that beliefs which are very strongly held may be mere prejudices. Negroes find it very hard to believe that water can become solid, because they have always lived in a warm climate. Is it not possible that we believe that nature as a whole will always act uniformly simply because the part of nature in which the human race has lived has happened to act so up to the present? All such beliefs then, however deeply rooted, call for criticism. The first duty of Philosophy is to state them clearly; and this can only be done when we have analysed and defined the concepts that they involve. Until you know exactly what you mean by *change* and *cause* you cannot know what is meant by the statement that *every change has a cause.* And not much weight can be attached to a person's most passionate beliefs if he does not know what precisely he is passionately believing. The next duty of Philosophy is to test such beliefs; and this can only be done by resolutely and honestly exposing them to every objection that one can think of oneself or find in the writings of others. We ought only to go on believing a proposition if, at the end of this process, we still find it impossible to doubt it. Even then of course it may not be true, but we have at least done our best.

These two branches of Philosophy—the analysis and definition of our fundamental concepts, and the clear statement and resolute criticism of our fundamental beliefs—I call *Critical Philosophy*. It is obviously a necessary and a possible task, and it is not performed by any other science. The other sciences *use* the concepts and *assume* the beliefs; Critical Philosophy tries to analyse the former and to criticise the latter. Thus, so long as science and Critical Philosophy keep to their own spheres, there is no possibility of conflict between them, since their subject-matter is

quite different. Philosophy claims to analyse the general concepts of substance and cause, *e.g.*; it does not claim to tell us about particular substances, like gold, or about particular laws of causation, as that *aqua regia* dissolves gold. Chemistry, on the other hand, tells us a great deal about the various kinds of substances in the world, and how changes in one cause changes in another. But it does not profess to analyse the general concepts of substance or causation, or to consider what right we have to assume that every event has a cause.

It should now be clear why the method of Philosophy is so different from that of the natural sciences. Experiments are not made, because they would be utterly useless. If you want to find out how one substance behaves in presence of another you naturally put the two together, vary the conditions, and note the results. But no experiment will clear up your ideas as to the meaning of *cause* in general or of *substance* in general. Again, all conclusions from experiments rest on some of those very assumptions which it is the business of Philosophy to state clearly and to criticise. The experimenter assumes that nature obeys uniform laws, and that similar results will follow always and everywhere from sufficiently similar conditions. This is one of the assumptions that Philosophy wants to consider critically. The method of Philosophy thus resembles that of pure mathematics, at least in the respect that neither has any use for experiment.

There is, however, a very important difference. In pure mathematics we start either from axioms which no one questions, or from premises which are quite explicitly assumed merely as hypotheses; and our main interest is to deduce remote consequences. Now most of the tacit assumptions of ordinary life and of natural science claim to be true and not merely to be hypotheses, and at the same time they are found to be neither clear nor self-evident when critically reflected upon. Most

mathematical axioms are very simple and clear, whilst most other propositions which men strongly believe are highly complex and confused. Philosophy is mainly concerned, not with remote conclusions, but with the analysis and appraisement of the original premises. For this purpose analytical power and a certain kind of insight are necessary, and the mathematical method is not of much use.

Now there is another kind of Philosophy; and, as this is more exciting, it is what laymen generally understand by the name. This is what I call *Speculative Philosophy*. It has a different object, is pursued by a different method, and leads to results of a different degree of certainty from Critical Philosophy. Its object is to take over the results of the various sciences, to add to them the results of the religious and ethical experiences of mankind, and then to reflect upon the whole. The hope is that, by this means, we may be able to reach some general conclusions as to the nature of the Universe, and as to our position and prospects in it.

There are several points to be noted about Speculative Philosophy. (i) If it is to be of the slightest use it must presuppose Critical Philosophy. It is useless to take over masses of uncriticised detail from the sciences and from the ethical and religious experiences of men. We do not know what they mean, or what degree of certainty they possess till they have been clarified and appraised by Critical Philosophy. It is thus quite possible that the time for Speculative Philosophy has not yet come; for Critical Philosophy may not have advanced far enough to supply it with a firm basis. In the past people have tended to rush on to Speculative Philosophy, because of its greater practical interest. The result has been the production of elaborate systems which may quite fairly be described as moonshine. The discredit which the general public quite rightly attaches to these hasty attempts at Speculative Philosophy is

reflected back on Critical Philosophy, and Philosophy as a whole thus falls into undeserved disrepute.

(ii) At the best Speculative Philosophy can only consist of more or less happy guesses, made on a very slender basis. There is no hope of its reaching the certainty which some parts of Critical Philosophy might quite well attain. Now speculative philosophers as a class have been the most dogmatic of men. They have been more certain of everything than they had a right to be of anything.

(iii) A man's final view of the Universe as a whole, and of the position and prospects of himself and his fellows, is peculiarly liable to be biased by his hopes and fears, his likes and dislikes, and his judgments of value. One's Speculative Philosophy tends to be influenced to an altogether undue extent by the state of one's liver and the amount of one's bank-balance. No doubt livers and bank-balances have their place in the Universe, and no view of it which fails to give them their due weight is ultimately satisfactory. But their due weight is considerably less than their influence on Speculative Philosophy might lead one to suspect. But, if we bear this in mind and try our hardest to be "ethically neutral," we are rather liable to go to the other extreme and entertain a theory of the Universe which renders the existence of our judgments of value unintelligible.

A large part of Critical Philosophy is almost exempt from this source of error. Our analysis of truth and falsehood, or of the nature of judgment, is not very likely to be influenced by our hopes and fears. Yet even here there is a slight danger of intellectual dishonesty. We sometimes do our Critical Philosophy, with half an eye on our Speculative Philosophy, and accept or reject beliefs, or analyse concepts in a certain way, because we feel that this will fit in better than any alternative with the view of Reality as a whole that we happen to like.

(iv) Nevertheless, if Speculative Philosophy remembers its limitations, it is of value to scientists, in its methods, if not in its results. The reason is this. In all the sciences except Psychology we deal with objects and their changes, and leave out of account as far as possible the mind which observes them. In Psychology, on the other hand, we deal with minds and their processes, and leave out of account as far as possible the objects that we get to know by means of them. A man who confines himself to either of these subjects is likely therefore to get a very one-sided view of the world. The pure natural scientist is liable to forget that minds exist, and that if it were not for them he could neither know nor act on physical objects. The pure psychologist is inclined to forget that the main business of minds is to know and act upon objects; that they are most intimately connected with certain portions of matter; and that they have apparently arisen gradually in a world which at one time contained nothing but matter. Materialism is the characteristic speculative philosophy of the pure natural scientist, and subjective idealism that of the pure psychologist. To the scientist subjective idealism seems a fairy tale, and to the psychologist materialism seems sheer lunacy. Both are right in their criticisms, but neither sees the weakness of his own position. The truth is that both these doctrines commit the fallacy of over-simplification; and we can hardly avoid falling into some form of this unless at some time we make a resolute attempt to think *synoptically* of all the facts. Our *results* may be trivial; but the *process* will at least remind us of the extreme complexity of the world, and teach us to reject any cheap and easy philosophical theory, such as popular materialism or popular theology.*

Before ending this chapter I will say a word about the three sciences which are commonly thought to be

* Theology, whether "natural" or "revealed," is a form of Speculative Philosophy, in our sense of the word. So, too, is Atheism.

specially philosophical. These are Logic, Ethics, and Psychology. Logic simply is the most fundamental part of Critical Philosophy. It deals with such concepts as *truth*, *implication*, *probability*, *class*, etc. In fact it may be defined as the science which deals with propositional forms, their parts, their qualities, and their relations. Its business is to analyse and classify forms, and to consider the formal relations that can subsist between them. Now all science *consists* of definite propositions, and each of these is of one of the forms which Logic studies ; but it is not the business of any other science explicitly to *discuss* propositional forms. Similarly all science is full of inferences, good and bad, and all inference depends on relations that are supposed to subsist between premises and conclusion. But it is for Logic, and for it alone, to decide what relations do in fact justify inference, and whether these relations do actually subsist in a given case. Thus Logic is that part of Critical Philosophy which deals with the most general and pervasive of all concepts, and with those fundamental beliefs which form the "connective tissue" of all knowledge.

The greater part of Ethics again is simply a branch of Critical Philosophy. It is a fact that we not only believe that such and such events happen, but that we also pass judgments of approval or disapproval on certain of them. Such judgments use peculiar concepts, like *good* and *bad*, *right* and *wrong*, *duty*, etc. A very important part of Ethics is the attempt to analyse and define these peculiarly obscure notions which we all use so gaily in everyday life. Again, there are a great many judgments of value which many people assume as certain ; *e.g.* Pleasure is good, It is wrong to tell lies, A man has a right to do what he likes with his own, and so on. Another important part of Ethics is the attempt to state such judgments clearly, and then to see what evidence, if any, there is for them. Thus, Ethics is that part of Critical

Philosophy which analyses the concepts and criticises the presuppositions that we use in our judgments of approval and disapproval.

Psychology, as it seems to me, is not a part of Philosophy at all, but is simply one of the special sciences. This is shown by the fact that, unlike Logic and Ethics, it argues inductively from experiment and observation, though the observation takes the peculiar form of introspection. It is, however, a very peculiar kind of special science. It is obvious that Chemistry and Physics are much more like each other than either of them is like Psychology. The reason is that the two former sciences treat two rather different but very pervasive sets of *material* properties, whilst the latter deals with minds, which apparently occupy a unique and strangely isolated position in the Universe. Or, again, we may say that Psychology deals with what is relatively private, whilst all the other natural sciences deal with what is relatively public. If, now it should be asked why Psychology has been supposed to be specially connected with Philosophy, I think that the following answers will be fairly satisfactory.

(i) Psychology supplies Critical Philosophy with a number of concepts as raw material for analysis and criticism. Such are the concepts of *mind*, *self*, *consciousness*, *instinct*, *sensation*, *perception*, etc. Now these notions we all admit to be highly confused and obscure, whereas we are inclined to think—until we learn better—that there is no particular difficulty about such concepts as *place*, *date*, *matter*, *cause*, etc., which we use in the other sciences. Thus a great part of any standard book on Psychology does in fact consist of attempts to analyse and define certain concepts, and this is of course Critical Philosophy.

(ii) When we try to clear up the meanings of physical concepts like *place*, *date*, *matter*, etc., we often find that a reference to the processes by which they come to be known is essential, and that they owe part

of their obscurity to the abstractions which science and common-sense have made. Thus, in doing Critical Philosophy, we do constantly have to appeal to facts which belong to Psychology, even when we are not primarily dealing with psychological concepts.*

(iii) In Speculative Philosophy we ought, no doubt, to take into account the results of *all* the sciences. But, owing to the unique subject-matter of Psychology, we shall go hopelessly wrong if we omit it, whilst we shall not go so hopelessly wrong if we omit one of the sciences of matter, such as Mineralogy or Botany.

For these reasons we may admit that Psychology is of peculiar importance to Philosophy, though we must deny that it is a part of Philosophy, like Logic and Ethics.

The present book deals wholly with Critical Philosophy, and only with a small part of that. It is concerned almost entirely with an attempt to clear up some of the concepts used in the natural sciences. It does not deal even with all these, *e.g.* very little is said about causation. The reason is that I did not want to deal with purely logical questions; and it is hardly possible to discuss causation adequately without going into the question of induction, in which causation is commonly thought to play an important part.

Additional works that may be consulted with profit:

F. H. BRADLEY, *Appearance and Reality*, *Introduction.*
H. SIDGWICK, *Philosophy: its Scope and Relations.*
B. A. W. RUSSELL, *Our Knowledge of the External World*, *Lectures I. and II.*
J. GROTE, *Exploratio Philosophica*, Part I. Caps. I. and II.
DESCARTES, *Rules for the Direction of the Mind.*
" *Discourse on Method.*

* It is also true that we cannot give a complete treatment of Logic (especially the subjects of Inference and Probability) without referring to minds and their special limitations.

CHAPTER I

> "When *I* use a word," Humpty-Dumpty said in rather a scornful tone, "it means just what I choose it to mean —neither more nor less."
>
> "The question is," said Alice, "whether you *can* make words mean so many different things."
>
> "The question is," said Humpty-Dumpty, "which is to be Master—that's all."
>
> (LEWIS CARROLL, *Through the Looking-Glass.*)

The Traditional Conception of Space, and the Principle of Extensive Abstraction

IT is not ultimately possible to treat Space, Time, and Matter, as used in physical science, in isolation from each other; for we shall see that they are closely bound together in their very natures. This is, however, a comparatively recent discovery; and the traditional view, with which most of us still work in daily life, is that Space and Time, at any rate, can be adequately analysed in isolation from each other and from matter. As this is the familiar view, it seems best to start from it and gradually to point out and remove its imperfections. In any case we must start *somewhere;* and the fact that the three concepts in question have so long been treated as separable without serious practical error shows that, to a great extent, they *are* separable. The truth is that what is logically most primitive in nature is not what is now most familiar to us, and therefore it is better for didactic purposes to start with the logically derivative but practically familiar, and work back to the logically primitive but practically unfamiliar. For example, the immediate data of sense, like coloured patches, are logically prior to the notion of physical objects, which

persist, and combine many qualities; yet the latter is much the more familiar notion to us. I shall start then from the traditional conception of Space.

Unquestionably we think of Space in ordinary life and in science as a single great box or container in which all physical objects are kept and in which all physical processes go on. It is true that many books on Mechanics do lip-service to a different view of Space, which makes it consist of relations between bits of matter. But this conception is forgotten as soon as the author has worked off that particular chapter, and ever afterwards he and his readers use the "box" theory of Space. We shall deal with this alternative view at a much later stage. Again, we shall see later that the notion of a single box needs overhauling, but we shall not be able to appreciate why this is so until we have considered the connexion of Space with Time.

For the present then, we shall take the common practical view of Space as a single box "with no sides to it," in which the things and events of the physical world move and have their being. The first point to notice is that, when people talk of Space and spaces, they may be using these correlative terms in two different senses. (i) When we talk of Berkeley Square as one space and Grosvenor Square as a different one, we simply mean that they are two different regions which do not overlap, but which are both *parts* of the single Space of nature. We do not mean that they are different *kinds* of Space. Neither Berkeley Square nor Grosvenor Square is a *Space*—for neither is a box containing the whole of nature; but each of them is a *space*, in the sense of a part of such a box.

(ii) On the other hand, when mathematicians talk of Euclidean and non-Euclidean Spaces, they are discussing different possible *kinds* of Space, and not different spaces like the two London Squares which are *parts* of the Space of nature, of whatever kind that may be. The word *Space* is thus used (*a*) as a proper name, in which

case it is equivalent to the phrase "*the* Space of nature, of whatever kind that may be"; and (*b*) as a general name, in which case it connotes the property of being a Space, and denotes all the various wholes of that kind, such as Euclidean Space, Lobatchewskian Space, and so on. Finally, every kind of Space has parts, which are spaces, but not of course Spaces.

As a matter of history the concept of Space in general sprang from the investigation of the Space of nature. Euclid certainly meant his axioms to describe the Space in which we live and move. But, on further reflection, two very important facts emerged. (i) The validity of Euclid's deductions does not depend in any way on this assumption being true. (ii) We can conceive of extended wholes which are continuous and have several dimensions, like the Space of nature, but which yet differ from the Euclidean kind of Space in many of their properties. We decide then to call any whole that sufficiently resembles the Space of nature *a Space*, but we allow that there are many possible wholes which agree to this extent and yet differ in their remaining properties. Mathematicians at first only made timid modifications in Euclid's axioms, but as boldness grew with familiarity, they gradually considered what, from the Euclidean point of view, were wilder and wilder kinds of Space.

The important thing for us to notice is that the propositions of any system of pure geometry are merely hypothetical. They simply state that such and such propositions follow from the axioms, when the terms employed are defined by the definitions and postulates of the system. We ought not to say that the angles of a triangle *are* together equal to two right angles, but that, if a triangle be in the Space defined by Euclid's axioms, this will follow. This fact is hidden from the beginner in mathematics, because (*a*) the Space of nature is commonly assumed to be Euclidean, and (*b*) figures are commonly used in proving pro-

positions. But the truth is that figures in geometry are used only as illustrations, like statistics in the late Mr Chamberlain's tariff-reform speeches. They play no logical part in the proof, as is shown by the fact that a proposition about circles can be proved just as conclusively with a rough circle drawn in chalk on a blackboard as with an accurate circle drawn with a pair of compasses. The real premises of the proof are the axioms of the system, and the definitions of the terms which we are arguing about.

When these facts are once grasped it is easy to see the connexion between the Space of physics and the Spaces of pure geometry. We have arrived, by whatever means, at the concept of one physical Space —the single sideless box in which all the phenomena of nature are kept. This has various characteristic properties, such as continuity, three dimensions, etc. From this the pure mathematician generalises. He takes a selection of these properties as the defining marks of Space in general; and then, by varying the remaining properties, conceives various kinds of Space and works out their geometry. At that stage, and not till then, the question can be put: "Of what kind is the Space of nature?" "Which of the various possible Spaces accords best with the Space of physics?"

This is the question: "In what kind of a box is nature contained?" It turns out not to be quite so simple as asking whether one's clothes are in a portmanteau, a trunk, or a Gladstone bag. In the first place, the actual entanglement of physical Space with Time and with Matter becomes highly relevant at this point. For instance, our geometry and our physics are constructed to deal with different but intimately connected factors in nature, which are not met with in isolation. It is therefore conceivable that several different systems of geometry will equally fit the spatial side of nature provided that suitable modifications be made in the forms of physical laws. Apart from this,

there is the purely mathematical question as to whether the difference between Euclidean and certain kinds of non-Euclidean geometry be not merely a difference in the conventions for measuring a single kind of Space. The first kind of complication is roughly comparable to the possibility of a box which changes its shape according to the way in which we pack our clothes in it. If some bluff, downright person (such as an Oxford tutor) then asks whether your box is a trunk or a portmanteau, and insists on "a plain answer to a plain question," there is likely to be misunderstanding. It is not so easy to illustrate the second kind of complication mentioned above, but perhaps the following analogy will be of use. The difference of temperature between two places might be defined either by the difference in length of a certain column of mercury when held at the two places, or by the difference in pressure of a certain volume of gas when it is transferred from one place to the other. When temperature-difference is measured by the first convention, two pairs of points may have the same temperature-difference; when it is measured by the second convention the same two pairs may have different temperature-differences. There is no question of right or wrong in the matter; we just take two different measures of temperature-difference, one of which is more convenient for one purpose and the second for another purpose. Substitute "distance between two points" for "temperature-difference between two places," and you have a case where two different systems of geometry mean, not two Spaces, but two alternative ways of measuring a single Space.

So much for the distinction between the one Space of the natural scientist and the many Spaces of the mathematician. Let us now ask ourselves: What is the irreducible minimum of properties that the ordinary scientist ascribes to the Space of nature? (i) He holds that it is in some sense continuous, and that it has three dimensions. We need not go into the accurate

mathematical definitions of continuity and dimensions. Roughly we mean by the former that any two spaces that do not overlap are at once separated and joined by another space, and that all these spaces are parts of the one big Space of nature. By saying that Space has three dimensions we roughly mean that three independent bits of information are needed to fix the position of a point.

(ii) Again, the scientist and the ordinary layman draw a sharp distinction between Space and the things in Space. They hold that Space, as such, never causes anything. *Mere* position has no effect on any property of matter. If we move a bit of matter about, it may of course change in shape or size. The mercury column of a thermometer will do this if we move it from outside the window to a place near the fire. But the traditional view is that the *mere* change in position is not enough to account for this. The length has changed because the mercury has altered its position with respect to certain *matter* in Space. The complete inactivity of Space is, I think, for the plain man *the* mark that distinguishes it from matter in Space. Whenever it seems to break down we feel perplexed and uncomfortable. I can illustrate this in two ways. (*a*) On the older theories of physics there was supposed to be a peculiar kind of matter, called Ether, that filled all Space. On these theories the Ether was supposed to produce all kinds of effects on ordinary matter, and it became almost a family pet with certain physicists. As physics has advanced, less and less has been found for the Ether to do. In proportion as this has happened physicists have begun to ask: "Do we mean by the Ether anything more than empty Space?" On Lorentz's theory of electro-dynamics, it is difficult to see that the Ether is anything but the concept of absolute Space; and that eminent scientist's attitude towards it recalls Mrs Micawber's statement that she "will never desert Mr Micawber."

(*b*) Conversely, many mathematicians have conceived Spaces in which difference of position does make a difference to the shapes and sizes of bodies, and have successfully explained physical phenomena thereby. Prof. Clifford is one example, and Einstein, in his theory of gravitation, is another. But we do not as yet feel comfortable with the theories of this type, however well they may explain the facts, because they seem to involve the action of Space on matter, and this seems to upset all means of distinguishing between the two. The average intelligent physicist will accept from the mathematician any kind of Space that fits the observable facts, so long as it does not act on matter. But the wilder kind of Spaces that the pure mathematician can offer him he refuses to accept as Spaces at all, because it is part of what he means by Space that it shall be indifferent to, and thus distinguishable from, its content. It may be that we ought not to accept this objection as ultimate, because the sharp separation between the three concepts of Space, Time, and Matter has all the appearance of being artificial; but in the present chapter we are confining ourselves to the traditional view.

Space then, at present, is to be thought of as a single infinite, three-dimensional receptacle, in which all the events of nature have their being, but which is indifferent to them. If we reflect, we shall see that the evidence for the existence of such an object is by no means obvious. We can neither see nor touch empty spaces; what we see and touch are bits of matter. Now of course most things in which scientists believe cannot be perceived by the senses; no one can see or touch a hydrogen atom or a light-wave. Such objects are inferred by the scientist from the perceptible effects which they are supposed to produce. But Space is not even in this position. For, as we saw, the essence of Space on the traditional view, is that it does not produce any effects. Obviously then the existence of Space cannot be inferred

from its supposed perceptible effects, since it is not supposed to have any. If then Space is neither perceived nor inferred, whence do we get the concept of it?

In dealing with both Space and Time there are two distinct sets of concepts used, which we might call *distributive* and *collective*. The collective properties of Space and Time are those that belong to them as individual wholes. Thus the questions of how we come to believe that there is one Space, that it is Euclidean, that it can be distinguished from the matter in it, and so on, are questions concerning collective properties of space. On the other hand, there are certain concepts that apply, not so much to Space as an individual whole, as to every bit of space. These are distributive properties, such as divisibility, order of points on lines, and so on. In this and the next chapter we shall confine ourselves to distributive properties of Space and Time respectively; it is only at a much later stage that the question of one Space or Time, and its distinction from things or events in it can be faced.

Now all the distributive properties that we ascribe to Space have their root in certain facts that we can directly observe in our fields of view, and to a less extent, in our fields of touch. Whenever I open my eyes I am aware of a variously coloured field. This is extended, or spread out, and this extendedness is the root of my notion of surfaces and volumes. Again, within the total field certain specially coloured patches will stand out against a background; *e.g.* there might be two green patches, which are in fact the visual appearances of a pair of trees. Such patches have shapes and sizes; and here we have the sensible basis of the concepts of definite figures. Then, between any two such outstanding patches there will always be an extended background with a different colour, which at once joins and separates the patches. If, *e.g.* we are in fact looking at two trees, standing up against a cloudless sky, our field of view will consist of two

characteristically shaped green patches separated and surrounded by a blue extension. In the visual field there is nothing to correspond to the notion of empty space, for the whole field is occupied by some colour or other. Still, the visual experience that we have been describing does suffice to give us, in a rough form, the distributive concepts of extension, shape, size, betweenness, and continuity. And it suggests, though it does not by itself actually give us, another concept. A field of view does not come sharply to an end at its edges. It fades gradually away, and the details become less and less definite the further they are from the centre. Thus there is nothing in the experience to suggest that the field of view is an independent complete whole ; it rather presents itself as a fragment of something bigger. This suggestion is strengthened by the fact that when we move our heads slightly the new field of view is only slightly different from the old one. Some details that were distinct have become less so, others that were indistinct have become clearer ; a little that was present has vanished and a little that was not present has been added at the extreme edges ; but the bulk of the field has scarcely altered. This confirms the feeling that any field of view is only a fragment of a larger whole, and I believe that it is one of the roots of the limitless character which we ascribe to Space.

Much the same concepts are crudely presented to us in our tactual fields. When I grasp anything it feels extended, and some things feel bigger than others. Again, if the thing has projections, I can feel them as standing out from a background of "feeling" in the same kind of way in which the green patches stand out from the blue background in the visual field. But there are certain peculiar facts connected with touch, and more especially with touch in conjunction with movement, which are the germ of the distinction between empty and filled spaces. Had we been confined to sight it is difficult to see how we could have reached

this distinction, since the visual field, as we have already said, is everywhere full of colour. (i) If I put my hand on the top of an open tin box I get a peculiar sensation. I feel a cold, sharp outline, and, although it would not be true to say that there is no felt background within and without this, yet it is true to say that it is neutral and indefinite as compared with the blue background of the visual field in our example. (ii) Suppose I move my fingers along the edge of a ruler. I have a series of kinæsthetic sensations accompanied by a series of tactual sensations. Suppose I continue the movement until my finger gets to the end of the ruler, and still continue it afterwards. The tactual sensations cease, but the kinæsthetic sensations go on just as before. The ceasing of the tactual sensations is the basis of the concept of emptiness; the persistence of the kinæsthetic sensations is the basis of the concept that extension goes on in spite of the absence of extended matter.

Many of these remarks, which are here just thrown out, will need to be more fully developed when we come to deal with the collective attributes of Space. In the meanwhile we notice that all the information gained in this way is extremely crude, as compared with the concepts that we use in geometry and apply in physics. We see and feel finite surfaces and lumps of complicated shapes, not the unextended points and the lines without breadth of the geometers. And the spatial relations that we can immediately recognise between outstanding patches in our fields of view are equally crude. They are not relations between points and straight lines, but between rough surfaces and volumes. All that I am maintaining is that these crude objects of sense-awareness do have properties that are evidently spatial, and that we can see in them the germs of the refined notions of points, straight lines, etc. The question is: "How are the refined terms and their accurately definable relations, which we use in our mathematics and physics,

but cannot perceive with our senses, connected with the crude lumps or surfaces and their rough relations, which we actually do sense?"

The real problem is this. The relations of rough finite volumes, such as we can perceive, are of unmanageable complexity. Again, the continuity and boundlessness of Space, as suggested to us by our sense-experiences, are vaguely felt, not intellectually grasped. In this state it is impossible to lay down their laws or to reason about them. What we want to do is to analyse finite figures and their fearfully complicated perceptible relations into sets of terms with simpler and more manageable relations. If we can do this successfully we shall have killed two birds with one stone. We shall have done full justice to the spatial properties of what we can perceive; for our analysis is supposed to be exhaustive. And, on the other hand, we shall be able to grasp these properties and to reason about them in a way that was impossible while they remained in the crude unanalysed state in which we meet them in sense-awareness. I will give examples of what I mean, starting with very crude ones, and gradually working up to more refined cases.

(i) If I want to measure an irregular piece of ground I first try to divide it up into triangles. Why? Because the triangle is a simple figure, and the areas of all triangles are connected with their linear dimensions by a single simple law. Moreover, I can exhaustively analyse any rectilinear figure into triangles. Thus, instead of having to apply a different principle of mensuration to every different rectilinear figure, I can treat them all by this analysis in accordance with one simple law.

(ii) The notion of the distance between two finite bodies is clearly indefinite; so too is that of the direction of the line joining them. For there is no one distance and no one direction in such a case. Yet evidently there is a certain relation between two such bodies,

which I can perceive, and should like to be able to treat mathematically. Two trees are at different perceptible distances from a third, and one pair of them may define a different perceptible direction from another pair. Thus there are crude perceptible relations of distance and direction, which we should like to be able to express accurately and to treat scientifically. Now we notice that the smaller we take our patches or lumps the less is the inaccuracy in the notion of *the* distance between them or *the* direction determined by them. Still, so long as they have *any* area or volume, the theoretical difficulty remains. What we should like to be able to do would be to cut up our finite areas and volumes into sets of parts of *no* size, as we cut up our irregular rectilinear figure into a set of triangles that exactly make it up, and to regard the crude complex relations between the finite wholes as compounded out of the simple and definite relations between these unextended parts.

Now this second example shows us an important general principle and an important general difficulty, both of which extend beyond Space and apply equally to Time and Matter. We find that the relations between objects become simpler and more manageable as the objects become smaller. We therefore want to analyse finite objects and their relations into smaller and smaller parts, and their simpler and simpler relations. But we find that when we try to pursue this course to the bitter end we land in a difficulty. The relations do not become really definite and manageable till we have come to parts with *no* size or events with *no* duration. And here we are faced with a discontinuity. What we perceive is always objects with some magnitude and duration, and the relations that our perception tells us about are always between such objects. Have we any right to believe that finite objects consist of parts of *no* magnitude, or that such parts, if they exist at all, will have relations in the least like those which hold between finite areas and

volumes? A point is something different in kind from a volume or area, however small. We know what we mean when we say that a big area can be cut up into smaller ones; but it is not at all clear what we mean when we say that it can be cut up into points. The one thing that is certain is that the sense in which points are parts of volumes must be different from the sense in which little volumes are parts of bigger ones. The latter sense of part and whole is one that we find exemplified among perceived objects. The former is not, and we are bound to define it before we can feel comfortable in using points and instants.

We commonly slur over this difficulty by entertaining two incompatible notions of points, and using them alternately as convenience requires. This expedient is not unfamiliar to theologians, and to business men returning their incomes for purposes of taxation. When we want to talk of an area as analysable into points we think of points as little volumes. If we feel qualms about this we usually suppress them with the excuse which Midshipman Easy's nurse gave for her baby, that "after all, it was a very little one." When we want to think of points as having exactly definite distances we take them to have "position but no magnitude," as Euclid put it. Now nothing will make these two conceptions of points consistent with each other. Either points are extended or they are not. If they are not, how can they fit together along their sides and edges (which they will not possess) to make a finite volume or area? If they are, in what sense can you talk of *the* distance between them, or of *the* direction determined by a pair of them? To call them infinitesimal volumes or areas only darkens counsel; for the word *infinitesimal* here only serves to cover the attempt to combine these two incompatible qualities.

The method by which such difficulties as these have been overcome is due to Whitehead, who has lately worked it out in full detail in his *Principles of*

Natural Knowledge, and his *Concept of Nature*, two epoch-making works. To explain it in full would take us into regions of mathematical logic which I do not propose to penetrate in the present book. But the problem is so important, and the method is of such general application in bridging the gaps between the crude facts of sense and the refined concepts of mathematical physics that I shall give a sketch of it.

The first thing to notice is that it does not in the least matter to science what is the *inner nature* of a term, provided it will do the work that is required of it. If we can give a definition of points which will make them fulfil a certain pair of conditions, it will not matter though points in themselves should turn out to be entities of a very different kind from what we had supposed them to be. The two conditions are (i) that points must have to each other the kind of relations which geometry demands; and (ii) that points must have to finite areas and volumes such a relation that a reasonable sense can be given to the statement that such areas and volumes can be exhaustively analysed into sets of points. *Any* entity that answers these conditions will do the work of a point, and may fairly be called a *point*, no matter what its other properties may be. This important fact, that what really matters to science is not the inner nature of objects but their mutual relations, and that any set of terms with the right mutual relations will answer all scientific purposes as well as any other set with the same sort of relations, was first recognised in pure mathematics. Whitehead's great merit is to have applied it to physics.

I will first illustrate it from pure mathematics, and then consider its application to our present problem. Consider such irrational numbers as $\sqrt{2}$ and $\sqrt{3}$. Why do we call them *numbers?* Simply because they obey the formal laws of addition and multiplication which integers, like 2 and 3, obey; *i.e.* because they have to each other relations with the same formal

properties as the relations that hold between integers. Now numbers like $\sqrt{2}$ and $\sqrt{3}$ were at first defined as the limits of certain series of rational numbers. Thus $\sqrt{2}$ was defined as the limit of the series of rational fractions whose squares are less than 2. Similarly $\sqrt{3}$ was defined as the limit of the series of rational fractions whose squares are less than 3. Then you can define what you are going to mean by the *addition* and *multiplication* of such limits. These will be new senses of addition and multiplication. The sign + does not stand for the same relation when we talk of $\sqrt{2}+\sqrt{3}$ as when we talk of $2+3$. But addition and multiplication, in the new senses, have the same formal properties as they have when used in the old sense. Thus, *e.g.* $\sqrt{2}+\sqrt{3}=\sqrt{3}+\sqrt{2}$ just as $2+3=3+2$. We have extended the meaning of *addition* and *multiplication*; but, as they have precisely the same logical properties in both senses, no harm is done by using the same name for both, and talking of the addition and multiplication of irrationals. Consequently there is no harm in calling $\sqrt{2}$ and $\sqrt{3}$ *numbers;* for we agreed that any set of entities were to count as numbers, provided they had to each other relations with the same logical properties as the relations between familiar numbers, like 2 and 3, possess. Now all reasoning depends entirely on the logical or formal properties of the objects reasoned about, and therefore we can henceforth reason about irrationals as if they were ordinary numbers.

In exactly the same way, if we can define objects which have to each other relations with the same formal properties as the relations between geometrical points, these objects will do all the work of points, and can be called points, whatever their internal structure may be. Once this is grasped an initial difficulty can be removed. We are apt to think of points as internally simple, because they are said to have no parts and no magnitude. But none of the uses to which we

put points in geometry or physics depend on this supposed internal simplicity. The usefulness of points depends entirely on the fact that any pair of them define a unique relation with very simple logical properties, viz., the straight line joining them. Now we see that *any* terms whatever that are related to each other by a relation with these properties will do this part of the work of points. Hence we must not be surprised if we should find that points are not really simple, but have a complex internal logical structure. This is what we *shall* find. But we shall also find that, in spite of the logical complexity of points, a clear sense can be given to the statement that they have no parts and no magnitude.

We can now go a step further. I said that irrationals *used* to be defined as the limits of certain series of rationals. They are not so defined nowadays. Why is this? The answer is that, if we define them in this way, it is not certain that there is anything answering to the definition. $\sqrt{2}$ is said to be the limit of the series of rationals whose squares are less than 2. But how do you know that this series has a limit at all; *i.e.* roughly speaking, how do you know that there exists a number which the series continually approaches, but never reaches? The fact is that we do not know it and cannot prove it. It follows that, if we define irrationals in this way, it is not certain that there are any irrationals; $\sqrt{2}$ might be a symbol which stands for nothing at all, like the phrase "The present King of France," which has a meaning but no application. We want therefore to get a definition that shall amount to much the same thing as the definition by limits, but shall not leave us in any doubt as to the existence of something answering to it.

Now very much the same difficulty arises over points. I will put it in this way. We are naturally tempted to define points as the limits of certain series of areas or volumes, just as we defined irrationals as the limits of

certain series of rationals. And these attempted definitions *are* steps in the right direction. But they are not ultimately satisfactory, because they leave the existence of points, as of irrationals, doubtful. Let me illustrate this with regard to points. We saw that, as we take smaller and smaller areas or volumes, the spatial relations between them become simpler and more definite. Now we can imagine a series of areas or volumes, one inside the other, like a nest of Chinese boxes. Suppose, *e.g.* that it was a set of concentric spheres. As you pass to smaller and smaller spheres in the series you get to things that have more and more approximately the relations which points have in geometry. You might therefore be tempted to define a point, such as the common centre of the spheres, as the limit of this series of spheres one inside the other. But at once the old difficulty would arise: "Is there any reason to suppose that this series has a limit?" Admittedly it has no last term; you can go on finding spheres within spheres indefinitely. But the mere fact that it does not have a last term is no proof that it does have a limit. The limit of an endless series might be described as the first term that comes after all the terms of the endless series. But this implies that the series in question forms part of some bigger series; otherwise there is no beyond. Now it is not at all obvious that our endless series of concentric spheres does form part of any bigger series, or that there is any term that comes after every sphere in it. Hence there is no certainty that points, defined as the limits of such series, exist.

How is such a difficulty to be overcome? It was first overcome for irrational numbers, and Whitehead then showed that it might be dealt with in the same way for points. The solution will at first sight strike those who are unfamiliar with it as a mere *tour de force;* nevertheless it is perfectly valid, and really does the trick. Instead of defining $\sqrt{2}$ as the *limit* of the series of rational numbers whose squares are less than 2, it is

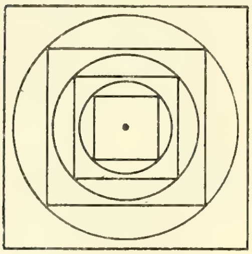
defined as *this series itself*. That is $\sqrt{2}$ is defined as the series of all rational numbers whose squares are less than 2. There is no doubt that there is such a thing as $\sqrt{2}$, so defined. For there certainly are rational numbers, like 1 and 1·2 and 2·5, and so on. And it is certain that the squares of some of them are less than 2, that the squares of others of them are greater than 2, and that the squares of none of them are equal to 2. It is therefore certain that there is a definite class of rationals whose squares are less than 2, and that it has an infinite number of members. It is equally certain that the numbers in this class form a series, when arranged in order of magnitude. Thus there is no doubt of the existence of the series which is said to be the meaning of $\sqrt{2}$.

But the difficulty that will be felt at first will be a different one. The reader will be inclined to say: "I don't doubt that $\sqrt{2}$, as defined by you, exists; what I very gravely doubt is whether, as defined by you, it is what I or anyone else mean by $\sqrt{2}$. By $\sqrt{2}$ I understand a certain number of a peculiar kind; I do not mean a series of numbers or of anything else." The answer to that difficulty is that series of this kind will serve every purpose for which irrationals, like $\sqrt{2}$ and $\sqrt{3}$, are used in mathematics. You can define addition and multiplication for such series, and they have exactly the same logical properties as the addition and multiplication of integers or of rational fractions. Lastly, taking this definition of $\sqrt{2}$, you can give a perfectly definite meaning to the statement that the length of the diagonal of a square, whose side is of unit length, is represented by $\sqrt{2}$. The position is therefore this. The definition of irrationals defines something that certainly exists. And this something has all the formal properties and will do all the work of irrationals. The sole objection to it is that it is paradoxical, in so far as it assigns a complex internal structure to irrationals which we did not suspect them of having. But that objection is really

unimportant, because of the general principle that in science it is only the logical properties of the relations between our terms that matter, and not their internal logical structure. The objection is just a prejudice to be got over, like our feeling that the inhabitants of Australia must be precariously hanging on to the earth by suction, like flies on a ceiling.

Now we deal with the difficulty about points in an exactly similar way. We should like to say that points are the limits of series of smaller and smaller volumes, one inside the other, like Chinese boxes. But we cannot feel any confidence that such series have limits and therefore that points, so defined, exist. Now there is no doubt that such series themselves exist; ordinary perception makes us acquainted with their earlier and bigger terms, and the assumption that Space is continuous guarantees the later ones. We see, on reflection, that it is of the very nature of any area or volume to have parts that are themselves areas or volumes. We, therefore, boldly define points, not as the limits of such series, but as such series themselves. This is exactly like the procedure adopted in defining irrationals.

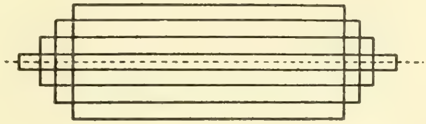
There are certain additional difficulties of detail in defining points, which do not arise in defining irrationals.



I will just indicate them and refer the reader to Whitehead for the complete solution of them. (i) There may be a great many different series of converging volumes which would all commonly be said to converge to the same point. This is illustrated for areas in the figure above, where the series of circles and the series of squares might equally be taken to define the point which is their common centre. Now, of course, the point cannot reasonably be identified with one of these series rather than with another. We, therefore, define the point, not

as any one of these series of converging volumes, but as the class of all the volumes in any of the series that would commonly be said to converge to the point. (ii) Not all series of converging volumes converge to points; some converge to lines, and others to areas. An example of a series of areas converging to a straight line is illustrated below. (It should be noticed that, although for simplicity of drawing I have always taken series of *areas* in my diagrams, the fundamental fact is series of *volumes*, and areas need definition, like points and lines.)

The general principle is, however, always the same. Points, straight lines and areas are all defined as series of converging volumes. But the series that define points



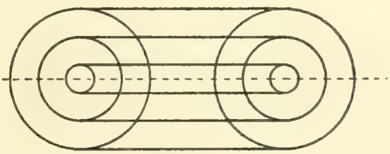
differ in certain assignable ways from those that define straight lines, and these in turn differ in certain assignable ways from those which define areas. Ordinary perception gives us examples of each kind of series, and the only difficulty is to state in formal logical terms these differences which we can all vaguely see and feel. To do this properly is, of course, a very hard job, but it can be and has been done. Many of these additional complications arise because Space has three dimensions, whilst the series of real numbers has only one. Consequently, as a matter of history, moments of Time were defined in this way before points of Space. Time forms a one-dimensional series, like the real numbers, and, therefore presents an easier problem than Space for this method.

Before going further I want to remove a legitimate ground of doubt which will probably be in the minds of most careful readers to whom the subject is new. Many will say: "This is no doubt highly ingenious,

but are we not merely moving in a circle? May not the theory be summed up by saying that points are those series of volumes that converge to points? If so, are we not plainly using the notion of point in order to define it?" This would of course be a fatal objection if it were well founded, but it is not. The theory may roughly be summed up in the statement that a point is a series of volumes that would *commonly be said to converge* to that point. The whole question is whether the common phrase "converging to the point p" really involves a reference to points. If it does the definition of points is circular and useless; if it does not there is no vicious circle in the theory. Now the essence of the theory is that it can state the meaning of such phrases as "converging to a point" in terms which involve nothing but volumes and their relations to each other. We see certain series of volumes which we say "converge to a point," *e.g.* series of concentric spheres. We see other series of volumes of which we do not say this. Here is a perceptible difference in perceptible objects. This difference, which can be seen and felt, must be expressible in terms of volumes and their relations to each other. It cannot really involve a relation to something that can neither be seen nor felt, such as a point. Thus a series of volumes is said to converge to a point simply and solely because of certain relations which hold between the volumes of the series. Another series of volumes is said not to converge to a point simply and solely because certain other relations exist between the volumes of this series. These relations, as well as their terms, are perceptible, and this is how we come to distinguish two such series. It only remains to state the differences of relation, which can thus be seen and felt, in definite terms that can be grasped by the intellect. This the present theory does. For example, a series of confocal conicoids could be defined as one whose members cut each other at right angles; a definition which makes no mention of their common focus, but

simply mentions a relation which the members of the series have to each other. There is thus no circularity in the definition of points by this method.

The method which we have been sketching, by which the accurate concepts of science are defined in terms of perceptible objects and their perceptible relations, is called by Whitehead the *Principle of Extensive Abstraction.* Our next question is: Do points, lines, etc., as defined by Extensive Abstraction, fulfil the conditions that we laid down for them at the beginning? The first was that they must have to each other the sort of relations that points, etc., are said to have to each other in geometry. For instance, two points must define a unique relation with certain logical properties, viz., the straight line that joins them. Intersecting straight lines must define planes, and so on. Points, straight lines, and planes, defined as above, do in fact have relations of this kind to each other. The detailed proof of this must here be taken on trust, but I shall take one example to indicate roughly the way in which these results come about. Take two different series of concentric spheres, one in one place and the other in another. Choose any sphere out of one set and any sphere out of another. There will be a certain crude perceptible relation between them. For instance, as shown in the diagram below, there will be a volume



which connects and contains both of them, which does not wholly contain any pair of larger spheres in the two series, but more than contains any pair of smaller spheres in the two series.

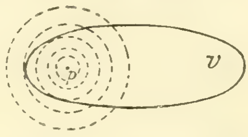
Let us call this the *containing volume* of the selected pair. As we take smaller and smaller pairs of spheres

from the two series it is easy to see that the corresponding containing volumes form a series of Chinese boxes of the usual kind. Now this series of containing volumes is obviously of the sort that defines a straight line. Our two series of spheres are of the sort that define points ; the points that they define are what we commonly call the centres of the two systems. And it is easy to see roughly that the line defined by the series of containing volumes is what we call the line joining the two centres. Of course, for accurate mathematical treatment, many more refinements are needed ; but I hope that the example will suffice to show in a rough way how points, as defined by us, determine straight lines, as defined by us.

The second condition which points had to fulfil was that it must be possible to give a clear meaning to the statement that finite volumes and areas can be completely analysed into sets of points. Now we can see at once that, *whatever* a point may be, it is certain that it cannot be part of a volume in the sense in which a little volume can be part of a bigger one. The latter is the fundamental relation ; it holds only between finite volumes, and it is perceptible. In this sense points, however defined, could not be parts of volumes. Divide a volume as long as you like and you will get nothing but smaller volumes. Put points together as much as you like (if this permission conveys anything to you) and you will not get any volume, however small. In fact the whole notion of "putting together" points is absurd, for it tries to apply to points a relation which can only hold between volumes or areas. To put together means to place so that the edges touch ; and a point, having no area or volume, has no edges. We see then that, whatever definition we give of points, we must not expect them to be parts of volumes in the plain straightforward sense in which the Great Court is part of the college buildings of Trinity. It is therefore no special objection to our definition of points that points, as

defined by us, could not be parts of volumes in the plain straightforward sense.

The sense in which a point p is contained in a volume v is roughly the following. We say that p is contained in v if, after a certain volume has been reached in the series that defines p, all subsequent volumes in this series are parts, in the plain straightforward sense, of the volume v. The diagram illustrates this definition.



The sense in which any volume can be exhaustively analysed into points is roughly the following: Any pair of volumes of which both are contained in v, but of which neither is wholly contained in the other, belong to series which define different points, both of which are contained in v in the sense just defined. Of course both these definitions need further refinements to cover all cases that can arise.

Now what precisely has been accomplished by all this? We have shown the exact connexion between what we can and do perceive, but cannot deal with mathematically, and what we can and do deal with mathematically, but cannot perceive. We perceive volumes and surfaces, and we perceive certain relations between them, viz., that they intersect, or that one is contained in the other, or that they are separated and both contained in some third volume or surface. We do not perceive the points without volume and the lines without breadth, in terms of which geometry and physics are stated and worked out. On the one hand, we cannot make geometry into a deductive science at all except in terms of points, etc. On the other hand, we want to be able to apply geometry to the actual world, and not to treat it as a mere mathematical fairy tale. It is essential therefore that the connexion between what we perceive, but cannot directly treat mathematically, and what we cannot perceive, but can treat mathematically,

should be made clear. This is what we have tried to do, following the method of Extensive Abstraction laid down and worked out by Whitehead.

It seems to me that the more we reflect the more clearly we see that something like the course that we have followed is necessary if the application of geometry (and also of rational mechanics) to the real world is to be justified. The world of pure mathematics with its points, straight lines, and planes, its particles, instants, and momentary configurations, has an appearance of unnatural smoothness and tidiness, as compared with the rough complexity of the perceptible world. Yet the laws of geometry and mechanics came out of the study of that world, and return to it in the form of applied mathematics. What I have tried to do is to show in rough outline how the two are connected, in the hope that the reader may be encouraged to consult the original authorities to learn how the same method establishes the connexion in the minutest details.

I think that possibly two difficulties may still remain in the reader's mind. (i) He may say: "Men used geometry for thousands of years, and applied it, and yet they knew nothing of these definitions of points, straight lines, and planes." I answer that this is perfectly true, and that it perfectly illustrates the difference between the special sciences and Critical Philosophy. Certainly people used the concepts of point and straight line, and used them correctly as the results show. But equally certainly they had the most confused ideas as to what they meant by points and straight lines, and could not have explained why a geometry stated in terms of these and their relations should apply so accurately to a world in which nothing of the kind was perceptible. It is the business of Critical Philosophy not to rest content with the successful use of such concepts, but to disentangle their meaning and thus determine the limits within which they can safely be employed.

(ii) The second question that may be asked is : "Do points, straight lines, etc., really exist in the same sense as volumes, or are they merely convenient and perhaps indispensable fictions?" This seems to me to be an important point, on which even authorities like Mr Russell often speak with a strangely uncertain voice. (Probably Mr Russell calls certain things, which he thinks can be defined in this kind of way, "fictions," from the same motives as led Mr Pope, according to Dr Johnson, to write the lines :—

> "Let modest FOSTER, if he will, excel
> Ten metropolitans in preaching well.")

The right answer to the question appears to me to be the following: Points, etc., as defined by us, are not fictions ; they are not made by our minds, but discovered by them, just as America was discovered, and not created, by Columbus's voyage. On the other hand, they do not exist in precisely the same sense in which finite volumes exist. They are real in their own kind, but it is a different kind from that of volumes. It is through no mere accidental limitation of our senses that we cannot perceive the points and straight lines of the geometers, whilst we can see and feel volumes. Only particulars can be perceived by the senses ; and points are not particulars. They are classes of series of volumes, or, to be more accurate, are the logical sums of such classes. The volumes and the series of volumes that define points exist quite literally, and the earlier and bigger terms of these series can be perceived. The points themselves are rather complicated logical functions of these. They exist in the sense that they are determinate functions of real series of actually existing particulars.

Perhaps an illustration from another region will make their mode of being clearer to some people. The curve called a *cycloid* is traced out by a point on the circumference of a circle when the latter rolls along a

straight line. Now the arches of Westminster Bridge are cycloidal, and can therefore be regarded as due to the rolling of a certain circle on a certain straight line. Now suppose we were asked whether this circle actually exists or is a mere fiction. In one sense I answer that it does not exist. So far as I know, no physical circle actually rolled at some date in the world's history on a physical straight-edge to produce the arches of Westminster Bridge. On the other hand, the circle is not a mere fiction. The cycloidal arches really do exist, and the circle corresponding to them is completely determined by the shape and size of these arches. This connexion is a real fact, absolutely independent of our minds and their operations. I therefore say that the circle exists, in the sense that it is a determinate function of the arches, which exist in the ordinary sense. Points, straight lines, etc., as defined by us, exist in the same sense as the circle determined by the arches of Westminster Bridge; the particular series of volumes which define points exist in the same sense as the arches themselves.

Additional works that may be consulted with profit:

A. N. WHITEHEAD, *Principles of Natural Knowledge*, Part III.
" *Concept of Nature*, Cap. IV.

CHAPTER II

> Alice sighed wearily. "I think you might do something better with the time," she said, "than waste it asking riddles with no answers."
>
> "If you knew Time as well as I do," said the Hatter, "you wouldn't talk about wasting *it*."
>
> (Lewis Carroll, *Alice in Wonderland*.)

The General Problem of Time and Change

We have now said as much about Space as can be said with profit before its relations to Time and Matter have been dealt with. We have shown at least how the concepts, such as points, lines, planes, etc., which are needed, whatever view we finally take of Space, are connected with the rough, untidy facts that we can perceive. We have not, however, explained why there is supposed to be one single Space in which all the events of nature are located, nor how things have places assigned to them in it. This can only be done at a later stage. In the meanwhile I propose to treat the concepts of Time and Change, as they appear at the same level of thought.

At first sight the problems of Time look very much like those of Space, except that the single dimension of Time, as compared with the three of Space, seems to promise greater simplicity. We shall point out these analogies at the beginning; but we shall find that they are somewhat superficial, and that Time and Change are extremely difficult subjects, in which spatial analogies help us but little.

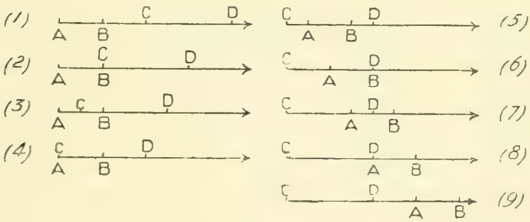
The physicist conceives Time in much the same way as he conceives Space. Just as he distinguishes Space

from the matter in it, so he distinguishes Time from events. Again, *mere* difference of position in Time is supposed to have no physical consequences. It is true that, if I go out without my overcoat at 2 A.M., I shall probably catch cold; whilst, if I do so at 2 P.M., I shall probably take no harm. But this difference is never ascribed to the mere difference in date, but to the fact that different conditions of temperature and dampness will be contemporary with my two expeditions. Again, Time, like Space, is supposed to be continuous, and physicists suppose (or did so until quite lately) that there is a single time-series in which all the events of nature take place. This series is of one dimension, so that, as far as appears at present, Time is like a very simple Space consisting of a single straight line.

Just as we treat our geometry in terms of unextended points and their relations, so we treat our chronometry in terms of moments without duration and *their* relations. Duration in Time corresponds to extension in Space. Now, just as we never perceive points or even unextended particles, so we are never aware of moments or of momentary events. What we are aware of is finite events of various durations. By an *event* I am going to mean anything that endures at all, no matter how long it lasts or whether it be qualitatively alike or qualitatively different at adjacent stages in its history. This is contrary to common usage, but common usage has nothing to recommend it in this matter. We usually call a flash of lightning or a motor accident an event, and refuse to apply this name to the history of the cliffs at Dover. Now the only relevant difference between the flash and the cliffs is that the former lasts for a short time and the latter for a long time. And the only relevant difference between the accident and the cliffs is that, if successive slices, each of one second long, be cut in the histories of both, the contents of a pair of adjacent slices may be very different in the first case and will be very similar in the second case. Such

merely quantitative differences as these give no good ground for calling one bit of history an event and refusing to call another bit of history by the same name.

Now the temporal relations which we perceive among events are similar to the relations of partial or complete overlapping which we can perceive in the case of two extended objects, like a pair of sticks. The possible time-relations between two events can be completely represented by taking a single straight line, letting "left-to-right" on this stand for "earlier and later," and taking two stretches on this line to represent a pair of finite events. Let AB and CD be two events of which the latter lasts the longer; then the possible temporal relations between the two are represented by the nine figures given below.



The most general kinds of relation between finite events are those of partial precedence and partial subsequence; the rest can be defined in terms of these. From these crude perceptible data and their crude perceptible relations the concepts of momentary events and moments can be obtained, and their exact relations determined, by the Method of Extensive Abstraction. I believe that, as a matter of history, one of the first successful applications of the method was made by Dr Norbert Wiener to this very problem.

The motives that lead us to apply Extensive Abstraction to Time are the same as those which lead us to apply it to Space. As scientists our main interest is to discover laws connecting events of one kind with events of other kinds at different times. Now, just

as the geometrical relations of finite volumes, as such, are of unmanageable complexity, so are the causal relations of events of finite duration. There is no simple relation between the contents of one hour and the contents of another. But the shorter we make our events the simpler become the relations between them. So, finally, we state our laws in terms of so-called "momentary events" and their exact relations, and we "analyse" finite events into sets of momentary ones, and explain their relations in terms of those of their momentary "parts." Everything that has been said of this procedure in geometry applies, *mutatis mutandis*, to its use in physics. Momentary "events" are not really events, any more than points are little volumes. A momentary event is not "part of" a finite one in the plain straightforward sense in which the event of a minute is part of the event that occupies a certain hour. The meanings of all these concepts, and their relations, have to be given in terms of perceptible entities and *their* relations, by means of Extensive Abstraction.

What we have been saying is most excellently illustrated by the science of Mechanics. What we want to deal with there is the movements of finite bodies, like wheels and planets; and we want to treat their changes of position and motion over long periods of time. To do this we have first to analyse the finite bodies into unextended particles, and then to analyse the finite events into momentary ones. The laws of Mechanics are only simple when they state relations between momentary configurations of one set of particles and a later or earlier configuration of the same or another set of particles. The gap between the perceptible facts, that we are trying to describe and predict, and the imperceptible concepts and relations, in terms of which we have to treat the facts, is bridged by Extensive Abstraction, applied both to extension in Space and to duration in Time. Mechanics is a

kind of geometry of events, which has to take account of both their spatial and their temporal characteristics. Geometry is the kind of mechanics which results when we confine ourselves to a single moment, and omit the temporal characteristics of events. These are, of course, only rough general statements; but they are perhaps illuminating, and they will be more fully explained later.

So far, the analogy between Time and Space has seemed to work very well. Duration has corresponded to length, before and after to right and left, and simultaneity to complete mutual overlapping. But, if we reflect a little more carefully, we shall see that the analogy between before and after and right and left is not so illuminating as it seems at first sight. The peculiarity of a series of events in Time is that it has not only an intrinsic *order* but also an intrinsic *sense*. Three points on a straight line have an intrinsic order, *i.e.* B is between A and C, or C is between B and A, or A is between C and B. This order is independent of any tacit reference to something traversing the line in a certain direction. By difference of sense I mean the sort of difference which there is between, say, ABC and CBA. Now the points on a straight line do not have an intrinsic sense. A sense is only assigned to them by correlation with the left and right hands of an imaginary observer, or by thinking of a moving body traversing the line in such a way that its presence at A is earlier than its presence at B, and the latter is earlier than its presence at C. In fact, if we want a spatial analogy to Time, it is not enough to use a straight line; we need a straight line with a fixed sense, *i.e.* the sort of thing which we usually represent by a line with an arrow-head on it. Now the points on straight lines do not have any intrinsic sense, and so the meaning of the arrow-head is only supplied by reference to something which is at one point *before* it gets to another. Thus to attempt

to understand before and after by analogy with a directed line is in the end circular, since the line only gets its sense through a tacit correlation with a series of events in Time.

Now the intrinsic sense of a series of events in Time is essentially bound up with the distinction between past, present, and future. A precedes B because A is past when B is present. We may begin by asking whether there is any spatial analogy to the distinction of past, present, and future. We shall find that there is, but that once more it is not ultimately useful, because it involves a reference to these very temporal characteristics on which it is supposed to throw light. The obvious analogy to *Now* in Time is *Here* in Space.

Here is primarily the name of a certain region in the continuum of possible positions that one's body can take up. When *Here* is used as a predicate, as when I say, "So and so is here," I mean that so and so is within a region whose boundaries I can reach with little or no walking. The peculiarity of *Here* is its peculiar kind of ambiguity. *Here*, as used by me, is understood to describe a different region from that which is described by the same word, as used by you. As used by me, it means "near me"; as used by you it means "near you." It is thus a word which has a partially different meaning as used by every different observer, simply because an essential part of its meaning is a relation to the particular observer who is using it.

We must notice, however, that *Here* has a second ambiguity. It not only has a different meaning as used by you and by me at the same time, it also has a different meaning as used by either of us at different times. By *Here* I always mean that region which is near me at the time of speaking. This difference of meaning at two moments *need* not betray itself by a difference of application, though it often does. If I stand still for five minutes the region which I call

Here at the end of the time will be the same as that which I called *Here* at the beginning; but, if I have moved, the difference in meaning will also be accompanied by a difference in application.

We can, of course, extract a general meaning of "hereness"; it means "nearness to an observer who uses the word *Here*, at the time when he uses it." But obviously *Here* is a descriptive phrase with a double ambiguity, since it refers both to a certain person and to a certain date in his history, and does not become definite till these two blanks have been filled in by the context.

It is evident then that *Here* is not going to help us to understand *Now*, since it contains an essential reference to *Now*. We must therefore treat past, present, and future on their own account, without expecting any help from spatial analogies. Now, the present does have a systematic ambiguity such as we noticed in *Here*. Whether it contains an essential reference to the particular observer who uses it I will not now discuss. The traditional view is that it is neutral as between various observers, but we shall later see reason to doubt this. However this may be, it is certainly ambiguous in another sense. Every place to which an observer's body can go is a possible *Here*. In the same way every event either is, has been or will be *Now*, on the ordinary view, provided it be short enough to fall into what psychologists call a *Specious Present*.

We are naturally tempted to regard the history of the world as existing eternally in a certain order of events. Along this, and in a fixed direction, we imagine the characteristic of presentness as moving, somewhat like the spot of light from a policeman's bull's-eye traversing the fronts of the houses in a street. What is illuminated is the present, what has been illuminated is the past, and what has not yet been illuminated is the future. The fact that the spot is of finite area expresses the fact that the Specious Present is not a

mere point but is of finite, though short, duration. Such analogies may be useful for some purposes, but it is clear that they explain nothing. On this view the series of events has an intrinsic order, but no intrinsic sense. It gains a sense, and we become able to talk of one event as earlier than another, and not merely of one event as between two others, because the attribute of presentness *moves* along the series in a fixed direction. But, in the first place, the lighting of the characteristic of presentness now on one event and now on another is itself an event, and ought therefore to be itself a part of the series of events, and not simply something that happens to the latter from outside. Again, if events have no intrinsic sense but only an intrinsic order, what meaning can we give to the assertion that the characteristic of presentness traverses the series of events *in a fixed direction?* All that we can mean is that this characteristic is *present* at B when it is *past* at A. Thus all the problems which the policeman's bull's-eye analogy was invented to solve are simply taken out of other events to be heaped on that particular series of events which is the movement of the bull's-eye.

The difficulties that we have found in this particular analogy are of very wide range. For instance, it is extremely tempting to try to resolve the difference between past, present, and future into differences in the cognitive relations of our minds to different events in a series which has intrinsic order but no intrinsic sense. Let us confine ourselves, for the sake of simplicity, to events that fall within the knowledge of a certain observer O. Undoubtedly O has a different kind of cognitive relation to those events which he calls *present* from that which he has to those which he calls *past* and to those which he calls *future*. About future events he can only guess or make inferences by analogy with the past. Some present events he can directly perceive with his senses. Some past events

he knows by direct memory, which is quite a different kind of experience from sense-perception. It is tempting to suppose that these are not simply interesting *facts about* past, present, and future, but are what we *mean* by these three temporal distinctions. Can such a theory be made to work?

Clearly we cannot simply define an event as present for O if O can perceive it or if it is contemporary with something that O can perceive. For we shall then have to define an event as past for O if O cannot perceive it but can either remember it or remember something contemporary with it. Now, of course, every event that falls within O's knowledge has these two incompatible relations to O; though, as we put it, it has them at different times. He can first perceive, but not remember the event, and can then remember but not perceive it. Hence these cognitive characteristics do not suffice to distinguish a past from a present event, since every event that O knows of has *both* these relations to him. If you add that an event always has the perceptual relation to O before it has the memory relation, you only mean that the event of remembering something is present when the event of perceiving it is past, and you have simply defined present and past for O's *objects* in terms of present and past for his cognitive *acts*. If you then try to define the latter in terms of different relations to O's acts of introspection, you simply start on an infinite regress, in which past and present remain obstinately undefined at any place where you choose to stop.

It does not of course follow that past and present in *external Nature* may not be reducible to certain relations between objective events and minds which observe them; but it does follow that these characteristics cannot be analysed away in this manner out of *Reality as a whole*, which of course includes observing minds as well as what they observe.

The difficulty about past, present, and future in

general can be summed up in two closely connected paradoxes. (i) Every event has all these characteristics, and yet they are inconsistent with each other. And (ii) *events* change in course of time with respect to these characteristics. Now we believe ourselves to understand change in *things*, but to talk of *events* changing seems almost unintelligible. The connexion between the two paradoxes is, of course, that we get into the second directly we take the obvious step to avoid the first.

We have plenty of experience of things which appear to have incompatible characteristics, such as redness and greenness, or greatness and smallness. As a rule we remove this apparent inconsistency by pointing out that the facts have been stated elliptically, and that really a relation is involved. In the first example we say that what has been omitted is a relation to two different times. The full statement is that the thing is red at one time and green at another, and there is no inconsistency in this. In the second example we have no need even to bring in a relation to two different times. It is enough to point out that the predicates great and small themselves tacitly assume relations; so that the full statement is that the thing is at once great as compared with one object and small as compared with another. In one of these two ways we always proceed when we have to deal with the apparent co-inherence of incompatible predicates in a single subject. We therefore naturally try one of these expedients to deal with the fact that every event is past, present, and future, and that these predicates are incompatible.

It seems natural and childishly simple to treat the problem in the way in which we treated the thing that was both red and green. We say: "Of course the event E has futurity for a certain stretch of time, then it has presentness for a short subsequent stretch, and it has pastness at all other moments." Now the

question at once arises: "Can we treat the change of an *event* in respect to its *temporal* qualities as just like the change of a *thing* with respect to qualities like red and green?"

To answer this question we must try to see what we mean when we say that a certain thing T changes from red to green. So far as I can see, our meaning is somewhat as follows: There is a certain long-lasting event in the history of the world. This stands out in a noticeable way from other events which overlap it wholly or partly. If successive short sections in time be taken of this long event, adjacent sections have spatial continuity with each other, and predominant qualitative resemblance to each other. On these grounds the whole long event is treated as the history of a single thing T. But, although adjacent short sections are *predominantly* alike in their qualities, there may be adjacent sections which differ very markedly in *some* quality, such as colour. If you can cut the history of the thing in a certain moment, such that a slice of its history before that is red and a slice after that is green, we say that the thing T has changed from red to green at that moment. To say that a thing changes, thus simply means that its history can be cut up into a series of adjacent short slices, and that two adjacent slices may have qualitative differences.

Can we treat the change of an event from futurity, through presentness, to pastness in the way in which we have treated the change of a thing (say a signal lamp) from red to green? I think it is certain that we cannot; for two closely connected reasons. In the first place, the attempt would be circular, because the change of things will be found on further analysis to involve the change of events in respect to their temporal characteristics. We have assumed that the history of our signal lamp can be analysed into a series of shorter adjacent events, and that it was true of a certain pair of these that the earlier was red and the later

green. But to say that this series of events passes from earlier to later (which is necessary if we are to distinguish between a change from red to green and a change from green to red) simply means that the red sections are past when the green ones are present and that the red ones are present when the green ones are future. Thus the notion of the history of the lamp as divisible into a series of sections, following each other in a certain direction, depends on the fact that each of these sections itself changes from future, through present, to past. It would therefore be circular to attempt to analyse changes in events in the way in which we have analysed changes in things, since the latter imply the former.

Apart from this objection, we can see directly that the change of events cannot be treated like the changes of things. Let us take a short section of the history of the lamp, small enough to fall into a Specious Present, and such that the light from the lamp is red throughout the whole of this section. This short event was future, became present, and then became past. If we try to analyse this change in the way in which we analysed the change of the lamp from red to green we shall have to proceed as follows: We shall have to divide this red event into shorter successive sections, and say that the latest of these have futurity, the middle ones presentness, and the earliest ones pastness. Now this analysis obviously does not fit the facts. For the fact is that *the whole* event was future, became present, and is now past. Clearly no analysis which splits up the event into successive sections with different characteristics is going to account for the change in the temporal attributes of the event as a whole.

We see then that the attempt to reconcile the incompatible temporal qualities of the same event by appealing to change, in the ordinary sense of the word, is both circular and ineffective. The circularity becomes specially glaring when put in the following way: The

changes of things are changes *in* Time; but the change of events or of moments from future, through present, to past, is a change *of* Time. We can hardly expect to reduce changes of Time to changes in Time, since Time would then need another Time to change in, and so on to infinity.

We seem, therefore, to be forced back to the other type of solution, viz., that the predicates, *past*, *present*, and *future*, are of their very nature relational, like *large* and *small*. Unfortunately we have already had occasion to look at some solutions of this type—the policeman's bull's-eye and the different cognitive relations—and the omens are not very favourable.

If we reflect, we shall notice that there are two quite different senses in which an entity can be said to change its relational properties. An example of the first is where Tom Smith, the son of John Smith, becomes taller than his father. An example of the second is where Tom Smith ceases to be the youngest son of John Smith, and becomes the last son but one. What is the difference between these two cases? In the first we have two partially overlapping life-histories, T and J. If we cut up both into successive short sections we find that the earlier sections of T have the relation of "shorter than" to the contemporary sections of J, whilst the later sections of T have the relation of "taller than" to the contemporary sections of J. In the second we have quite a different state of affairs. When we say that T is the youngest son of J we mean that there is no entity in the universe of which it is true to say both that it is a son of J and that it is younger than T. When we say that T has ceased to be the youngest son of J we mean that the universe does contain an entity of which it is true to say both that it is a son of J and that it is younger than T. In the first case then, we simply have a difference of relation between different corresponding sections of two existing long events. In the latter, the difference is that a certain

entity has changed its relational properties because a second entity, which did not formerly exist (and therefore could stand in *no* relation whatever to T), has begun to exist, and consequently to stand in certain relations to T, who is a member of the same universe as it.

Now it is obvious that the change that happens to an event when it ceases to be present and becomes past is like the change of Tom Smith when he ceases to be the youngest son of John Smith; and the continuous retreat of an event into the more and more remote past is like the successive departure of Tom from being the "baby" of the family, as John Smith (moved by the earnest exhortations of the Bishop of London) produces more and more children. A Specious Present of mine is just the last thin slice that has joined up to my life-history. When it ceases to be present and becomes past this does not mean that it has changed its relations to anything to which it was related when it was present. It will simply mean that other slices have been tacked on to my life-history, and, with their existence, relations have begun to hold, which could not hold before these slices existed to be terms to these relations. To put the matter in another way: When an event, which was present, becomes past, it does not change or lose any of the relations which it had before; it simply acquires in addition new relations which it *could* not have before, because the terms to which it now has these relations were then simply non-entities.

It will be observed that such a theory as this accepts the reality of the present and the past, but holds that the future is simply nothing at all. Nothing has happened to the present by becoming past except that fresh slices of existence have been added to the total history of the world. The past is thus as real as the present. On the other hand, the essence of a present event is, not that it precedes future events, but that there is quite literally *nothing* to which it has the relation of precedence. The sum total of existence is always

increasing, and it is this which gives the time-series a sense as well as an order. A moment t is later than a moment t' if the sum total of existence at t includes the sum total of existence at t' together with something more.

We are too liable to treat change from future to present as if it were analogous to change from present to past or from the less to the more remote past. This is, I believe, a profound mistake. I think that we must recognise that the word "change" is used in three distinct senses, of which the third is the most fundamental. These are (i) Change in the attributes of things, as where the signal lamp changes from red to green; (ii) Change in events with respect to pastness, as where a certain event ceases to be present and moves into the more and more remote past; and (iii) Change from future to present. I have already given an analysis of the first two kinds of change. It is clear that they both depend on the third kind. We analysed the change in colour of the signal lamp to mean that a red section of its history was followed by a green section of its history. This is sufficient analysis for a past change of quality, dealt with reflectively in retrospect. But, when we say that the red section precedes the green section, we mean that there was a moment when the sum total of existence included the red event and did not include the green one, and that there was another moment at which the sum total of existence included all that was included at the first moment and also the green event. Thus a complete analysis of the qualitative changes of things is found to involve the coming into existence of events.

Similarly we have seen that the second kind of change involves the third. For the change of an event from present to past turned out to depend on the fact the sum total of existence increases beyond the limits which it had when our given event came into existence.

Let us call the third kind of change *Becoming*. It

is now quite evident that becoming cannot be analysed into either of the two other kinds of change, since they both involve it. Moreover, we can see by direct inspection that becoming is of so peculiar a character that it is misleading to call it change. When we say that a thing changes in quality, or that an event changes in pastness, we are talking of entities that exist both before and after the moment at which the change takes place. But, when an event becomes, it *comes into existence*; and it was not anything at all until it had become. You cannot say that a future event is one that succeeds the present; for a present event is defined as one that is succeeded by nothing. We can put the matter, at choice, in one of two ways. We can either say that, since future events are non-entities, they cannot stand in any relations to anything, and therefore cannot stand in the relation of succession to present events. Or, conversely, we can say that, if future events succeeded present events, they would have the contradictory property of succeeding something that has no successor, and therefore they cannot be real.

It has long been recognised that there are two unique and irreducible, though intimately connected types of judgment. The first asserts that S is or exists; and is called an *existential* judgment. The second asserts that S is so and so, or has such and such a characteristic. This may be called a *characterising* judgment. The connexion between the two is that a thing cannot be so and so without *being*, and that it cannot be without being *so and so*.* Meinong, with the resources of the German tongue at his disposal, coins the convenient words *Sein* and *Sosein*. Now it seems to me that we have got to recognise a third equally fundamental and irreducible type of judgment, viz., one of the form: S becomes or comes into existence. Let us call these *genetic* judgments. I think that much of the trouble about Time and Change comes from our obstinate

* *Über die Stellung der Gegenstandstheorie*, and elsewhere.

attempts to reduce such judgments to the characterising form. Any judgment can be *verbally* reduced to this form. We can reduce "S is" to "S is existent." But the reduction is purely verbal, and those who take it seriously land in the sloughs of the Ontological Argument. Similarly "S is future" is verbally a judgment that ascribes a characteristic to an event S. But, if we are right, this must be a mistake; since to have a characteristic implies to exist (at any rate in the case of particulars, like events), and the future does not exist so long as it is future.

Before passing on there is one more verbal ambiguity to be noted. The same word *is* is used absolutely in the existential judgment "S is," and as a connective tie in the characterising judgment "S is P." Much the same is true of the word *becomes*. We say "S becomes," and we say "S becomes P." The latter type of judgment expresses qualitative change, the former expresses coming into existence.

The relation between existence and becoming (and consequently between characterisation and becoming) is very intimate. Whatever is has become, and the sum total of the existent is continually augmented by becoming. There is no such thing as *ceasing* to exist; what has become exists henceforth for ever. When we say that something has ceased to *exist* we only mean that it has ceased to be *present;* and this only means that the sum total of existence has increased since any part of the history of the thing became, and that the later additions contain no events sufficiently alike to and sufficiently continuous with the history of the thing in question to count as a continuation of it. For complete accuracy a slight modification ought to be made in the statement that "whatever is has become." Long events do not become bodily, only events short enough to fall in Specious Presents become, as wholes. Thus the becoming of a long event is just the successive becoming of its shorter sections. We shall have to go

more fully into the question of Specious Presents at a later stage.

We are left with two problems which we may hope that the previous discussions will help us to solve. (i) If the future, so long as it is future, be literally nothing at all, what are we to say of judgments which profess to be about the future? And (ii) What, in the end, is our answer to the original difficulty that every event is past, present, and future, and that these characteristics are mutually incompatible?

(i) Undoubtedly we do constantly make judgments which profess to be about the future. Weather forecasts, nautical almanacs, and railway time-tables, are full of such judgments. Admittedly no judgment about the future is absolutely certain (with the possible exception of the judgment that there will always be events of some kind or other); but this is irrelevant for our present purpose. No historical judgment about the past is absolutely certain either; and, in any case, our question is not whether we can have *certain* knowledge about the future, but is the prior question: What are we really *talking about* when we profess to make judgments about the future, and what do we *mean* by the truth or falsity of such judgments?

We cannot attempt to answer these questions till we have cleared up certain points about the nature of judgments in general. First, we must notice that the question: "What is a certain judgment about?" is ambiguous. It may mean: "What is the subject or subjects of the judgment?" or: "To what fact does the judgment refer?" The fact to which a judgment refers is the fact that renders it true or false. It is true, if it has the peculiar relation of concordance to the fact to which it refers; and false, if it has the relation of discordance to this fact. Discordance, I think, is a positive relation which is incompatible with concordance; it is not the mere absence of concordance. I see no reason to suppose that the reference of a

judgment to a fact is a third independent relation over and above the relations of concordance and discordance. I take it to be just the disjunction "concordance-or-discordance"; and I suppose that to say that J refers to F simply means that F is the fact which either makes J true by concording with it or false by discording with it.

Now people make many judgments, which have nothing to do with the future, but are nevertheless apparently about objects which do not, in fact, exist. Many English peasants, in the Middle Ages, must have made the judgments "Puck exists" or "Puck has turned the milk." And the latter of these, of course, implies the former. I will assume (in spite of Sir Conan Doyle) that Puck does not in fact exist. What were these men referring to, in our sense of the word? To answer this we have simply to ask: What fact made their judgments false? The answer is that it is the negative fact that no part of the universe was characterised by the set of characteristics by which they described Puck to themselves. Their judgment boils down to the assertion that some part of the existent is characterised by this set of characteristics, and it is false because it discords with the negative fact that the set in question characterises no part of the universe. Naturally they did not know that this was what their judgment referred to, or they would not have made it. But, in our sense of reference, there is no reason why a person who makes a judgment should know what it refers to.

Now it would obviously be absurd to say that what these men were *talking about* was the negative fact that no part of the universe has the characteristics which they ascribe to Puck. Hence we see the need of distinguishing between what a judgment refers to and what the person who makes the judgment is talking about. What they were talking about was a certain set of characteristics, viz., those by which they described

Puck to themselves. This may be called the logical subject of their judgment. It is something real and independent of the judging mind; having the kind of reality and independence which is characteristic of universals, and not, of course, that which is characteristic of particular existents. Thus, although there is no such being as Puck, people who profess to be judging about him are not judging about nothing (for they are judging about a set of characteristics which is itself real, though it does not happen to characterise any particular existent). Nor are they referring to nothing (for they are referring—though they do not know it—to an important negative fact about the existent).

Since the non-existence of Puck is compatible with the fact that the judgment "Puck exists" is an intelligible statement about something real, we may hope that the non-existence of the future may prove to be compatible with the existence and intelligibility of judgments which profess to be about the future. Up to a point the two kinds of judgment can be treated in much the same way. The judgment which is *grammatically* about "Puck" proves to be *logically* about the set of characteristics by which the assertor describes Puck to himself. Similarly the judgment, "To-morrow will be wet," which is grammatically about "to-morrow," is logically about the characteristic of wetness. The non-existence of to-morrow is therefore consistent with the fact that the judgment is about something.

Still there is one very important difference between the two kinds of judgment. Judgments like "Puck exists" are not only *about* something; they also *refer to* some fact which makes them true or false. This fact may be negative, but it is a real fact about the existent world. If we ask what fact judgments ostensibly about the future refer to, we must answer that there is no such fact. If I judge to-day that to-morrow will be wet, the only fact which this judgment can refer to, in our sense of the word, is the fact which renders

it true or false. Now it is obvious that this fact is the wetness or fineness of to-morrow when to-morrow comes. To-day, when I make the judgment, there is no such fact as the wetness of to-morrow and there is no such fact as the fineness of to-morrow. For these facts can neither of them begin to be till to-morrow begins to be, which does not happen till to-morrow becomes to-day. Thus judgments which profess to be about the future do not refer to any fact, whether positive or negative, at the time when they are made. They are therefore at that time neither true nor false. They will become true or false when there is a fact for them to refer to; and after this they will remain true or false, as the case may be, for ever and ever. If you choose to define the word *judgment* in such a way that nothing is to be called a judgment unless it be either true or false, you must not, of course, count "judgments" that profess to be about the future as judgments. If you accept the latter, you must say that the Law of Excluded Middle does not apply to all judgments. If you reject them, you may say that the Law of Excluded Middle applies to all genuine judgments; but you must add that "judgments" which profess to be about the future are not genuine judgments when they are made, but merely enjoy a courtesy title by anticipation, like the eldest sons of the higher nobility during the lifetime of their fathers. For convenience, I shall continue to speak of them as judgments.

So far then, we have determined two facts about judgments which profess to be concerned with the future. (*a*) They are about something, viz., some characteristic or set of characteristics; and (*b*) they do not refer to any fact at the time when they are made. This is clearly not a complete analysis. Two further points need to be cleared up. (*a*) If such judgments when made do not refer to anything, how is it that, if certain events become, the judgment is verified, and, if other events become, it is refuted? (*b*) If such judg-

ments are about characteristics, what precisely is it that they assert about these characteristics?

(*a*) Suppose I judge to-day that to-morrow will be wet. Nothing that may happen to-morrow will be relevant to this judgment except the state of the weather, and nothing will then make it true except the wetness of the weather. This is true enough, but it does not prove that the judgment refers to any fact, in our sense of reference. With *any* judgment we can tell what *kind* of fact will verify or refute it, as soon as we know what the judgment is about and what kind of assertion it makes. But no amount of inspection of a judgment itself will show us *the particular fact* which makes it true if it is true and false if it is false. There is therefore no inconsistency between the statement that we can know at once what *kind of fact* would verify a judgment about the future, and the statement that such judgments do not refer to any *fact* when made.

(*b*) As regards any judgment we have to consider not only what it is about, but also what it asserts about its subject or subjects. These two questions are not altogether free from ambiguity, and this ambiguity must be cleared up before we consider the special question as to what judgments that profess to be about the future assert. (1) There is the confusion between what a judgment is about and what it refers to. This we have already dealt with. (2) There is the distinction between what a judgment is ostensibly about and what it is really about. If you had asked a peasant, who said that Puck had turned the milk, what he was talking about, he would have said that he was talking about a certain individual fairy. This is what the judgment professes to be about. What it is really about is a certain set of characteristics. Roughly speaking, we may say that what a judgment professes to be about can be determined by a grammatical analysis of the sentence in which the judgment is expressed. Although there is always a connexion between

the grammatical structure of a sentence and the logical structure of a judgment, it is highly dangerous to suppose that what the sentence is grammatically about is the name of what the judgment is logically about. (3) When these two confusions have been set aside and we are quite definitely dealing with the *judgment*, and neither with the *fact* to which it refers nor the *sentence* which expresses it, there is still a difficulty as to how much is to be included under the head of what the judgment is about and how much is to be included under the head of what the judgment asserts. Take first a very simple characterising judgment, like "3 is a prime." What is this about, and what does it assert? We should all agree that it is at any rate about the number 3. But is it about the characteristic of primeness too? If you say Yes, what is there left for it to assert? If you say No, how can you face the obviously equivalent judgment "Primeness is a characteristic of 3"? Exactly the same kind of difficulty arises over a relational proposition, like "3 is greater than 2." We should all at this time of day agree that it is at least about the numbers 2 and 3. But is it or is it not about the relation of greater? I think that we must say that the former judgment is about primeness as much as it is about the number 3, and that the latter is about the relation of greater as much as it is about the numbers 2 and 2. Really it is as misleading to say that the first asserts primeness as to say that it asserts 3. The minimum that it asserts is the primeness of 3. Similar remarks apply to the second. If we like to use the useful word *tie*, which Mr W. E. Johnson* has lately introduced into logic, we might say that the first judgment is about the number 3 and the characteristic of primeness, and asserts that they are connected by the characterising tie. The second is about the numbers 3 and 2 and the relation greater, and asserts that they are connected by the relational

* *Logic*, vol. i.

tie in the order 3 to 2. But we might equally well distinguish different kinds of assertion, and say that the first is about the number 3 and the characteristic of primeness, and makes a characterising assertion about them. In the case of the second we should talk of a relating assertion.

So far we have purposely chosen examples which are about timeless objects, like numbers. Let us now take the series of judgments: "It has rained," "It is raining," and "It will rain," which are about events, and contain an essential reference to time. The first may be analysed as follows: "There is an event which is characterised by raininess, and the sum total of existence when the judgment is made includes all and more than all which it includes when this event becomes." The second may be analysed as follows: "There is an event which is characterised by raininess, and the sum total of existence is the same when this event becomes and when the judgment is made." Thus judgments about the past and the present can be analysed into judgments which involve the four familiar types of assertion—the existential, the characterising, the genetic, and the relational. But the judgment that it will rain cannot be analysed in a similar way. It cannot mean anything that begins with the statement: "There *is* an event," for the only events that there are are the events that have become up to the time when the assertion is made; the sum total of existence does not contain future events. We can only restate the judgment in the form: "The sum total of existence will increase beyond what it is when the judgment is made, and some part of what will become will be characterised by raininess." We cannot then analyse *will* away, as we can *has been* and *is now*. Every judgment that professes to be about the future would seem then to involve two peculiar and not further analysable kinds of assertion. One of these is about becoming; it asserts that further events will become. The other is about some characteristic; it asserts that

this will characterise some of the events which will become. If then we ask: What are judgments which profess to be about future events really about? the answer would seem to be that they are about some characteristic and about becoming. And if it be asked: What do such judgments assert? the only answer that I can give is that they assert that the sum total of existence will increase through becoming, and that the characteristic in question will characterise some part of what will become. These answers are compatible with the non-existence of the future. The only "constituents" of the judgment, when it is made, are the characteristic — which has the kind of reality which universals possess — and the concept of becoming. About these the judgment makes certain assertions of a quite peculiar and not further analysable kind. Something called *to-morrow* is not a constituent of judgments which are grammatically about "to-morrow," any more than an individual called *Puck* is a constituent of judgments which profess to be about "Puck."

I have thus tried to show that there is an extreme difference between judgments which profess to be about future events and these which are about past or present events. The former, when made, do not refer to anything, and therefore are not literally true or false, though it is possible for anyone who understands their meaning to see what kind of fact *will* eventually make them true or false as the case may be. Again, *is now* and *has been* need not be taken as new and ultimate types of assertion, but *will be* apparently must be so taken. Nevertheless, although the future is nothing and although judgments which profess to be about future events refer to nothing, they are not about nothing. They are about some characteristic and about becoming; and, so far as I can see, they make an unique and not further analysable kind of assertion about these terms.

There are just two points that I want to make before

leaving this subject. (*a*) Of course there are plenty of *ex post facto* statements which nominally involve the existence of future events. We can say that the Battle of Hastings *was* future to Edward the Confessor. Such statements need no special analysis. We merely mean that the sum total of existence now includes the Battle of Hastings, and that when Edward the Confessor's death became it did not include this battle. We, who live after both events, are dealing with two parts of the existent, which can and do stand in various relations to each other; and so there is no kind of difficulty in giving a meaning to the statement.

(*b*) It is commonly held that there can be no certain knowledge about the future, but that all judgments which profess to be about it consist of more or less probable conjectures made by analogy with the past. Now we do not always recognise how odd our certainty about this is on the assumption that the future really is something that has "future existence" as the past really is something that has "past existence." We have immediate, and not merely inferential, knowledge about some past events by direct memory. Hence mere difference in date between the act of cognition and an event does not necessarily prevent the event from being an object to the act. If the future exist, and be just that part of the existent which succeeds the present, it is difficult to see why a present act of cognition might not know an event which is later than itself, just as it can know some events which are earlier than itself. Why should we not have direct anticipations of some future events, just as we have direct memories of some past ones, if the future were of the same general nature as the past, and simply differed from it by standing in the converse temporal relation to the present? Still more, why should all claims to direct knowledge of future events be regarded as so wildly paradoxical?

These facts become plausible on two theories about the future, one of which we have rejected, and the other

of which is our own. Obviously if to be future just *means* to be incapable of being directly cognised, direct cognition of future events could be ruled out as a contradiction in terms. We have, however, examined and rejected this view of the future. But the impossibility of absolutely certain knowledge about the future follows equally from our theory. We can be absolutely certain that an event has the characteristic C only if we are directly acquainted with this event and can notice the characteristic in it. Now we can be directly acquainted only with *something*, not with a mere non-entity. On our view we cannot stand in the relation of direct acquaintance to future events, for the same reason which prevents us from robbing a Highlander of his breeks. We can stand in this relation to present events (in sense-awareness) and to past events (in genuine memory), because such events are parts of the sum total of existence when the cognition in question takes place.

(ii) The last question that we have to deal with is the alleged difficulty that every event is past, present, and future; that these characteristics are incompatible; and that there is no way of reconciling them which does not either involve an infinite regress, in which the same difficulty recurs at every stage, or a vicious circle. This argument has been used by Dr M'Taggart* as a ground for denying the reality of Time. It is certainly the best of the arguments which have been used for this purpose; since it really does turn on features which are peculiar to Time, and not, like most of the others, on difficulties about continuity and infinity which vanish with a knowledge of the relevant mathematical work on the subject. Do the results of our earlier discussions in this chapter help us to remove this supposed contradiction?

Let us take M'Taggart's example of the death of Queen Anne, as an event which is supposed to combine the incompatible characteristics of pastness, presentness,

* *The Unreality of Time*, MIND, N.S., 1908.

and futurity. In the first place, we may say at once that, on our view, futurity is not and never has been literally a characteristic of the event which is characterised as the death of Queen Anne. Before Anne died there was no such event as Anne's death, and "nothing" can have no characteristics. After Anne died the sum total of existent reality does contain Anne's death, but this event then has the characteristic of pastness. No doubt I can say "Anne's death *was* future to William III." But I simply mean that, so long as William III was alive, there was no event characterised as the death of Anne; and that afterwards, as the sum total of existence increased by becoming, it contained both the events of William's life and the event of Anne's death. Anne's death succeeded William's life so soon as Anne's death existed at all, and it succeeds it henceforth for ever; but it did not succeed it while William was alive, because it had not become, was not anything, and therefore could not have any characteristics or stand in any relations. But it might be said that Anne herself or William III might have made the judgment: "Queen Anne's death *is* future"; that this is a true judgment on their parts; and that it cannot be explained in the same way as my *ex post facto* judgment that Queen Anne's death *was* future. To this I answer that the existence and the truth of William's judgment do not imply that there ever was an event which has the two characteristics of futurity and of being the death of Anne. When William made this judgment there was no event for it to refer to; for the event which afterwards became, and was the death of Anne, had not then become and was not anything. What William did was to make a peculiar kind of assertion about becoming and about the characteristic of being the death of Queen Anne. He asserted that the sum total of existence would increase by further becoming, and that some part of what would thus be added would be characterised as the death of his sister-in-law. He

was neither talking about nor referring to that particular event which did in fact eventually become, and which, when it became, was in fact characterised as the death of Anne. For, when he made his judgment, there was no such event in the whole of reality for him to talk about or to refer to. Thus the first thing that we have to say with regard to M'Taggart's argument is that no event ever does have the characteristic of futurity. When we say that a certain event is future, the *sentence* which expresses our judgment is no doubt of the same form as when we say that a certain book is green. We are therefore tempted to treat the former judgment as a characterising judgment, like the latter; and to suppose that the only difference between them is that one asserts the characteristic of "futurity" whilst the other asserts the characteristic of greenness. From what has gone before we conclude that the former judgment is not really a characterising judgment at all, and that there is no characteristic of "futurity." Judgments which appear to characterise events as future make a peculiar kind of *assertion* about some *ordinary characteristic* (*e.g.* wetness or fineness); they do not make an ordinary characterising assertion about a certain *event* and a peculiar kind of *characteristic* (viz., "futurity").

Is there anything contradictory in the fact that Queen Anne's death has been present and is now past? There very well might be if we had to take the change of an event in respect to the characteristics of presentness and pastness as analogous to the change of a signal lamp in respect to the characteristics of red and green. But we have seen that this cannot be done, and that the second kind of change depends on the first. When Queen Anne's death became, it came into relations with all that had already become, and to nothing else, because there was nothing else for it to be related to. All these relations it retains henceforth for ever. As more events become it acquires

further relations, which it did not have, and could not have had while those events were non-existent. This is all that ever happens to the event in question. Suppose we now ask ourselves the question: "Does anything that was true of Anne's death when it first became get false of it afterwards, through further becoming? And, if so, does this raise any logical difficulty?" Here we must draw a distinction. (1) All the relations which Anne's death entered into with the sum total of reality, as it was when this event first became, persist eternally for ever afterwards, and are wholly unaffected by anything else that may be added on to this sum total by further becoming. Hence no true proposition about these will ever become false, and no false proposition about them will ever become true. (2) As further events become they automatically enter into various relations with Anne's death, which thus acquires additional relations and becomes a constituent in additional facts. If *e.g.* my Lord Bolingbroke swore when he heard of Anne's death, it is clear that something subsequently became true of the death which was not true of it when it first became. When Lord Bolingbroke had sworn it became true of Queen Anne's death that it caused a certain event in his lordship's life. And this was not true of Queen Anne's death before Lord Bolingbroke had heard of it, and had thereby been caused to swear. Thus something, which was not true of Queen Anne's death when it became, is afterwards rendered true of it by the becoming of Lord Bolingbroke's oath.

Now we are inclined to think that to say that something, which *was not true* of an event, subsequently became true of it, is equivalent to saying that something which *was false* of the event, became true of it. This is, I think, a mistake; for "not-true" is a wider term than "false." Suppose we compare the two statements: "It is not true that Queen Anne's death caused the earthquake at Lisbon," and: "It is not true that Queen

Anne's death, when it happened, had caused Lord Bolingbroke to swear." In the former "not-true" is equivalent to "false." For it means that there is a certain negative fact (containing both the death and the earthquake as constituents) which discords with the judgment that the first caused the second. But the latter does not mean that at the time of Anne's death there was a negative fact, containing Anne's death and Bolingbroke's oath as constituents, and discording with the judgment that the death causes the oath. For, when Anne's death became, there was no such entity as Lord Bolingbroke's oath, and therefore no fact of which this is a constituent. What happens when Lord Bolingbroke swears is not that something which was false of Anne's death becomes true of it, but that something becomes true of Anne's death which was before neither true nor false of it.

Now I do not think that the laws of logic have anything to say against this kind of change; and, if they have, so much the worse for the laws of logic, for it is certainly a fact. What the laws of identity, contradiction, and excluded middle, between them assert is that any proposition is either true or false, cannot be both, and cannot alter in this respect. They do not assert (and, if they do, they must be amended) that the *number* of propositions, is eternally fixed; they only assert that it cannot be diminished. But it may be increased, and it *is* continually increased by the process of becoming which continually augments the sum total of existence and thereby the sum total of positive and negative facts. Or, to put it in another way, the laws of logic apply to a fixed universe of discourse, and we can at any moment get a fixed universe of discourse by taking the sum total of reality up to that moment. But the universe of actual fact is continually increasing through the becoming of fresh events; and changes in truth, which are mere increases in the *number of truths* through this cause, are logically unobjectionable.

I can hardly hope that what I have been saying about Time and Change will satisfy most of my readers, or indeed, that it is more than a shadow of the truth, if that. It is admitted that this is the hardest knot in the whole of philosophy. The Dean of Carlisle judiciously remarks that "we cannot understand Time, but we shall not understand it better by talking nonsense about it." In the hope that I have not darkened counsel by words without understanding, I leave this most difficult subject, to return at a later stage to the questions of one or many time series, the entanglement of Time with Space, and the placing and dating of events.

Additional works which may be consulted with profit :

B. A. W. RUSSELL, *Our Knowledge of the External World*, Lecture IV.

A. N. WHITEHEAD, *Concept of Nature*, Cap. III.

J. M. E. M'TAGGART, *The Relation of Time and Eternity* (MIND, N.S., vol. xviii. No. 71).

„ *The Unreality of Time* (MIND, N.S., xvii., 1908).

H. BERGSON, *Time and Free-Will.*

„ *Matter and Memory.*

CHAPTER III

> "Its eyebrows (of a vivid green)
> Have never, never yet been seen;
> But Scientists, who ought to know,
> Assure us that it *must* be so.
> Oh, let us never, never doubt
> What no one can be sure about!"
> (H. Belloc, *The Microbe*.)

The Traditional Kinematics, and its Gradual Modification in the Region of Physics. (1) The Absolute and the Relational Theories

We have now dealt with the traditional concepts of Space and Time, and we might turn next either to Matter or to Motion. I propose to treat the classical doctrine of Motion before touching the problem of Matter. As we all know, the concept of Motion has been the subject of constant discussion by physicists and mathematicians for centuries, and in recent years the classical kinematics has been profoundly modified, owing to circumstances that have arisen within the region of Physics itself. The older arguments between supporters of Absolute and Relative Motion, and the later ones about the Theory of Relativity, are essentially pieces of Critical Philosophy in our sense of the word. Thus we may fairly say that, as regards Motion, physicists have been their own philosophers, forced into this unwelcome position by their own domestic difficulties. Now this is not so in the case of Matter. The difficulties about Matter, which show the need for radical philosophic criticism of that concept, are not indigenous to Physics itself. They arise in the main when we begin to take into account the way

in which we get to know matter through sensation. It is the apparent conflict between what our sensations tell us and what Physics teaches about matter, combined with the fact that our sensations are after all the only ultimate source of all our alleged information on the subject, which compels us to indulge in philosophical criticism. The moment we begin this criticism we find that it will lead us very far afield, and that we cannot stop till we have profoundly modified the traditional concepts of Space, Time, and Motion too. Now I hope to be able to show that these modifications, which are forced on us as philosophers when we begin to deal with the concept of Matter, are of somewhat the same kind as those which Physicists have had to make for purely domestic reasons. If this can be shown, even in rough outline, it will greatly strengthen the case for the newer views of Space, Time, Motion, and Matter. There is much in these views which is at first sight highly paradoxical and upsetting to common-sense, so that it is of some advantage even to the scientist to know that they can be justified on wider grounds than the special needs of his science. On the other hand, it is always a comfort to the philosopher to know that he is not simply *bombinans in vacuo*, but is working on lines which have been found to lead to useful results in some concrete region of science.

This book is written primarily for scientists who are interested in philosophy, and secondarily for philosophers who are interested in science. It has therefore been my plan to diverge as gradually as possible from the concepts that are most familiar to scientists. Now, for the reasons given, the philosophic criticism of the concept of Motion is more familiar to most scientists than the criticism of the concept of Matter. It therefore seems right to treat the former before the latter. I am going, then, to deal at present with the purely physical arguments which have gradually undermined the traditional Kinematics and replaced it by that

of the Theory of Relativity. In spite of many excellent (and more, execrable) popular works which have appeared in the last few years, I think there is still room for a restatement of these arguments. To many scientific readers they will of course be perfectly familiar, but it will do no harm to the reader who is primarily a philosopher to put himself *au courant* with the present position in Physics before going further. At a considerably later stage, when we have seen what modifications in the traditional concepts of Space and Time are forced on us by our criticisms of the traditional concept of Matter, we shall return to the present subject, and try to connect the physical with the philosophical doctrines.

We have at least four general kinematic concepts to consider, viz., the Absolute Theory of Motion, the Relational Theory of Motion, the Special Theory of Relativity, and the General Theory of Relativity. This is approximately the *historical* order in which these concepts have arisen in Physics since the Renaissance. We must remember, however, that the controversy between the Absolute and the Relational Theories of Motion had a long history before ever modern Mechanics was founded by Galileo, Descartes, Huyghens, Newton, and Leibniz. This controversy was inherited by Mechanics, and the opposite sides were upheld by two such eminent contemporaries as Newton and Leibniz. I shall treat the concepts in their historical order, putting the Absolute Theory before the Relational Theory of Motion. But, when the various theories have been clearly stated and the *pros* and *cons* have been weighed, a further task will confront us, viz., to try to exhibit their *logical* order and interconnexions. I must confess that I have not seen a satisfactory account of this point in any work on the subject. It seems commonly to be assumed that the logical order has been the same as the historical, and that the successive kinematic concepts have represented a steady development of the doctrine that motion is purely relative. Yet some of

the chief exponents of the General Theory of Relativity, which is the latest phase of kinematics, use language which seems to imply a thoroughly Absolute Theory. We hear of "kinks" in Space or in Space-Time, and we are told that they modify the motions of matter, or that matter consists of such "kinks." All this is extremely puzzling after one has been led to believe by the same writers that the General Theory of Relativity is the final triumph of the Relational Theory of Motion. I think we shall find that the logical connexions are not so simple as we have been told; and it will certainly be useful to do our best to throw some light on this dim spot. We cannot, however, profitably discuss this question until we have seen what precisely the various theories assert.

The Absolute and Relational Theories of Motion. In the last two chapters we have been discussing the traditional concepts of Space and Time. Now the kinematic concept which strictly corresponds to these is that of Absolute Motion. In accordance with the traditional concepts of Space, Time, and Matter, the three are largely independent entities. The traditional view does not as a rule go very deeply into the question of their mutual relations, but I think the following would be a fair statement of what it tacitly assumes on this subject: Time could have existed without Space or Matter; Space could not have existed without Time, but it could have existed without Matter; Matter could not have existed without both Space and Time. Space needs Time in order to endure, but the only connexion is that all points of Space endure unchanged for ever. Matter needs Time in order to endure, and it needs Space in order to have place and shape, which are essential to it. With Matter there begins the possibility of Motion; Matter need not have moved, but as a fact it does so from time to time.

The alternative between the Absolutist and the Relationist Theory of Time may be illustrated as follows:

We say that the Battle of Hastings precedes the Battle of Waterloo by a certain amount, viz., 749 years. The two battles are events in the world's history, and the Absolutist and the Relationist agree that a certain temporal relation subsists between them, and that it has a certain measure in terms of the usual units. The whole question between them as to Time is the following: Is this relation simple, direct, and unanalysable, connecting the two events in question and nothing else, or is it a complex compounded out of other relations which involve other terms in addition to the two events? The former alternative is taken by the Relationist, the latter by the Absolutist. On the former view there is not something called Time which could exist even though there had been no events; Time just consists of the relations of before and after among events. These relations have various magnitudes which can be measured by comparison with the temporal relation between some standard pair of events, such as the successive occupations of the same position on a dial by the hands of a suitably standardised clock.

The Absolutist, on the other hand, holds that the temporal relations between events are not direct and unanalysable; they are really compounded out of relations of two wholly different kinds. On this view there is something called Time which is composed of simple entities called moments; and it is only moments which can strictly be said to be before or after each other. There is further a certain peculiar relation between events and moments which is denoted by the word *at*. *At* is a many-one relation, *i.e.* many different events can be at the same moment but no momentary event can be at more than one moment. The Absolutist analyses the statement that the Battle of Hastings precedes the Battle of Waterloo by 749 years into the three following propositions: (1) The Battle of Hastings happened at a certain moment t_1. (2) The Battle of Waterloo happened at a certain moment t_2.

(3) The moment t_1 eternally precedes the moment t_2 by 749 years. (I am neglecting the fact that both battles took up a finite time and therefore did not literally happen at two *moments*. This is not important for the present purpose, and can quite easily be dealt with on either theory.)

It is important to notice that the traditional Absolutist and the traditional Relationist agree in holding that there is something that can be called *the* dates of the two battles and something that can be called *the* time-lapse between them. Neither of them would admit that the same pair of events could stand in several different temporal relations; that, for instance, they might be both contemporary and yet one earlier than the other, or again that they might precede each other by several different amounts. They agree that there is one and only one temporal relation between a given pair of events, and they only differ as to the right analysis of this relation. It is important to notice this, because it is here that the Theory of Relativity differs from both of them. For, as we shall see, this theory denies that there is a single relation which can be called *the* time-lapse between a given pair of events.

Now that we have got the difference between the Absolute Theory of Time and the Relational Theory clear we can briefly consider the arguments between them. These fall into two classes, viz., those which apply directly to Time and those which apply to it only indirectly through the question of Motion. Absolute motion implies absolute Time and Space, though there will, of course, be relative motion even if there be absolute Time and Space. The Absolute Theory does not deny relative motion, but simply asserts that all relative motion is the appearance of absolute motions. The arguments for and against these theories, which depend on motion, may be reserved for the moment, and we will now consider those which apply directly to Time.

The main merit of the Relational Theory is that it is simpler and keeps closer to the observable facts. We can observe events, and if two events fall into the same specious present, or if one is sensed and the other remembered, we can directly observe the temporal relation between them. We cannot perceive moments of Time. Nor can we say that they are hypothetical entities, like atoms and electrons, which we also cannot perceive. We accept the latter because there are certain sensible facts which we can explain with them and cannot easily explain without them. But, whilst electrons are supposed to be causes with sensible effects, bare moments are not supposed to do anything or to produce any effects, sensible or otherwise.

Undoubtedly there is something more than mere relations in Time. We have already seen that the Time series has a definite intrinsic sense, and that this arises because there is a continual addition to the sum total of existence, whilst nothing that has ever existed ceases to do so save in a derivative and analysable sense. Even though there were no "change" in the ordinary sense of the word, *i.e.* if every fresh slice of existence were qualitatively indistinguishable from all its predecessors, there would be this continual becoming. But, so long as this absolute feature in Time is recognised, there seems no objection to the Relative Theory as such. If it has to be rejected, it will not be in favour of the Absolute Theory but in favour of something still more relativistic than itself.

A minor objection to the Relational Theory of Time, as stated in most mechanics books, is that it is incomplete. Relativists, as well as other people, constantly talk in practice of moments and of several events happening at the same moment. For the Absolutist, of course, such statements are literal expressions of fact. For the Relativist they cannot be so, since he does not literally believe in the existence of moments. It is therefore his duty to give a definition of what he means

by "moments," which shall (a) be compatible with his theory, and (b) compatible with the common usage of this word by himself and others. This duty he invariably shirks. The problem can, however, be solved by the Method of Extensive Abstraction. Two applications of it will be needed: (1) to define momentary events in terms of finite events and their relations of partial overlapping, and (2) to define moments. A moment is eventually defined as a class of contemporary momentary events. Thus the objection under discussion is not intrinsic to the Relative Theory of Time, but only to the common presentment of it.

Let us now consider the difference between the Absolute and the Relational theories of Space. This is much the same as the difference between the two theories of Time. It is, I think, harder to accept a purely relative theory of Space, because of certain additional complications which turn up here. On the Relational Theory spatial relations directly connect bits of matter, *e.g.* the theory says that Cambridge is 60 miles N.N.E. of London, and takes this to be a direct relation between the two towns. The Absolute theory would analyse the fact into three propositions, viz.: (1) London is at a certain point p_1 of Absolute Space; (2) Cambridge is at a certain point p_2 of Absolute Space; and (3) p_2 is 60 miles N.N.E. of p_1.* The Absolute Theory thus assumes certain entities, which may be called *geometrical points*, in addition to matter; spatial relations directly connect these. They only indirectly connect pieces of matter in so far as these are *at* the geometrical points in question.

Now there is an additional complication in the case of Space which is not present with Time. Events always have the same temporal relations to each other; the Battle of Hastings always precedes the Battle of Waterloo by 749 years when the latter Battle has once become. But bits of matter move about; consequently

* I am neglecting here the motion of the earth.

statements about *the* distance from one bit of matter to another or about *the* relative position of two bits of matter are ambiguous. A train travelling from London to Edinburgh by the East Coast Route is sometimes to the East of London and sometimes to the West of it, and is constantly at different distances from it. The way in which the Absolute Theory deals with these facts is the following: It holds that the points of Absolute Space have to each other purely geometrical relations which are wholly independent of Time. It puts the burden of change on the relation *at*, which connects bits of matter with points of Space. What it says is that *at*, in the present sense, is a three-term relation which always connects a bit of matter, a geometrical point, and a moment of Time. The simplest statement that you can make about the position of a bit of matter is that it is at such and such a point at such and such a moment. Another way of putting it is that the presence of a bit of matter at a geometrical point is an event, and that, like all events, this occupies a certain moment of Absolute Time. The relation of being at a point at a moment is held to have certain properties, which are just worth mentioning. (1) Two bits of matter cannot be at the same point at the same moment. This property expresses the impenetrability of matter. (2) One bit of matter cannot be at two different points at the same moment. (The only alleged exception to this is the Body and Blood of Christ in the Celebration of the Eucharist.) (3) If one bit of matter is at two different points at two different moments it must be at a continuous series of intermediate points at the intermediate moments. This expresses the fact that bits of matter do not suddenly leave one place and afterwards turn up at another without following a path from the first to the second. (4) Every bit of matter is at some point or other at all moments. This expresses the indestructibility of matter.

Now all these propositions certainly express im-

portant alleged facts which are commonly believed to be true of matter, and any theory must contain them in some form. On the Relational Theory of Space it is clear that they will need a great deal of reinterpretation, since that theory believes neither in geometrical points, nor in moments, in the literal senses of those words. It follows that if the Relational Theory of Space is to be of the slightest use, it must give meanings to all these statements which (*a*) shall not imply the literal existence of points or moments, and (*b*) shall nevertheless be equivalent in practice to these propositions. I need scarcely say that writers of mechanics books, who start by telling their readers that Space is relative, never attempt to recast these statements in terms of their theory, and never even mention or apparently recognise the need of doing so.

Now this fact, that things move about, at once introduces a difficulty into the notion of distance and relative position on the Relational Theory. We very often need to know the distance between one thing at one moment and another thing at another moment. When we try to measure the velocity of anything it is evidently necessary to know the distance between one piece of matter at the time of starting and another piece of matter at the time of arrival. Again, if we use a measuring rod which has to be taken up and laid down several times between A and B, it is clear that what we directly measure is neither the distance between A and B at t_1 (the moment when we begin to measure) nor the distance between A and B at t_2 (the moment when we cease to measure). If in certain cases the measured distance is held to agree with the momentary distance this must be a matter of inference, and it will be necessary for the Relational Theory to state and justify the assumptions made and the conventions used in drawing these inferences.

Now the Absolute Theory can, of course, give a perfectly definite *meaning* to the distance between a

body at one moment and the same or a different body at another moment. What it says is that the distance required is the distance between the place where the one body was at the first moment and the place where the other body is at the second moment. In ordinary life we do constantly use this phraseology; but we forget that, whilst it has a literal meaning on the Absolute Theory, it needs to be *given* a meaning on the Relative Theory. For, on that theory, the primary meaning of distance is distance between two bodies at the same moment. And, as soon as this is seen, we see further that the relative theory of Space cannot be complete without some criterion of simultaneity at different places. This example brings out rather well the characteristic merits and defects of each type of theory. The Absolute Theory does give a definite *meaning* to the notion of distance between two bodies at different moments; but, since we certainly cannot perceive points of Absolute Space, it fails to explain how we ever know that we are measuring distance in the sense defined. On the other hand the Relational Theory gives a clear meaning only to the notion of distance between two bodies at the same moment; and this is not enough for practical or scientific purposes. But it does stick to bodies, that is to things that we can actually perceive and deal with.

It is pretty evident that the Relational Theory suffers from not being thorough enough, and not fully recognising its responsibilities. It ought to start with events, and to take the relation of distance between contemporary events as fundamental. The notion of bodies and of the distances between bodies at different times will have to be built on this basis; you cannot take either Space or Time or Matter as something given. There is a common matrix out of which the concepts of all three are developed by experience and reflection thereon. The Relational Theory needs to define some sense of Space, which shall still be relative but shall not

be merely momentary. Science and common-sense require a Space which shall be timeless, in the sense of enduring unchanged throughout Time: a collection of momentary Spaces is not enough. It is one of the great merits of Whitehead to have grasped this point. The Absolute Theory does offer us a timeless Space; but, as this can neither be perceived nor inferred causally from what is perceptible, it is rather like the offer of a gold brick or a Castle in Spain. The Relational Theory (whatever may be its pretensions) only offers us a collection of momentary Spaces. This has at least two disadvantages: (1) that strictly momentary relations between bodies can no more be directly observed than distances between points of Absolute Space; and (2) that motion becomes, not change of position within a Space, but a movement out of one momentary Space into another momentary Space. The Relational Theory can hardly solve these unsettled problems without raising precisely those questions which lead on to the Special Theory of Relativity.

We will now desert the subject of Absolute *v.* Relative Space, as such, for the present, and consider those arguments on the subject which depend on the question of Absolute *v.* Relative Motion. It is doubtful whether people would ever have worried their heads greatly about Absolute Space and Time, had it not been that there seemed to be very grave difficulties about purely relative motion. The question has really arisen twice in the history of modern physics, first at the foundation of the classical dynamics by Galileo and Newton, and then again in connexion with electrodynamics in quite recent years.

It is usual for scientific writers with a tincture of philosophy to talk as if plain common-sense unhesitatingly holds motion to be purely relative, and as if it were only persons debauched by metaphysics who believe in absolute motion. This is of course a profound mistake. It is indeed true that the plain man

does not mean by *motion* absolute motion as defined by Newton. But he is perhaps even more shocked by the theory that all motion is purely relative, when once the logical consequences of that theory are explained to him. Naturally, the scientific theories both of absolute and of relative motion are highly abstract intellectual analyses of facts which the plain man is content to see and feel without analysing. Still, it would not be going too far to say that the analysis offered by the absolute theory seems to common-sense nearer to the facts than that proposed by the Relationists. This is hidden by the very half-hearted and obscure way in which most Relationists state their views; in practice it is almost as difficult to take a consistently relational view about motion as it is to bear constantly in mind the fact that men at the antipodes do not have the uncomfortable feeling that we should have if we were hanging head downwards with our feet fixed to the ceiling. Let us then try to state the two theories clearly and to draw their logical consequences. Absolute motion is the passing of a body from one point of Absolute Space to another. Absolute rest is the remaining of a body at a point of Absolute Space. Relative motion has the same meaning on both theories; it is just a change in the relative positions of two bodies. The difference about it is that the Relationists say that all motion simply is a change in the spatial relations of one body to others, whilst the Absolutists say that there is absolute as well as relative motion and that the two must be distinguished from each other. On the Absolute Theory all relative motion implies absolute motion, and is the appearance of it to us, but a knowledge of relative motion does not suffice to determine unambiguously the absolute motions involved. Thus, suppose that A and B are two bodies, and that u is the rate at which the distance between them is increasing. Then u is a relative velocity. The Absolutist says that it must be due to absolute motions

in A or in B or in both, and that all that we can say about them is that their difference is equal to u.

Now the point at which the purely relative theory of motion conflicts with common-sense is that it will never allow you to say of any two bodies that one is moving and that the other is at rest. Distance between A and B is a perfectly mutual relation; if the distance between A and B increases at a certain rate the distance between B and A *ipso facto* increases at the same rate. If then motion just means rate of change of distance between bodies there is no sense in saying that A moves and B stands still. Suppose now that I am the body A and that B is the wall of the room. Common-sense is perfectly sure that I move and that the wall stands still. But for the consistent Relativist this is simply nonsense; it is true in precisely the same sense, and in the only sense in which he admits motion, that the wall moves towards me. Thus common-sense seems here to be much more on the side of the Absolutist than on that of the Relationist. It quite admits that, in particular cases, it is difficult or impossible to tell in what proportions a particular relative motion ought to be divided between the two bodies, but it is quite convinced that in every case there is a genuine meaning in the question: What is the real velocity of each body? This question, as we have seen, has a perfectly definite meaning on the Absolute Theory, but its meaning is not obvious on the Relational Theory.

Of course I do not regard this common-sense objection as at all conclusive, for I think that the Relationist can make a fairly satisfactory answer to it. He will say: "You think that certain bodies are absolutely at rest and others in motion, not because there is really anything but relative motion, but because you tacitly assume a certain body for relating all others to." This body, for the ordinary man, is the earth. He says that the wall is at rest because it does not move relatively to the surface of the earth; he says that he himself

moves because he does change his position with respect to this body of reference. It is very easy to forget about a relation altogether if we always tacitly relate to the same term in a whole series of judgments. If our common-sense friend replies that when he moves he gets tired, whilst when other things move and he stands still he does not get tired, the Relativist can easily deal with this objection. He will say: "All motion is relative, and all relative motions are equally genuine facts; but they do not all have the same effects. When you and the earth move relatively to each other effects are produced in your body, but when you rest relatively to the earth and merely move with respect to other things which are themselves in motion with respect to the earth, such as tram-cars, no such effects are produced. This is just a law of nature which we have to recognise."

So far the Relationist has a perfectly good case. It is when we come to deal with mechanics, and particularly with rotation, that his difficulties begin to accumulate. We will deal with rotation first, because it can be discussed without any knowledge of the laws of mechanics, and because it furnished Newton with one of his strongest arguments in favour of absolute rotation. Suppose that you take a pail of water and hang it up by a string, then twist the string a number of times and let it untwist itself. The pail will, of course, spin rapidly round its axis. At first the water will not spin, but gradually it will take up the spinning movement of the pail, and eventually the water and the pail will be spinning as one rigid body. Now stop the pail. The water will go on spinning for some time till it is gradually brought to rest by friction. Now what we have to notice is this: At the beginning of the experiment, *i.e.* when, in ordinary language, the bucket is spinning and the water is still at rest, the water has its maximum velocity of rotation with respect to the pail. And at this stage the surface of the water

is quite flat. At the second stage of the experiment, when, in ordinary language, we should say that the water had picked up the speed of rotation of the pail, the water has no rate of rotation with respect to the pail. Yet at this stage the surface of the water is depressed in the middle, so that it becomes a paraboloid of revolution. Now we all say that this depression is due to the rotation of the water. But, if we confine ourselves to relative rotation, we see that the depression was *nil* when the relative rotation was a maximum, and that it was a maximum when the relative rotation is *nil*. If we now pass to the next stage of the experiment, where, in ordinary language, the pail has been brought to rest and the water is still rotating, we have again a maximum rate of relative rotation, but this is now accompanied by a maximum depression in the surface of the water. Thus there seems to be no regular connexion between relative rotation and depression at all; for the depression can be a maximum both when there is no relative rotation and when the relative rotation is a maximum, and the depression can be *nil* both when there is maximum relative rotation—as at the beginning—and when there is no relative rotation—as at the end of the experiment.

These are the facts which led Newton to hold that we must distinguish between absolute and relative rotation. The argument comes to this: If we take all rotation to be simply and solely the rotation of one body with respect to another we can find no general law connecting rotation with depression. Yet we are all agreed that in some sense the depression is due to the rotation. Newton's suggestion was that absolute rotation, and it alone, produces physical changes like the depression of the water in the pail and the flattening of the earth at the poles. It is true that we can observe only the relative rotations of bodies; but these are appearances of absolute rotations, and by studying and measuring such physical consequences as depression

and flattening we can ascribe to each of the bodies its proper amount of absolute motion.

Now of course the facts on which Newton based his argument are genuine and very important. But they certainly do not necessitate Newton's conclusion, although that is no doubt one way of explaining them. They can equally well be explained without recourse to absolute motion. If we reflect, we shall see that it is logically impossible that premises which are wholly about bodies, such as water and pails, and about their shapes and relative motions, could necessitate conclusions about something entirely different, viz., Absolute Space and Absolute Time. By a logical argument you may learn of new relations between the terms that are mentioned in the premises, but you cannot possibly learn about the existence of other terms of a quite different kind from any that were mentioned in the premises. So we can see at once, from purely logical considerations, that Newton's argument cannot necessitate a belief in absolute motion. What we can legitimately argue is that, *if* there be such things as absolute Space, Time, and Motion, it is in rotation that they first disclose themselves by producing observable effects in matter, and that by studying these phenomena we may be able to detect the presence and measure the magnitude of the absolute motion of each body.

But, as I have said, the Relationist can interpret the pail experiment in terms of his theory. If we reflect carefully on the results of that experiment, we see that all that it tells us is that *one particular* relative rotation is not connected by any simple law with the depression of the water in the pail. It shows that the relative rotation of water and bucket is irrelevant. It does not in the least follow that *no* relative rotation is relevant. At the beginning of the experiment the water was at rest relatively to the fixed stars, at the middle it was rotating, and at the end it was again at rest with respect to them. What the Relationist must say is therefore

the following: "There is nothing but relative rotation, and any body that you choose to mention has at one and the same time all sorts of different relative rotations; for instance, the water at the beginning is rotating with respect to the pail and is at rest with respect to the fixed stars. Each of these states of motion is equally real and there is no incompatibility between them, because they are not properties of the water alone but are relations between it and other things. It is no more unreasonable to say that the water is at once at rest and in motion than it is to say that a man is at once a father and a son; it only seems odd because we are haunted by the ghost of the Absolute Theory. But of all these various equally real and co-existing motions some only are connected by simple laws with physical changes in the water. Relative rotation between the water and the fixed stars causes depression of the surface of the latter; relative rotation between the water and the walls of the pail causes no such depression if the water be at rest with respect to the fixed stars." This answer of the Relationist seems to me to be perfectly compatible with all the facts of the pail experiment and to be perfectly consistent with itself.

I will now consider certain objections which have been brought against this interpretation of the facts.

(1) It is sometimes said: Suppose the water stayed still and that the fixed stars rotated round it; the water would be moving relatively to the fixed stars. On the above explanation the water ought to be depressed. Is it reasonable to suppose that the mere rotation of the fixed stars would have any effect on the water in the pail? This objection is merely silly and circular. It is based on an assumption which has a meaning on the Absolute Theory and no meaning at all on the Relational Theory. On the Absolute Theory there is a sense in distinguishing between the case where the water rotates and the stars keep still and the case where the stars rotate and the water keeps still. But the dis-

tinction is meaningless on the Relational Theory. The argument in question is therefore irrelevant as opposed to the Relational Theory. It is really circular, for its premise only has a meaning for a man who has already rejected the Relative Theory, and, therefore, it cannot consistently be used as an argument against this theory.

(2) A stronger objection is the following: Even if the sky had always been covered with thick clouds, so that the fixed stars had never been observed, we could still have discovered that the earth rotates, have determined its axis, and have measured its rate of rotation by means of the gyrostatic compass and Foucault's pendulum. What is it that we discover and measure in such cases if it be not the absolute rotation of the earth? How can it be the rotation of the earth relative to the fixed stars, since they do not come into the question at all? I think that this objection is fallacious, but it needs a little reflection to answer it. I will take the case of Foucault's pendulum; and neglect the gyrostatic compass, which is harder to discuss without mathematics. It will suffice to say that the answer that I shall give about Foucault's pendulum, if valid at all, will apply equally to the gyrostatic compass.

To simplify matters we will suppose that the compass is hung up at the North Pole and started swinging. Make a chalk mark on the ground where the plane in which the pendulum starts swinging cuts the earth. As time goes on you will find that the pendulum no longer swings in this plane; if you draw another such chalk line it will make an angle with the first. In fact, the plane will slowly rotate, and the time of its rotation will be twenty-four hours. If this experiment be done anywhere else on the earth, analogous results will be got. The actual measurements will depend on the latitude, and it will be found that they are all connected with each other and with the latitude by a

simple law. The fact to be noticed is that what has been measured in all cases is a *relative* rotation between the plane of swing of the pendulum and the earth's surface. Let us suppose that the sky were always covered with thick clouds so that the fixed stars could never be seen. What people would probably have said would be the following: "All pendula slowly rotate their planes of rotation with respect to the earth, and the way in which they do this at different places follows a simple law."

Now, if motion be purely relative, this is precisely equivalent to saying that the surface of the earth rotates with respect to the planes of swinging pendula. It follows that a perfectly clear meaning could have been given to the rotation of the earth on the Relative Theory, even if no stars had ever been observed. Suppose some speculative scientist had said: "There may be other bodies beyond those thick clouds; if so, does the earth rotate at the same rate with respect to them?" Of course, no answer could have been given. We who can see the fixed stars know that the planes in which pendula swing do not rotate with respect to them, and we therefore know that the rotation of the earth or of any other body with respect to the plane of swing of a pendulum is the same as its rotation with respect to the fixed stars. This particular fact of nature would, of course, have been hidden from us if we had never seen the stars; but otherwise we should be in exactly the same position as we are in now. *We* can say: "The earth rotates at such and such a rate both with respect to the fixed stars and with respect to the planes of pendula." Men who had never seen the fixed stars could only make the latter part of this assertion. We know an extra fact which they do not, but what each of us knows is equally about relative rotation.

(3) The third objection is one that is constantly mixed up with the one that has just been discussed,

but really is quite different from it. It is said: "If there were no fixed stars the earth could not be rotating with respect to *them*. Now you say that it is rotation with respect to the fixed stars which causes the flattening of the earth at the poles and the depression of the water in a rotating pail. Can you seriously maintain that, if the fixed stars were annihilated, the earth would become perfectly spherical and the water in the pail perfectly flat? You certainly ought to hold this. For you say that the cause of the depression of the water is its rotation with respect to the fixed stars. If the fixed stars ceased to exist, this relative rotation would *ipso facto* vanish too. The alleged cause of the depression having thus ceased to exist, we may presume that the depression itself would cease too."

Before discussing this argument, I want to point out its precise connexion with the previous one, and the cause of the frequent confusion between the two. The present argument deals with the physical causation of such phenomena as the flattening of the earth at the poles, and the depression of the water in a spinning pail. It points out an implication of the Relational Theory which its supporters are very liable to forget. The theory says that the cause of such phenomena is the rotation of the earth or the pail with respect to some other body or bodies. Now, if this is to be literally true, it would seem that the existence of *one at least* of the assigned bodies of reference must be an essential part of the cause of the physical phenomena in question. Relationists are inclined to regard the fixed stars, or whatever frame of reference they may happen to use, as *mere* axes of reference, and in no sense causal factors. The present argument shows that this is inconsistent. To square the Relational Theory with the facts, it is necessary to hold that certain relative motions stand out from all others in producing observable physical consequences. Now these outstanding relative motions are those which bodies have

with respect to *certain* bits of matter, such as the fixed stars. These particular bits of matter are thus put in a unique position among all other bodies. Motion with respect to any one of this particular set of bodies produces physical phenomena; otherwise similar motions with respect to other bodies do not produce similar physical consequences. Thus the existence of this privileged set of bodies is an essential factor in the production of these particular physical phenomena, and we have no right to suppose that these phenomena would continue to happen if all the bodies in this set were annihilated. (It is not necessary to suppose that the existence of any *one* member of the set, *e.g.* the fixed stars, is essential. What does seem to be essential is that there should be *at least* one member of the set, though it is immaterial which particular one it may be.) This is the basis of the present argument, and the force of it is that it is hard to believe that the existence of a certain privileged set of bodies is a necessary condition of the flattening of the earth or the depression of the water.

Now the previous argument was not about physical causation, but was about the meaning of and the evidence for the statement that the earth rotates. It suggested that, since we could know that the earth rotates and measure the rate at which it does so, even though we had never seen the fixed stars, we cannot *mean* by the statement that the earth rotates simply that it does so with respect to the fixed stars. And it concluded from this that, when we talk of the rotation of the earth, we must mean absolute rotation, and that we must be able to detect and measure it by observations made on purely terrestrial bodies. As we have seen, the premise of this argument and the first part of its conclusion are true, but its final conclusion does not follow. What we observe in these purely terrestrial experiments is still relative rotation, and what men who could not see the fixed stars would mean when they

said that the earth revolved, would be that it does so with respect to the plane of a swinging pendulum. We who can observe the fixed stars have found out the additional fact that the rotation of the earth with respect to them is the same as its rotation with respect to a pendulum swinging at the North Pole.

The arguments, then, are entirely different. Why is it that they are so often mixed up? I think the reason is the following: It is thought that, since you could find out the rotation of the earth without knowing anything about the fixed stars, therefore the fixed stars cannot be an essential part of the cause of such phenomena as the flattening of the earth. This is, however, a very bad argument. We can find out a good deal about the symptoms and treatment of influenza, though no one has ever seen an influenza germ. This does not prove that these symptoms do not depend on a germ, or that they would not cease altogether if the germ were exterminated.

Having cleared up the connexions, real and imaginary, between these two arguments, let us consider the second of them. Several answers might be made to it. The first, which was made by Mach,* seems to me to be logically sound, and to contain an important truth, though—as I shall point out later—it does not altogether satisfy our physical instincts. The argument that we are discussing appeals to our conviction that such remote bodies as the fixed stars cannot really be essential factors in the causation of purely terrestrial phenomena like the flattening of the earth and the depression of the water in the pail. Now Mach's answer is to say that this conviction is a mere prejudice, and to point out how this prejudice arose. Mach says that we have really not the least idea what would happen if the fixed stars were annihilated, and that therefore we have no right to suppose that the earth would still be flattened and the water still depressed

* *Science of Mechanics.*

after such a cosmic upheaval. Mach's grounds for this assertion seem to me to be sound. They are as follows: The laws of motion and all other scientific laws have been discovered and verified in a world which, as a matter of fact, does contain the fixed stars. Our laws do not make explicit mention of these bodies, because they have been a constant factor, and are assumed to be going to be a constant factor in all predictions which we make by means of these laws. But, though constant factors need not be mentioned, it does not follow that they are causally irrelevant. We say that gas lights when you put a match to it; and we do not as a rule mention that air must be present, because it practically always is present when we strike matches and attempt to light gas. Nevertheless this constant factor is as relevant as the matches and the gas, and if we argued that the absence of air would make no difference, we should be wrong. You can never safely assume that any factor which has been present in all cases under which a law has been verified is irrelevant to the truth of the law, until you have produced a definite negative instance in which this factor was absent and the law was nevertheless found still to hold. Now we obviously cannot remove the fixed stars, spin a bucket, and see whether the water is still depressed in the middle. Therefore we have no right to feel so sure that it still would be depressed in the middle if there were no fixed stars.

I will now point out why this argument, though logically sound and based on an important general principle, is liable to leave us dissatisfied as physicists. Mach's answer accepts the view that the flattening of the earth and the depression of the water depend on motion relative to the *fixed stars*, and that therefore the existence of these bodies is an essential factor in the causation of such phenomena. Now we must notice that, if this be true, a very peculiar kind of physical causation is introduced. It is of such a kind that, if

there were much of it in the world, physics and all other experimental sciences would be impossible. It is a fundamental assumption in all our practical work that the more distant a body is the less difference it makes to the physical phenomena in a given region. The chemist assumes that practically everything that goes on outside his laboratory, and most things that go on outside his test-tube, are irrelevant to the phenomena inside his test-tube. We are, of course, prepared to admit that possibly everything that happens anywhere has *some* influence on everything else, and that the more delicate we make our experiments the less we can afford to treat anything as irrelevant. But, unless very distant things could on the whole be safely neglected, and neglected with greater safety the further they are away, all experimental research would be hopeless, because no phenomenon would be even approximately isolable from the rest of the world. If gravitational, electric, and magnetic forces varied directly instead of inversely with the square of the distance, there would be what Mr Mookerjee very justly termed "a rare hullaballoo or pretty kettle of fish." Now Mach's answer does introduce a sort of physical causation which is of just this objectionable kind. The fixed stars are the most distant bodies that we know of, and yet they are an essential factor in causing the flattening of the earth and the depression of the water. This is why I said that the implications of Mach's answer contradicted our physical instincts. Of course it is quite possible that here our physical instincts are mere prejudices. It may well be that all the known laws of nature, when fully expressed, involve two factors, viz., those that we actually mention and measure on the one hand, and the general structure of the stellar universe on the other. The latter has kept fairly constant up to the present, and so we have come to no harm as yet by neglecting it and confining ourselves entirely to the first factor.

I now turn to a second possible answer to the present

objection to the Relational Theory of motion. I am inclined to think that Mach's answer concedes more than is necessary to the opponent. The opponent confines himself to the fixed stars, argues that it is only rotations with respect to *them* that produce physical consequences on the Relational Theory, and therefore confronts the Relationist with the conclusion that the existence of the fixed stars must be an essential factor in the production of these physical phenomena. Mach accepts this as a fair consequence of the Relational Theory, and simply argues that it is unobjectionable for the reasons given above. This seems to me too big a concession. I pointed out that every body has at one and the same time many different relative motions, all equally real, just as any town has at one and the same time any number of different "distances." There is no kind of contradiction or inconsistency in this unless we tacitly smuggle in the idea of absolute motion. Now, if the laws of Mechanics be true, all the motions of all other bodies relative to (say) the fixed stars obey a certain set of rules, viz., Newton's laws of motion, or whatever modification of them may be found to be necessary. Suppose that a whole set of bodies B_1, B_2, . . . B_n obey Newton's laws for all their motions with respect to the fixed stars. Let us select any body B_r out of this set. Then the motions of any other, such as B_1, with respect to B_r, could be compounded out of the motions of B_1 and B_r with respect to the fixed stars. But, by hypothesis, the motions of both B_1 and B_r with respect to the fixed stars obey Newton's laws. Hence the motions of B_1 with respect to B_r must obey laws which are merely mathematical transformations of Newton's. Precisely the same remarks apply to the motions of any of the other B's with respect to B_r. The standard body B_r might be as wild as we like, it might be a midge dancing in the sunlight; still, if it and all other bodies obey a certain set of rules for all their movements with respect to the fixed stars, all other

bodies will obey a set of rules for their movements with respect to *it*. No doubt these rules would be of perfectly awful complexity if we had chosen a midge instead of the fixed stars as our body of reference; but what does this prove? Only, so far as I can see, that we should probably never have *discovered* that all motions are subject to laws if we had not had the fixed stars available as bodies of reference. When we say: "It is only motions relative to certain bodies (of which the fixed stars are typical) which obey the laws of Mechanics," this is true in one sense and false in another. It is true that only such motions obey even approximately the simple and familiar laws of motion discovered by Galileo and Newton. It is not true that motions with respect to other bodies obey *no* laws, or that the laws which they obey are incompatible with or independent of Newton's. The laws of such motions must be just mathematical transformations, often of unmanageable complexity, of the familiar and simple laws which govern motions with respect to the fixed stars. This seems to be a necessary consequence of the two facts (*a*) that all motions with respect to the fixed stars are subject to Newton's laws, and (*b*) that the motions of any body with respect to any other can be compounded out of the motions of both with respect to the fixed stars.

If this argument be sound, we can now give an answer to the present objection to the Relational Theory, which shall accept all that is true in Mach's answer and shall not shock our physical instincts or prejudices. The objection, I may once more remind the reader, was this: If the earth be flattened and water in a spinning pail depressed only through rotation with respect to the fixed stars, then, if there were no fixed stars, the earth would not be flattened nor the water depressed. We can now see that this consequence does not really follow from the Relational Theory of Motion. If you twisted the pail in the absence of the

fixed stars there would still be relative motion between it and other things. It is true that these other relative motions would not be connected with the depression of the water by the same simple laws which connect that depression with the rotation of the pail relative to the fixed stars. But the depression would be connected with these other relative motions by laws which are mathematical transformations of these simpler ones. In that sense it would be true to say that the annihilation of the fixed stars would not *necessarily* make any difference to the phenomena. On the other hand, we can still admit with Mach that it would not be safe to assume that laws which have been discovered and verified in the presence of the fixed stars would necessarily continue to hold when such a large and important part of the material universe as the fixed stars had been annihilated. The difference between our answer and Mach's comes to this: Mach accepts it as a necessary consequence of the Relational Theory that the existence of the fixed stars is an essential condition of the phenomena under discussion; he then devotes himself to showing that we ought not to be surprised at the disappearance of these phenomena in the absence of the fixed stars, and therefore that this consequence of the Relational Theory is no objection to it. We argue that this is not a necessary consequence of the theory, but add that we too should not be surprised if laws which had been ascertained in the presence of the fixed stars should be found to break down after so huge a change as the annihilation of those bodies.

The upshot of the discussion seems to me to be that there is no conclusive objection to the view that all motion is relative, and that all arguments which have been produced to show that we must recognise, and can indirectly measure, absolute motion, are fallacious. This being so, I think there are strong reasons for rejecting the Absolute Theory. After all, the laws of motion are empirical laws, discovered by observing and

reflecting upon the actual movements of actual bodies. Now, all that we can observe in the way of motion is the change in position of one body with respect to others. It were strange indeed if such observations could lead to laws about something which is, from its very nature, unobservable, and stranger still if such laws enabled us to control and predict the movements of bodies in nature. Absolute Space, Time, and Motion have all the appearance of being mathematical devices, and not substantial constituents of nature, and a theory is to be preferred which reduces such mathematical scaffolding to a minimum, provided of course that it is adequate to all the facts with which it professes to deal. I think that mathematicians and writers on dynamics have been justified in rejecting the Relational Theory in the forms under which it has been commonly presented in the past ; but I think that this is because it has been badly and inadequately stated, and not because it is impossible to make it fit all the facts.

This is about as far as we can go when we confine the discussion to ordinary mechanical phenomena. But the whole question arose again in recent years over electro-dynamics, and it has been found that reflection on the facts of this region of phenomena necessitates a still more radical overhauling of the traditional concepts of kinematics. This leads to the Theory of Relativity, which I shall deal with in the next chapter.

The following additional works may be consulted with advantage :

LEIBNIZ, *Correspondence with Clarke.*

E. MACH, *Science of Mechanics*, Chap. II., § vi., Appendix XX. and XXII.

B. A. W. RUSSELL, *Principles of Mathematics*, vol. i., Chap. LVIII.

A. MÜLLER, *Das Problem des absoluten Raumes.* (Vieweg. Braunschweig., 1911.)

P. PAINLEVÉ, *Les Axiomes de la Mécanique.* (Gauthier-Villars, Paris, 1922.)

H. POINCARÉ, *Science et Hypothèse*, Chap. VII. (Flammanon, Paris.)

H. POINCARÉ, *Science et Méthode*, Part II., Chap. I.

CHAPTER IV

"Ah! that accounts for it," said the Hatter. "He won't stand beating. Now, if you only kept on good terms with Time, he'd do almost anything you liked with the clock. . . . You could keep it to half-past one as long as you liked."

(LEWIS CARROLL, *Alice's Adventures in Wonderland.*)

Modification of the Traditional Kinematics in the Region of Physics (*continued*). (2) The Special Theory of Relativity

THE older controversies between Absolutists and Relationists, which we have discussed in the last chapter, took place wholly within the region of dynamics, *i.e.* they dealt with the movements of bodies and with the changes of shape, such as flattening and depression, which some of these movements produce. It is clear, however, that the same kind of question could be raised over anything whatever that moves, and over any kind of effects which movement may seem to produce. Now there is good evidence—some of which will be mentioned in a later chapter—for the view that light travels out from its sources with a very great but finite velocity; and this velocity has been measured. Again, the motions of charged bodies produce magnetic effects which vary with the velocities of the bodies.

Thus in theory the whole question between the Absolute and the Relational views of Motion might be argued out again in the regions of light and electromagnetics. A wave of light might be expected to have all sorts of different *relative* velocities, and the question might be raised: Which, if any of these, is what the physicist means by *the* velocity of light? The Absolutist

might here step in and say that by *the* velocity of light we must mean, not any of its relative velocities, but its absolute velocity, in the sense discussed in the last chapter. Similarly, we might ask: Which, if any, of the numerous different relative velocities of any charged piece of matter produces magnetic effects? And the Absolutist might say that *no* relative velocity has this effect, but only the absolute velocity of the charged body. I do not think that these additional facts really make any difference in principle to the conclusions which we reached about the Absolute and the Relational Theories in the last chapter. I will try to justify this statement before going on to discuss what modifications the new facts do make in the traditional kinematics.

The subject is a little confused at the outset through the introduction of a new friend—the Luminiferous Ether—which did not enter into the purely dynamical arguments. Thus we get an apparently intermediate view, put forward by physicists who reject Absolute Space, Time, and Motion with righteous horror as metaphysical figments, and tell us that what is important in light and electro-magnetics is motion, not with respect to this or that *body*, but with respect to the Luminiferous Ether. It seems to me that for the present purpose there is no important difference between the Ether and Absolute Space. A distinction was originally drawn, because various physical properties, such as elasticity and density, used to be ascribed to the ether, and because it was supposed to produce various effects on ordinary matter. This is inconsistent with the traditional view that Space does nothing, has no physical properties, and is thus distinguished from Matter. But there are two circumstances which make the distinction between the Ether of the modern physicist and the Absolute Space of the older Mechanics so slight as not to be worth keeping. On the one hand, the Absolutist has really no right to say that Absolute Space does nothing to matter. For it is of the essence

of his view that absolute motion produces flattening and other mechanical effects on matter; and, since Absolute Space is involved in Absolute Motion, it is clear that he ought to hold that it is an essential factor in the production of these effects. On the other hand, as we shall see, the Ether has proved to be a more and more retiring entity, until it is difficult to discover that it plays any part in physics except that which Absolute Space played in the older Mechanics. Thus I do not regard the two views that *the* velocity of light means its absolute velocity and that it means its velocity relative to the Ether as genuine alternatives. The Ether just is Absolute Space *plus* some hypothesis as to its filling, and this latter addition is irrelevant for our present purpose.

Having cleared this complication out of the way, we can see fairly easily that the facts about light and electro-magnetism make no difference in principle to the question of Absolute *versus* purely Relative Motion. When the velocity of light was measured, and when the fundamental equations of the electro-magnetic field were laid down, writers did not as a rule state very clearly what frames of reference they were assuming. But it is certain that they were, in fact, assuming the familiar frame of reference with respect to which Newton's laws of motion hold. If this be Absolute Space, then they were talking about Absolute Motion, and if it be the fixed stars, then they were talking about motions with respect to the fixed stars. Every reason that there is for taking the latter alternative as regards ordinary dynamics exists for doing the same with regard to light and electro-magnetics. The velocity of light is something that has been experimentally measured, and what has been measured must have been the time that a wave of light took to get from one *body* to another (or rather from one body to a second and then back again to the first). Clearly it was the velocity of light relative to these bodies that was measured, and not the time

that it took to get from one point of Absolute Space or one bit of the Ether to another. Similarly the laws of electro-magnetics were discovered and verified by experiments on *bodies*, and the velocities that were observed were the velocities of these bodies relative to others. Again, all the arguments that could be produced to show that in light and electro-dynamics we must be dealing with absolute motions, and that we have the means of indirectly measuring them, are precisely parallel to the arguments to prove the same conclusion from the phenomena of rotation. And they could be met in precisely the same way. Thus the new sciences which have developed since Newton's time leave the question between the Absolutists and the Relationists exactly where it was ; and that is, if we are right, they leave the Relationists in possession of the field, provided they state their case carefully enough.

I do not suppose that any physicist would deny *one* side of the above statement, viz., that the facts about light and electro-magnetics lend no fresh support to the Absolute Theory. But he might be inclined to think that they do provide additional grounds for the Relational Theory. I do not think this is strictly true ; but it is plausible, and an explanation of why it is so will carry us into the heart of our present subject.

In the purely dynamical arguments between Absolutists and Relationists the Absolutist staked his case on absolute *acceleration* and absolute *rotation*. He did not profess to be able to produce any direct empirical evidence for absolute *rectilinear velocity ;* though, of course, if he could prove the existence of absolute acceleration, that of absolute velocity would be proved indirectly. It follows at once from the form of Newton's laws of motion that absolute rectilinear velocity, even if it exists, will not show itself by any dynamical consequences ; for it is acceleration, and not velocity in a straight line, which Newton's laws connect with force, and therefore with possible deformations of bodies.

Now, when we come to deal with light and electro-magnetics, there is a real difference in this respect. If what is called *the* velocity of light be its absolute velocity (or its velocity with respect to the "stagnant ether," if you prefer that expression) we might expect to be able to measure the absolute velocity of a body like the earth by finding the velocity of light with respect to it and noticing how much greater or less it was than *the* velocity of light. The absolute velocity of the earth in its orbit would presumably be the difference between the *absolute* velocity of light and the velocity of a wave of light *as measured from the moving earth*, given that the earth and the wave of light were moving in the same direction when the measurement was made. Again, various observable electro-magnetic effects depend on the velocities of charged moving bodies. If it be the absolute velocity of the charged body that is relevant to these effects, we ought to be able to discover what part of the observed relative velocity of a moving charged body is due to its own absolute velocity and what part is due to the absolute velocity of our axes of reference, for it will be only the former that will be responsible for the electro-magnetic effects which we measure.

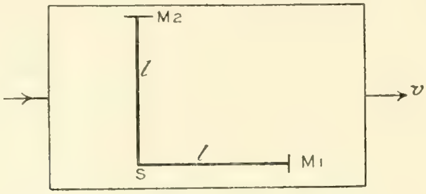
Now it is a fact, and a very important one, as we shall see in detail in a moment, that all attempts to find the absolute velocities of bodies by these means have failed, although the experiments were quite delicate enough to detect the effects which were being looked for, if they had really happened. We can now see what amount of truth there is in the popular view that the new facts about light and electro-magnetics have produced strong additional arguments for the Relationist and against the Absolutist view of Motion. It *is* true that light and electro-magnetics seemed to offer for the first time a means of detecting and measuring absolute *rectilinear velocities*, and that when the experiments were done the results were always wholly negative. But the

negative results of these experiments are just as paradoxical on the traditional Relationist Theory as on the traditional Absolutist Theory. They cannot therefore be taken as arguing for the former and against the latter. It is clear that neither theory, as it stands, is fitted to deal with the facts. Of course, if it should be found that the Relationist Theory can, and the Absolutist Theory cannot, be so modified as to fit the facts of light and electro-magnetics, we may say that *ultimately* these facts furnish a conclusive argument against the Absolute Theory. But at present we must hold that their *immediate* consequence is simply to show the need of modifying both theories. To this modification we will now turn.

I shall confine myself to the question of the velocity of light, and not touch on purely electro-magnetic experiments. The argument in the former case can be followed by any person who takes a little trouble and is acquainted with the first book of Euclid and with algebra up to simple equations; whilst the electro-magnetic experiments cannot be understood without a fair knowledge of mathematical physics. And there is no loss of generality in restricting ourselves to the simple case of light, for light is really an electro-magnetic phenomenon. All that the reader needs to remember here is that the paradoxical result which we are going to explain about the velocity of light is not an isolated phenomenon, but is exactly paralleled by every electro-magnetic experiment that has ever been done with a view to detecting the absolute velocity of the earth or other bodies.

The Michelson-Morley Experiment. I shall state the argument here in terms of the Absolute Theory, because, with our scientific traditions, this makes it more easy to follow. But I shall show at the end that this does not mean that the argument implies the truth of the Absolute Theory, or that it would be inconsistent to use the conclusion as the premise of an argument against

that theory. Suppose we had a platform moving through the "stagnant Ether" (which, as we have seen, is practically the same thing as Absolute Space) in a certain direction with a constant velocity v. On this platform let there be an observer, a source of light, and a couple of plane mirrors. Draw a straight line on the platform through the source of light and parallel to the direction of motion of the platform. Draw another straight line on the platform through the source and at right angles to the first line. Measure off equal distances from the source along the two lines. At the points thus obtained place the two mirrors, each one normally to its line. The illustration below will show the arrangement.



At a certain moment let the source S give out a flash of light and let part of this go to the mirror M_1, and another part to the mirror M_2. Let us first consider the part that travels to M_1. This will have to travel further through the ether than the marked distance l between S and M_1, for M_1 will have travelled a certain distance through the ether while the light is moving towards it, and therefore the light will have to overtake it. Now let the light be reflected back along its old path to the source. It will now have to travel less than the marked distance through the ether, because the source is moving towards it. Suppose the light left S at time O, reached M_1 at t_1, was reflected instantaneously, and got back to S at t_2. Let c be the absolute velocity of light, *i.e.* its velocity through the "stagnant ether." It is then clear that

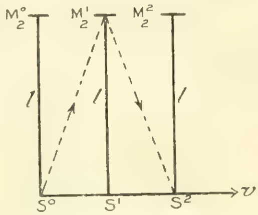
$$l + vt_1 = ct_1$$

and

$$l - v(t_2 - t_1) = c(t_2 - t_1)$$

whence it follows that $t_2 = 2lc/(c^2 - v^2)$. This then is the total time that elapses between the emission of this part of the light and its return to the source after its double journey.

Let us now deal with the light which travels to the other mirror M_2 and is reflected back from it to the source. This light must not travel out in the direction SM_2, as marked on the platform, or it will never reach M_2. For M_2 will have moved to the right by the time such light had got to where it was when the light started. We have therefore to consider light which strikes the mirror at a point in the ether equidistant between the point where the source *was* when the light left it and the point where the source *will be* when the light returns to it. The diagram above will make this quite clear.



The actual course of the light in the ether is the line $S^0M^1_2S^2$. If T_2 be the time when this light gets back to S it is easy to see that

$$c^2\frac{T_2^2}{4} = l^2 + v^2\frac{T_2^2}{4}$$

whence $$T_2 = \frac{2l}{\sqrt{c^2 - v^2}}.$$

Thus the two parts of the original beam of light do not get back to the source at the same time; or, to put it in a different but equivalent way, light which gets back to the source at the same time from the two mirrors must have started from the source at different times. Now, under these conditions, there ought to be a shifting of the position of the interference bands which always arise when the two beams of light which have travelled by different paths from the same source meet again. And from the shift of the bands it would be possible to find the difference between t_2 and T_2. From

this we could calculate v, the absolute velocity of the platform, in terms of c, the absolute velocity of light, by using the two formulæ just proved.

An experiment of this kind was done with great care by Michelson and Morley. Their moving platform was the earth. The velocity v was the tangential velocity of the earth in its yearly motion round the sun. Their apparatus was quite delicate enough to detect smaller shifts in the interference bands than those which were expected. Yet not the slightest trace of any shifting at all was detected. A great many other experiments have been tried in which electro-magnetic effects were looked for as a result of the earth's motion through the ether; in every case the results have been *nil*. This negative fact, that no effect due to the uniform rectilinear motion of a body through the ether has ever been detected, although it had been predicted, and although the apparatus used was quite delicate enough to detect and measure it if it were present, is the basis of the first Theory of Relativity.

Before going any further I want to impress on the reader the extremely paradoxical nature of this fact, and to point out that it is as embarrassing to the traditional Relational Theory of Motion as to the additional Absolute Theory. If I travel in a slow local train, and an express passes me going in the same direction on the main line, I expect to find and I do find that the express moves more slowly relative to me than it would if I were standing on the platform of a station. It is obvious that the express takes longer to pass me under the former circumstances than under the latter. Now we should certainly expect this to happen for all kinds of motion, and this is common ground to the traditional Absolutist and the traditional Relationist. Yet the negative result of the Michelson-Morley and the electro-magnetic experiments might quite fairly be summed up as follows: The velocity of light with respect to various bodies is the same, even

though these bodies be moving with various velocities in the same direction as the light or in the opposite direction to it. In the Michelson-Morley experiment the earth in its orbit corresponds to a slow local train, and the light which goes from S to M_1 corresponds to a very fast express moving in the same direction on a parallel line. The result is as if an express train should appear to be going just as fast to observers in the local train as to observers standing on a station platform. The paradox can be stated just as well in terms of the Absolute and in terms of the Relational Theory. In terms of the Absolute Theory we can say that, although the earth is moving with an absolute velocity through the ether in the same direction as the light, this does not diminish the velocity of the light with respect to the earth; everything goes on as if the earth were absolutely at rest in the ether. In terms of the Relational Theory we can say that the relative velocities of a wave of light, with respect to a number of bodies *which are moving relatively to each other* in the same direction as the light, *are nevertheless all the same.*

Naturally the first thing to do is to see whether any *physical* explanation can be given for this paradox, without modifying the traditional views of Space and Time which are common to the older Absolute and Relational Theories. What physical assumptions were made in the argument which led to the formulæ of the Michelson-Morley experiment? We assumed (*a*) that the ether is not dragged along by the moving platform, as water would be by a stick that was trailed through it; (*b*) that the absolute velocity of light in the "stagnant ether" is the same in all directions; (*c*) that the reflection at the mirrors takes place practically instantaneously; and (*d*) that the fact that a source, which emits light, is itself in motion through the ether makes no difference to the velocity of the emitted light. Would it be reasonable to account for the negative result of the Michelson-Morley experiment by rejecting

or modifying any of these physical assumptions? As regards (*a*) any modification will bring us into immediate conflict with another set of well-established experimental facts, viz., the aberration of light from distant stars, due to the yearly movement of the earth in its orbit. We shall have occasion to refer again to this phenomenon in a later chapter. For the present we may say that the amount of aberration will vary according to the extent to which the earth drags the ether along with it. The actually observed aberration corresponds to the hypothesis that there is no dragging at all, which is what we assumed in our argument.

The assumption (*b*) seems to be the only reasonable one to make on the subject. Nor would it help us to reject it. For the earth is moving in its orbit in different directions at different times of year. It follows that the assumption that the velocity of light in the ether is different in different absolute directions, even if it be intelligible, could only account for the negative result of the Michelson-Morley experiment at one time of year. At other seasons the discrepancy between prediction and observation would be worse than before.

The assumption (*c*) is needlessly sweeping; all that we need to assume is that, whatever time the reflection may take, it is the same for both mirrors. It were surely absolutely arbitrary to suppose that reflection at M_2 always takes up a different amount of time from reflection at M_1, and that this difference is always exactly such as to neutralise the expected difference in the times of arrival of the two beams of light at the source.

(*d*) On the wave theory of light there is no reason why the velocity of a source at the moment of emission should have any effect on the velocity with which the emitted disturbance afterwards travels through the ether. If we held the corpuscular theory of light, matters would be different; for a corpuscle shot out of a moving source would presumably have a velocity compounded of that of the source and that due to the emitting impulse.

But the cumulative evidence for the wave theory and against the corpuscular theory is so strong that it seems idle to try to explain the negative result of the experiment by a hypothesis which is only plausible on the latter view.

Interpretation of the Michelson-Morley Result in terms of the Absolute Theory. It is clear then that no ordinary modification in our physical assumptions will explain the negative result of the Michelson-Morley experiment without bringing us into still worse collision with well-established facts. We are therefore forced to consider the assumptions that were tacitly made in our measuring of distances and time-lapses. This brings us, as regards Space, to the *Lorentz-Fitzgerald Contraction*, and, as regards Time, to the notion of *Local Time*.

I shall still confine myself in my exposition to the terminology of the Absolute Theory, and we shall now be seeing what assumptions as to our measurements of distance and time-lapse have to be made in order to square the results with that theory. It will be remembered that we measured off on our platform two lines at right angles to each other, each of which had the measured length l. This means that our measuring rod had to be laid down exactly l times (if it was of unit length) before we made our mark on each line. Now, on the assumption that identity of *measure* means identity of *physical distance*, we saw that the times taken by the two beams to get back to the source were t_2, for the one that travelled parallel to the direction of motion of the platform, and T_2 for the other. The physical distances travelled by the two, on the present assumption, will, of course, be ct_2 and cT_2 respectively. The first of these is $\dfrac{2l}{1-\frac{v^2}{c^2}}$ and the second is $\dfrac{2l}{\sqrt{1-\frac{v^2}{c^2}}}$.

Now actually the two get back at the same time instead of the two different times t_2 and T_2. It therefore

is necessary to suppose that really they travelled the same physical distance through the ether. We can only explain this on the assumption that, although our *measurements* in the two mutually normal directions on the platform were the same, the *physical distances* measured were not the same. This is equivalent to assuming that our measuring rod does not remain of the same physical length when it is turned in different directions on the moving platform. If we suppose that the physical distance at right angles to the direction of motion really is l, whilst that in the direction of the motion is only $l\sqrt{1-\frac{v^2}{c^2}}$, we can account for the negative result of the experiment. For, in that case, both beams will have traversed the same physical distance through the ether, viz.: $\frac{2l}{\sqrt{1-\frac{v^2}{c^2}}}$; and, as they travel with the same velocity c, they will get back at exactly the same time. What we have to assume then is that a measuring rod, which is of unit physical length when held broadways on to the direction of motion of the platform through the ether, shrinks to a physical length $\sqrt{1-\frac{v^2}{c^2}}$ when laid down on the platform in the direction of its motion. This is what is called the *Lorentz - Fitzgerald Contraction*. It is not, of course, supposed to be confined to one particular rod, but is common to the platform and everything on it. The result is that it cannot be detected by the use of another measuring rod, because that will contract in precisely the same way as the first when you lay it alongside the first.

We can now deal with the question of Local Time. We have supposed that the velocity of light in the stagnant ether is c units of length per second. Now, assuming the Lorentz-Fitzgerald Contraction, we have seen that the distance travelled in the ether by either

beam of light from source to mirror and back again to source is $\dfrac{2l}{\sqrt{1-\dfrac{v^2}{c^2}}}$ units of length. It is clear then that a clock at the source, which marked zero when the flash started ought to mark $\dfrac{2l}{\sqrt{1-\dfrac{v^2}{c^2}}} \div c$ seconds when the flash returns to the source, if it is set in such a way that it accurately measures seconds of physical time-lapse. Now the distance travelled by the light *relatively to the platform* is $2l$ units of length. Therefore the measured velocity of the light *relatively to the platform* will be $2l \div \dfrac{2l}{c\sqrt{1-\dfrac{v^2}{c^2}}}$ or $c\sqrt{1-\dfrac{v^2}{c^2}}$ units of length per second, assuming that the clock at the source is going at such a rate that a second, as measured by it, really does represent a physical time-lapse of one second. The relative velocity of light *would* therefore vary with the velocity of the platform. But this is exactly what we do not find, although we might have expected to do so. We actually find that the measured velocity of the light does *not* depend on the velocity of the source, the observer, or his instruments. It is therefore evident that some further explanation beside the Lorentz-Fitzgerald Contraction is needed to account for the facts. It is evident that this further assumption must be concerned with our clocks, since we have already dealt with our measuring rods. Suppose that, when one second of physical time has elapsed, the clock at the source only indicates $\sqrt{1-\dfrac{v^2}{c^2}}$ seconds, *i.e.* that it is a little slow. Then when $\dfrac{2l}{c\sqrt{1-\dfrac{v^2}{c^2}}}$ seconds have really elapsed the clock at the source will only indicate $\dfrac{2l}{c\sqrt{1-\dfrac{v^2}{c^2}}} \times \sqrt{1-\dfrac{v^2}{c^2}}$,

i.e. $2l/c$ seconds. The measured distance travelled by the light relatively to the platform is, as before, $2l$. Thus the measured relative velocity of the light will now be c, and will thus be independent of the motion of the platform. This, as we saw, is the result which is actually found by experiment. We must therefore accept it as a fact that the clock at the source on the moving platform goes more slowly than it would do if the platform were at rest in the ratio of $\sqrt{1-\frac{v^2}{c^2}}$ to 1. This assumption is of course additional to the Lorentz-Fitzgerald Contraction, and makes no difference to it.

But we are not yet out of our difficulties about the measurement of time. So far we have dealt only with the case of a single clock in a single place on the platform; for the light came back in the end to the place whence it started, and the time-lapse was measured wholly by the clock there. This of course does correspond to the way in which the velocity of light is measured in purely terrestrial experiments, such as that of Fizeau and Foucault. Still, it is clear that we often want to compare the time at which one event happens in one place with the time at which another event happens in some other place. In order to do this we must have some reason to believe that the clocks in the two places are, not merely going at the same rate, but also that they agree in their zeros. Now the mere fact that they agreed in these respects when they were together is no guarantee that they will continue to do so when one has been taken away to a distance. In the case of a pair of ordinary clocks, for instance, the shaking that one of them gets on its journey, the possibly different average temperature of the region to which it has been moved, the different gravitational attraction at different parts of the earth, and many other factors, combine to make it most unsafe to argue that, because the two agreed when they were together, they will continue to do so now that they have been separated.

It is thus absolutely necessary to have some criterion of sameness of rate and sameness of zero which can be applied to widely separated clocks whilst they remain *in situ*. The only method that seems possible is that of signals which travel from one to the other. Let a signal be sent out from clock A when this marks t_A and received at clock B when this marks t_B. Let another be sent out when the first clock marks t'_A and received when the second marks t'_B. If it is found that $t'_A - t_A = t'_B - t_B$, we say that the two clocks are going at the same rate. Again, if a signal leaves A at t_A, reaches B when the clock there marks t_B, is immediately reflected back to A, and reaches there when the local clock marks t'_A, it seems reasonable to conclude that the zeros of the two clocks agree, provided that $t_B = \frac{1}{2}(t_A + t'_A)$. This would obviously be the right criterion to adopt on the Absolute Theory, provided the platform were at rest in the ether. But, we have seen, whether the platform be at rest in the ether or not, there is no observable phenomenon by which the observers on it can detect its absolute motion or rest. Hence, in any case, they are forced to use this criterion *faute de mieux*. Moreover, with this criterion and with it alone, the observers on the platform will find the same value for the velocity of light relative to the platform whether they measure it by observations all made with a single clock in one place, or by observations made with two different clocks in two different places. We can easily show this, as follows: We have seen that the velocity of light, as determined by observations with a single clock, is found to have the same value c, no matter what may be the velocity of the platform through the ether. Now let the clock B be put where the mirror M_1 was in the Michelson-Morley experiment. Let a flash leave the source (where the clock A is) when this clock marks O, reach the clock B when this marks t_B, be immediately reflected back, and reach A again when this marks t'_A. Then, if the two clocks have been set by our criterion,

$t_B = \frac{1}{2}(O + t'_A) = \frac{1}{2}t'_A$. Now we know that the velocity of light relative to the platform, as measured entirely by observations made at A with the clock there, is c. And the measured distance that this light has travelled relatively to the platform is $2l$, *i.e.* the measured distance on the platform backwards and forwards between A and B (or S and M_1 in the diagram to illustrate the Michelson-Morley experiment). Hence $t'_A = 2l/c$. Hence t_B, which is $\frac{1}{2}t'_A$, is l/c. That is, a beam of light which left A when A's clock marked O and travelled the distance l relative to the platform to the point B, will reach B when the clock there marks l/c. Thus the observers at A and B on comparing notes will again conclude that the velocity of light with respect to the platform is c, which is exactly the same conclusion as experimenters who had confined themselves to making observations at A with A's clock had already reached. So that the conventions just laid down for standardising distant clocks are not only those which are practically forced on the observers by their inability to detect the movement of their platform through the ether, they are also the only conventions which will lead to the same measure for the velocity of light relative to the platform, when two different but equally reasonable methods of measurement are adopted. (It ought to be remarked that the last point is of merely theoretical interest, since the only practical method of measuring the velocity of light by terrestrial experiments is by observations made in a single place.)

Now these conventions, reasonable and inevitable as they seem, lead to the result that on a moving platform clocks which are set by them do not "really" agree in their zeros. This means, in terms of the Absolute Theory, that identity of clock-readings in different places does not imply identity of physical date, if the clocks have been standardised by these conventions and are dotted about a platform which is in absolute motion through the ether. This we will now show. We have

just seen that, with these conventions, if a flash leaves A when the clock there reads O, it will get to B when the clock *there* reads l/c. If there were nothing wrong with the clocks except the systematic slowness which we have already had to assume, this clock-reading would mean a physical time-lapse of amount $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{l}{c}$. Now actually the light which left A and went to B has travelled (*a*) a distance $l\sqrt{1-\frac{v^2}{c^2}}$ (allowing for the Lorentz-Fitzgerald Contraction of the platform and the rod with which it is measured), and (*b*) has had further to catch up B, which is itself travelling through the ether in the same direction with a velocity v. A very simple calculation of exactly the same kind as that given on p. 120 will show that the actual amount of time that has elapsed between leaving A and reaching B is $\frac{l\sqrt{1-\frac{v^2}{c^2}}}{c-v}$. Now we have seen that, if we only allow for the systematic slowness of all the clocks on the moving platform, the physical time-lapse would be $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{l}{c}$.

These two quantities are not equal, and the one that we have just obtained by direct calculation is the right one. Hence the clock at B is not merely going somewhat too slowly, like the clock at A; it is also not really in agreement with A as to its zero, *i.e.* identity of readings between the two clocks do not represent identity of physical dates. When the clock at B reads l/c the true physical time-lapse is $\frac{l\sqrt{1-\frac{v^2}{c^2}}}{c-v}$. This equals

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{l\left(1-\frac{v^2}{c^2}\right)}{c-v}\right)=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(\frac{l}{c}+\frac{vl}{c^2}\right).$$

In general, if the clock at B marks t_B, and the measured distance of B from the source in the direction of motion of the platform be denoted by x_B, the physical time-lapse corresponding to the reading t_B is given by the equation

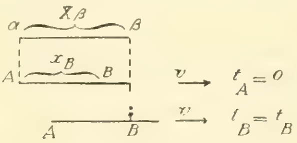
$$t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left(t_B + \frac{vx_B}{c^2}\right) \qquad (1)$$

We see then, that if clocks be dotted about a platform which is moving through the ether with uniform velocity in a straight line, and if these clocks be standardised by means of light signals, and we want to pass from the readings of any clock to the corresponding physical time-lapse, we must not merely divide the reading by $\sqrt{1 - \frac{v^2}{c^2}}$. Before doing this we must add to the reading a quantity $\frac{vx_B}{c^2}$, where x_B is the measured distance from the standard clock to the given clock, in the direction of motion of the platform. Not only are all the clocks slow, in the sense that they all take more than an hour of physical time to make a complete rotation ; in addition to this the hands of the various clocks are pushed back from the very start by amounts which increase the further they are away from the standard clock in the direction of motion of the platform. Clock-readings, like t_B, are called *Local Times*, because they vary with the position of the clocks on the platform, even when the absolute time is the same.

It is usual, for convenience, to denote the fraction $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ by k. We can then say that the Lorentz-Fitzgerald Contraction means that a measured length x in the direction of motion of the platform represents a physical length of only x/k. And the equation just reached tells us that the absolute time is connected with the local time of a clock on a moving platform by the formula

$$t = k(t_B + vx_B/c^2), \qquad (1)$$

assuming that the clocks have been set by light signals according to the conventions laid down on p. 129. We want one more equation before we can get any further. Suppose that when the standard clock on the platform marked O it was opposite to a point α in Absolute Space. When the clock B marks t_B let that clock be opposite to a point β of Absolute Space. The co-ordinate of β, in the direction of motion of the platform and relative to the platform, will of course simply be x_B, the distance as measured along the platform in this direction from the standard clock to the clock B. How will this be related to X_β, the physical distance in Absolute Space between the point β and the point α, which the standard clock was opposite to at the beginning? The diagram above will illustrate the problem.



We have two factors to consider. (1) Owing to the Lorentz-Fitzgerald Contraction the measured length x_B only represents a physical length x_B/k. (2) The platform has moved through the ether for the physical time-lapse that corresponds to the local time t_B. If this lapse be t the platform has moved a physical distance vt. But, by equation (1), $t = k\left(t_B + \frac{vx_B}{c^2}\right)$. Hence

$$\begin{aligned} X_\beta &= vt + \frac{x_B}{k} \\ &= vk\left(t_B + \frac{vx_B}{c^2}\right) + \frac{x_B}{k} \\ &= k\left\{x_B\left(\frac{1}{k^2} + \frac{v^2}{c^2}\right) + vt_B\right\} \\ &= k(x_B + vt_B). \qquad (2) \end{aligned}$$

This is the other fundamental equation of the subject, for it connects the *physical* distance of two points in *Absolute Space* with the *measured magnitude* of their co-ordinates *relative to a moving platform*. The k factor

enters through the Contraction and the Local Time, the v factor through the ordinary rules of relative motion.

We can now sum up the results of the Michelson-Morley experiment in terms of the Absolute Theory. To explain the negative results of that experiment, whilst preserving the Absolute Theory, we have had to make three assumptions. Two of these involve action between Space and Matter; the third is merely the explicit recognition of a convention. (1) We have had to assume that Absolute Motion of a body produces a contraction in the direction of motion. (2) We have had to assume that all clocks on a platform, which moves through the ether, are thereby made to go more slowly. These are both definite assertions as to the action of Absolute Space (or ether) on matter. (3) We saw that the conventions which we use to judge of identity of zero in scattered clocks are not justified if the clocks be in motion through the ether. This is not a new physical assumption, but is in accordance with common-sense. What is new is that we must still go on using this convention, because we can never tell whether we are in motion or not through the ether. It will be seen then that the results of the Michelson-Morley Experiment *can* be dealt with in terms of the Absolute Theory, provided we are prepared to make suitable physical assumptions as to the effect of absolute motion on clocks and measuring rods. Thus, it cannot be said that the newer facts definitely settle the old question between Absolutists and Relationists in favour of the latter. Nevertheless, I think that reflection on the newer facts does strengthen the case of the Relationists by making the Absolute Theory seem more and more arbitrary and improbable. Before going further I will point out why I think this. (1) In order to explain the fact that motion through the stagnant ether does not produce the observable effects which one might reasonably expect it to do, the Absolutist has to assume that it *does* produce two different effects on matter, and that

the combination of these exactly neutralises the expected phenomena. If a student, when taxed with not showing up an essay, were to reply that he had written it and then upset the ink over it, we should perhaps feel a little doubtful, and ask him to let us see the paper. If he then said that, by a strange coincidence, as the ink dried it faded, so that it was now impossible to see anything on the paper, even the Charity which "believeth all things" would be severely strained. Yet this is about the position in which the Absolute Theory finds itself when dealing with the Michelson-Morley experiment. (2) The alleged physical effects of motion through the ether are of the most extraordinary kind. For instance, the Lorentz-Fitzgerald Contraction, if taken as a physical fact, affects all kinds of matter equally. A rod of steel contracts as much as a bit of india-rubber. We might at least expect that such a contraction would be accompanied by strains, and that these would show themselves in the usual way by leading to phenomena, such as double refraction, in otherwise isotropic transparent materials like glass. Such effects have been carefully looked for * and have never been found. Similar remarks apply to the systematic slowing of the clocks. In fact we may fairly say that the assumptions which the Absolute Theory has to make to square itself with the results of the Michelson-Morley experiment are so "fishy" as to cast additional grave doubt on that theory. Let us then try to interpret the Michelson-Morley result in terms of the Relational Theory.

Interpretation of the Michelson-Morley Result in terms of the Relational Theory. The two transformation equations which we reached in the last section contain unobservable factors which we must now try to eliminate. On their left-hand sides they contain absolute time-lapses and absolute distances. On their right-hand sides they contain v, the supposed absolute velocity

* In particular, by Rayleigh and Brace.

of the platform through the ether, which it is admitted we cannot detect. This occurs both explicitly, and also implicitly in the term k. We want to get equations which will contain nothing but relative velocities, actual clock-readings, and measured distances. This is not difficult to do. First of all we must take two platforms, p_1 and p_2. Let us still talk in terms of the Absolute Theory, and suppose that p_1 has an absolute velocity v_1 and p_2 an absolute velocity v_2 in the same direction. Let this common direction, as before, be taken as the x-axis. The first thing that we must find is the measured relative velocity v_{21} which the platform p_2 has with respect to observers on p_1, who measure it with their own clocks and rods. Let a certain point on the platform p_2 be opposite to the standard clock of p_1 when this reads O. Let the same point of p_2 be opposite to B in p_1 when the clock there reads t_B. The velocity of p_2 relative to p_1 as measured by the observers on p_1 will then obviously be x_B/t_B. This is v_{21}. Now from equations (1) and (2) we can easily derive the equations

$$t_B = k(t - v_1 X_\beta / c^2) \qquad (1)$$

and $$x = k(X_\beta - v_1 t). \qquad (2)$$

Hence $$v_{21} = \frac{X_\beta - v_1 t}{t - \frac{v_1 X_\beta}{c^2}}$$

Now $\frac{X_\beta}{t} = v_2$, the absolute velocity of p_2.

$\therefore$, dividing through by t, we get

$$v_{21} = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}. \qquad (3)$$

This formula is both intrinsically interesting, and essential for the next stage of our work. Let us put

$$k_1 = \frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}}, \quad k_2 = \frac{1}{\sqrt{1 - \frac{v_2^2}{c^2}}}, \text{ and } \quad k_{21} = \frac{1}{\sqrt{1 - \frac{v_{21}^2}{c^2}}}.$$

We have $t = k_1\left(t_1 + \frac{v_1 x_1}{c^2}\right) = k_2\left(t_2 + \frac{v_2 x_2}{c^2}\right)$

and $x = k_1(x_1 + v_1 t_1) = k_2(x_2 + v_2 t_2)$,

where x_1 and t_1 are the measured co-ordinate and the clock-reading on p_1 which correspond to physical distance x and absolute time-lapse t respectively, whilst x_2 and t_2 are the measured co-ordinate and clock-reading that correspond on p_2 to the same physical quantities. From these equations we can at once show that

$$t_1 = k_1 k_2\left(1 - \frac{v_1 v_2}{c^2}\right)\left(t_2 + \frac{x_2}{c^2}\frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}}\right)$$

$$= k_1 k_2\left(1 - \frac{v_1 v_2}{c^2}\right)\left(t_2 + \frac{v_{21} x_2}{c^2}\right) \qquad \text{by (3).}$$

Now it is easy to prove that $k_{21} = k_1 k_2\left(1 - \frac{v_1 v_2}{c^2}\right)$; whence

$$t_1 = k_{21}\left(t_2 + \frac{v_{21} x_2}{c^2}\right). \tag{4}$$

In the same way we can prove that

$$x_1 = k_{21}(x_2 + v_{21} t_2). \tag{5}$$

These equations are absolutely symmetrical as between t_1 and t_2, x_1 and x_2. For it follows from them that

$$t_2 = k_{21}\left(t_1 - \frac{v_{21} x_1}{c^2}\right)$$

and $$x_2 = k_{21}(x_1 - v_{21} t_1).$$

But $k_{21} = k_{12}$ and $v_{21} = -v_{12}$, whence

$$t_2 = k_{12}\left(t_1 + \frac{v_{12} x_1}{c^2}\right) \tag{4^1}$$

and $$x_2 = k_{12}(x_1 + v_{12} t_1) \tag{5^1}$$

which are of precisely the same form as (4) and (5) respectively.

We have thus eliminated almost the last trace of anything "absolute" and unobservable. Our equations

now contain only clock-readings; measured distances; relative velocities of one platform to another; and the velocity of light with respect to the two platforms, which the Michelson-Morley experiment shows to have the same value for all platforms, even though they be in motion relatively to each other, provided the motion be rectilinear and uniform. The equations now tell us what co-ordinates and dates observers on one platform will ascribe to an event, provided we know what co-ordinates and dates the observers on any other platform ascribe to the same event, and also know the measured velocity of the one platform with respect to the other. The only trace of "absoluteness" that is left is the proviso that the platforms must be moving in straight lines, and with uniform velocities in the ether. This must be left till we come to the General Theory of Relativity in Chapter VI.

In the meanwhile the reader may be inclined to raise a purely logical question, which ought to be settled before we go any further. He may say: "You have just been deducing certain transformation equations from the assumption of absolute motion through the stagnant ether, and in this connexion you have assumed a real physical contraction in moving bodies and a real physical slowing down of moving clocks. It is true that you have at last deduced a set of equations which are entirely in terms of measured distances, clock-readings, and measured relative velocities. But even these were deduced from the assumption of two platforms moving with different absolute velocities through the stagnant ether. Would it not be a gross inconsistency if you were finally to make these equations the basis of a purely Relational Theory of Space, Time, and Motion? Would you not obviously be using your conclusions to prove something which directly contradicts the premises from which you derived those conclusions? And is this not plainly inconsistent?"

This objection is invalid, as I shall now show. To

some people this fact may be obvious, and they may think the whole objection far fetched. I can assure them, however, that it is fetched from no farther than the University of Oxford ; and respect for the difficulties felt by that learned body induces me to make the logical position perfectly clear. To say that p is *the* premise from which we deduce q means more than to say that p implies q, though of course it involves this. It means in addition that our belief in p is our *only* ground for believing in q. When p and q are related in this way we cease to have any ground for believing in q so soon as we cease to believe in p. But p may imply q, though p is false and q is true. And, provided that we have other grounds for believing q, there is not the least logical objection to our first getting to know q as an implication of p and then using our belief in q as an argument against p. A foreigner might come to believe the true proposition that the Prime Minister of Great Britain in 1921 was a Welshman because he mistakenly believed that Mr Asquith was Prime Minister at that date and that Mr Asquith was a Welshman. He might then find other grounds for believing that the Prime Minister was a Welshman ; he might, *e.g.*, read in the papers that the Prime Minister had delivered a moving address in Welsh to the Free Calvinistic Anabaptists of Llanfairpwllgwyn. . . . On subsequently comparing the Welsh national characteristics with what he could learn about those of Mr Asquith he might begin to feel a legitimate doubt as to his original belief that Mr Asquith was Welsh. Yet he would commit no inconsistency if he continued to believe that the Prime Minister in 1921 was Welsh. He *would* have been inconsistent if he had *never* had any other reason for thinking that the Prime Minister was Welsh except the belief that Mr Asquith was Welsh and was Prime Minister ; but we are assuming that this was only his *original* ground for his conclusion, and that he subsequently found *other* reasons to support it.

Now this is precisely the position about the transformation equations. They do not begin to be directly verifiable till they are got in the purely relational forms (4) and (5). Once they are in these forms they contain nothing but what is observable, and the evidence for them is that they, and they alone, fit all the known facts. They do indeed follow from the Absolute Theory, together with the physical assumptions about contractions and clocks. This is not surprising, since those assumptions were made precisely in order to square the Absolute Theory with such facts as the negative result of the Michelson-Morley experiment. But, once they have been reached, by whatever means, the evidence for or against them is direct and inductive. The Absolute Theory is not *the* premise of them, and there is thus no inconsistency in using them to cast doubt on the Absolute Theory. We do this just because the Absolute Theory *only* leads to them when supplemented by certain physical assumptions which are intrinsically very improbable. If q be known to be true, and p only leads to q when supplemented by the very improbable premise p', the truth of q reflects the improbability of p' back on to p. This I think settles the purely logical question. In future the transformation equations in the relational forms (4) and (5) are to be accepted on their own merits, and without regard to the particular way in which it happens to be convenient to introduce them to the notice of readers brought up (as most of us are) on Absolutist traditions.

There is, however, a real logical incoherence in a good many expositions of the Theory of Relativity. The Lorentz-Fitzgerald Contraction and the slowing of the clocks on a moving platform are first introduced as physical changes due to absolute motion. Later on the Absolute Theory is rejected. But the Lorentz-Fitzgerald Contraction is still recognised as a fact, and the same is true of the slowing down of the clocks. There is an apparent inconsistency here which is very

puzzling to the student of the subject. It is clear that, if the Contraction and the slowing of the clocks are still to be recognised, they must be *reinterpreted*, and this is what is actually intended but not always clearly brought out. Let us then reinterpret them in purely Relational terms.

We have two platforms, p_1 and p_2, of which the second moves in a straight line along the x-axis of the first with a uniform measured relative velocity of v_{21}. A rod is lying on p_2 along the x-axis. The people on p_2 measure it and find that their unit measure goes into it l_2 times. What measure will the people on p_1 ascribe to this rod? They cannot, of course, measure it *directly* so long as it remains on p_2, so they will have to adopt the following expedient. Suppose that one end of the rod is opposite to a point B of p_1 when the clock there marks t_{1B}. Suppose that the other end is opposite to a point C of p_1 when the clock there marks t_{1C}. Let $t_{1B}=t_{1C}$. Then the people on p_1 will say that the distance BC on their platform, as measured by themselves, is the length of the rod which is fixed in p_2. For it is the distance between the points in p_1 which were opposite the two ends of the rod at the same moment, as judged by the clocks on p_1. The length, as measured by them, will therefore be $x_{1C}-x_{1B}$. Now, by equation (5),

$$x_{1C}=k_{21}(x_{2C}+v_{21}t_{2C})$$

and $$x_{1B}=k_{21}(x_{2B}+v_{21}t_{2B})$$

$$\therefore \quad x_{1C}-x_{1B}=k_{21}\{(x_{2C}-x_{2B})+v_{21}(t_{2C}-t_{2B})\}.$$

By equation (4),

$$t_{1B}=k_{21}\left(t_{2B}+\frac{v_{21}x_{2B}}{c^2}\right)$$

and $$t_{1C}=k_{21}\left(t_{2C}+\frac{v_{21}x_{2C}}{c^2}\right).$$

Now $t_{1B}=t_{1C}$, by hypothesis,

$$\therefore \quad t_{2C}-t_{2B}=-\frac{v_{21}}{c^2}(x_{2C}-x_{2B}).$$

Hence $$x_{1a} - x_{1b} = k_{21}(x_{2a} - x_{2b})\left(1 - \frac{v_{21}^2}{c^2}\right)$$

$$= \frac{1}{k_{21}}(x_{2a} - x_{2b})$$

$$i.e. \quad l_1 = \frac{1}{k_{21}} l_2 = \sqrt{1 - \frac{v_{21}^2}{c^2}}\, l_2 \qquad (6).$$

Thus we see that a rod whose length is l_2, as measured by observers who are at rest relatively to it, has a shorter length as measured by observers relatively to whom it moves with a uniform rectilinear velocity. If the two sets of observers can communicate with each other, those on p_1 will say that moving bodies are shortened in the direction in which they are moving, and the amount of shortening is that given by the Lorentz-Fitzgerald formula. Suppose now that the rod were transferred from p_2 to p_1, and the observers on p_1 now measured it directly, whilst those on p_2 now measured it in the same indirect way which the p_1 observers had to use before. The observers on p_1 would now find that the rod had the measured length l_2, whilst those on p_2 would ascribe to it the measured length $\frac{l_2}{k_{12}}$, which is the same as $\frac{l_2}{k_{21}}$ since $k_{12} = k_{21}$. The observers on p_2 would put the case to themselves as follows: They would say that the rod, which was formerly at rest, has now acquired the velocity v_{12} (which is equal to $-v_{21}$), and that this makes it contract in the proportion given by the Lorentz-Fitzgerald formula. Thus both parties would agree that motion causes contraction, and both would agree in the formula which connects contraction with velocity. Both get the same measure when the rod is at rest on their platforms and they can measure it directly. This measure is l_2. Both get the same measure when the rod is moving relatively to their platform and they can only measure it indirectly. This measure is $\frac{l_2}{k_{12}}$, or, what is

the same, $\frac{l_2}{k_{21}}$. The contraction is thus no longer a *physical* change caused by absolute motion through the stagnant ether; it is simply a change in the *measure of length* of the same body, according as it is at rest relatively to the observers and can be measured directly, or is in uniform motion with respect to the observers and can only be measured indirectly. The measurements of the two sets of observers are perfectly concordant with each other, whenever the conditions under which they are made are precisely similar. And there is nothing particularly shocking in the fact that the measurements by two different sets of observers of the same body are not concordant when the conditions under which they measure it are not precisely similar. It is not even inconvenient, since the transformation equations tell us how to pass from the one measure to the other.

We can now deal with the interpretation of the facts about the clocks in terms of the Relational Theory. Let the clock at the point B on p_2 first read t_{2B} and later on let it read T_{2B}. The time-lapse as measured by observers on p_2 will, of course, be $T_{2B}-t_{2B}$. Let the clock which is opposite to B in p_1 on the first occasion read t_{1B}, and the clock which is opposite to B in p_1 on the second occasion read T_{1B}. Then we have

$$T_{1B}=k_{21}\left(T_{2B}+\frac{v_{21}x_{2B}}{c^2}\right)$$

and $$t_{1B}=k_{21}\left(t_{2B}+\frac{v_{21}x_{2B}}{c^2}\right)$$

Whence $$T_{1B}-t_{1B}=k_{21}(T_{2B}-t_{2B})=\frac{1}{\sqrt{1-\frac{v_{21}^2}{c^2}}}(T_{2B}-t_{2B}) \qquad (7).$$

Thus the time-lapse, as measured indirectly from p_1, is greater than the time-lapse as measured directly on p_2. The people on p_1, on communicating with those on p_2, will therefore say that the clocks on p_2 are

rendered slow by the motion of p_2. If, however, a clock from p_2 were transferred to p_1 and the time-lapse were measured with it directly by people on p_1 and indirectly by people on p_2, the latter would say that their old clock was now going more slowly, and would ascribe this to its transference to the moving body p_1. Thus both parties would agree that rectilinear motion slows clocks, and both would agree as to the connexion between this slowing and the relative velocity. But, once again, the slowing is not now a *physical* effect, due to absolute motion through the ether. It is simply a change in the *measure of time-lapse*, according as it is measured by the readings of a *single* clock which is fixed in the place where the time-lapse is measured, or by the readings of *two different* clocks which successively face this place in the course of their motion with respect to it. The measurements of the two sets of observers are again quite concordant, whenever they are carried out under precisely similar conditions; and when the conditions of the two observations differ in the way described above, we can always pass from the one measured time-lapse to the other by using the equations.

We might sum up these results as follows: (1) There is a direct and an indirect way of measuring length. The former can only be applied to bodies that are at rest relatively to the person who is making the measurement, and consists of the familiar process of applying a measuring rod and seeing how many times it has to be laid down before it reaches the other end of the body. When the body to be measured is moving relatively to the observer this method cannot be applied. What has to be done then is for *two observers* on the same platform to note what points on the platform the two ends of the moving body face *at the same moment as judged by the clocks on their platform.* They then measure this distance directly, and take it as the measure of the length of the moving body. These

two methods lead to the same measure for the same body (assuming that clocks have been standardised on the two platforms by the principles laid down earlier in the chapter) if and only if the two platforms be at rest relatively to each other. If the two platforms be in uniform rectilinear relative motion, the two methods do not lead to the same measure for the same body. The two measures are then connected with each other and with the measured relative velocity by the Lorentz-Fitzgerald formula. It will be noticed—and this is very important—that the indirect method of measuring *length* necessarily involves a reference to *time*, since we measure the distance between those two points which the two ends of the moving body are judged to face *simultaneously*. Whether the direct method of measurement also implicitly involves a reference to time we will not discuss at present, though we shall have to do so later.

(2) There is a direct and an indirect way of measuring the time that elapses between two successive events which happen at the same point on a platform. The former can only be applied by observers who are and remain at this place on the platform, and it consists of the familiar process of noting how far the hands of the clock there have turned between the two events. When the two events happen on a body which is moving relatively to the observer this method cannot be used. What has to be done then is for *two observers* to note the readings of their clocks when the first event happens opposite to one and the second event happens opposite to the other. The difference between the readings of these two separated clocks is then taken as the measure of the time-lapse between the two events on the moving body. These two methods lead to the same measure for the time-lapse between the same pair of events (assuming that both sets of clocks have been standardised by the principles already laid down) if and only if the two platforms be at rest relatively

to each other. If the two platforms be in uniform rectilinear relative motion, the two methods do not lead to the same measure of the time-lapse between the same pair of events. The two measures are then connected with each other and with the measured relative velocity by the formula (7). It is important to notice that the indirect measure of *time-lapse* is essentially bound up with *distance*. For the two events which happen in the same place with respect to the one platform happen in different places with respect to the other. The greater the relative velocity of the two platforms the greater the spatial separation of the two events will be, and the greater will be the discrepancy between the two measures of the time-lapse.

This connexion between the spatial and temporal separations of a pair of events comes out still more clearly when we consider a more general case, which must anyhow be treated for the sake of completeness. We have assumed so far that the two events whose temporal separation was to be measured happened at the same point on one of the platforms. Let us now suppose that a certain event happens at B on p_2 when the clock there reads t_{2B}. Let a second event happen at C on p_2 when the clock there marks t_{2C}.

Then the time-lapse as measured on p_2 is $t_{2C}-t_{2B}$. But

$$t_{1B}=k_{21}\left(t_{2B}+\frac{v_{21}x_{2B}}{c^2}\right)$$

and
$$t_{1C}=k_{21}\left(t_{2C}+\frac{v_{21}x_{2C}}{c^2}\right).$$

Whence $$t_{1C}-t_{1B}=k_{21}\{(t_{2C}-t_{2B})+\frac{v_{21}}{c^2}(x_{2C}-x_{2B})\}. \qquad (8).$$

Now
$$x_{2B}=k_{12}(x_{1B}+v_{12}t_{1B})$$
$$x_{2C}=k_{12}(x_{1}\ +v_{12}t_{1C}).$$

Whence
$$\begin{aligned} x_{2C}-x_{2B} &= k_{12}\{(x_{1C}-x_{1B})+v_{12}(t_{1C}-t_{1B})\} \\ &= k_{21}\{(x_{1C}-x_{1B})-v_{21}(t_{1C}-t_{1B})\}. \end{aligned}$$

Whence $$t_{1C}-t_1 = k_{21}\left\{(t_2 - t_{2B}) + \frac{k_{21}v_{21}}{c^2}(x_{1C}-x_{1B}) - \frac{k_{21}v_{21}^2}{c^2}(t_1 - t_{1B})\right\}$$

Whence $$k_{21}^2(t_1 - t_{1B}) = k_{21}(t_2 - t_{2B}) + \frac{k_{21}^2 v_{21}}{c^2}(x_{1C}-x_{1B}),$$

or $$t_1 - t_{1B} = \frac{1}{k_{21}}(t_{2C} - t_{2B}) + \frac{v_{21}}{c^2}(x_{1C}-x_{1B}). \qquad (9).$$

Thus the time-lapse between two remote events has a different measure according to whether it is determined by clocks which are at rest relatively to the events, or by clocks which are in uniform rectilinear motion relatively to them. The discrepancy between the two measures depends on the spatial separation between the two events, in the direction of relative motion of the two platforms. Equation (8) expresses the relation in terms of the spatial separation, as measured by observers who are at rest relatively to the two events; equation (9) expresses it in terms of the spatial separation as measured by observers who are in uniform rectilinear motion relatively to the two events. In particular, let us suppose that the two events are contemporary as judged by the clocks of their own platform. This means that $t_{2B}=t_2$. Then they will not be contemporary as judged by the clocks on the other platform, for $t_{1C}-t_{1B}$ will be equal to $\frac{v_{21}}{c^2}(x_{1C}-x_{1B})$. Thus the temporal separation with respect to p will increase with the spatial separation.

The upshot of the whole matter is to show how inextricably our measurements of distance and of time-lapse are bound up with each other. It is now quite evident that any attempt to measure lengths of bodies which are moving relatively to us involves judgments of simultaneity. On the other hand, a pair of events which are simultaneous with respect to a certain platform, and are separated in space with respect to that

platform, will be successive with respect to any platform that moves relatively to the first; and the time-lapse between them with respect to the second platform will depend on the spatial separation of the two events. It is only pairs of events that happen *both* at the same place and at the same date with respect to *some* platform which will happen at the same place and date with respect to *all* platforms that move with uniform rectilinear velocities relative to the first. A pair of contemporary events, which occupy different places with respect to the platform in which they are contemporary, will be successive in all other platforms that move relatively to the first. A pair of successive events, which occupy the same place with respect to a certain platform, will occupy different places with respect to all other platforms which move relatively to the first. The latter fact was familiar enough before the Theory of Relativity was developed. If I travel to Scotland and eat my lunch in the dining-car, the two events of eating my soup and drinking my coffee are successive; and they happen in the same place relatively to the train, viz., at my seat in the dining-car. But, with respect to the earth, they happen at different places, *e.g.*, at Grantham and at York. The fact which has only lately been recognised is that the same applies to the *dates* of events which happen in different *places*. If the watches of the travellers and the officials on the train had been set, by the same principles as clocks are set on the earth, while the train was in motion, we should have the following result: My neighbour and I might each take a mouthful of soup at the same time, as judged by our watches; but, as judged by the clocks on the earth, his mouthful would happen a little later than mine, if I were facing the engine and he had his back to it. And the difference in date would be proportional to the width of the table at which we were both sitting. The reason why this point has long been obvious about Space but has needed very delicate experiments to force it on our attention as regards Time is

the following: The separation between Grantham and York is gross and unmistakable. But the separation in time between my mouthful and my neighbour's, as judged by clocks on the earth, is proportional to the ratio of the velocity of the train to the square of the velocity of light (see equations 8 and 9). Now the velocity of light is enormous as compared with that of the trains on even so efficient a railway as the Great Northern, and so the temporal separation is negligible and can only be detected indirectly through the negative results of such delicate experiments as the Michelson-Morley.

We see then that, in the long run, the Theory of Relativity *is* more whole-heartedly relational than the traditional Relational Theory of Motion which we discussed in the last chapter. For, according to it, not only is the spatial separation of successive events relative to the system of co-ordinates chosen, but also the temporal separation of two events in different places is relative to the system of co-ordinates and the clocks associated with them.

The Restricted Physical Principle of Relativity. I will end this chapter by trying to state this physical principle clearly, and then to explain it. It may be stated as follows: The laws of any physical phenomenon have the same mathematical form, whether they have been discovered and verified by observers who were at rest relatively to this phenomenon or by observers who were moving relatively to it with a uniform rectilinear velocity. Let us now try to see exactly what this means. The law of any phenomenon, when expressed in mathematical form, is a differential equation connecting some measured quantity which is observed in a certain place at a certain time with some other measured quantity which is observed in some other (or it may be the same) place at some other (or it may be the same) time. The law will also involve the distance between the two places and the time-lapse between the two

dates. Maxwell's equations are a perfect example of a physical law. Now it is clear that such laws are, in the end, verifiable only in so far as they express relations between actually measured magnitudes, such as clock-readings, deflexions of galvanometers or magnetometers, number of weights put into a balance, number of times that a certain rod has to be laid down to get from one place to another, and so on. We may *take* these measures to represent so much time-lapse, so great a current or magnetic force, such and such a gravitational attraction, so much length, etc.; and we may, if we like (and if we can make clear what we mean), raise the question whether these actual measures which we read off our instruments "truly" represent the "real" physical magnitudes in question. But, so far as our laws and their verification are concerned, the *measured* magnitudes are the important things, and the question of what they stand for in the physical world is a secondary matter of theoretical interpretation. *E.g.*, Maxwell's equations, so far as they can be verified, state relations between the readings of electrometers, magnetometers and galvanometers in various places; the readings of clocks in these places; and the number of times rods have to be laid down to get from one place to another.

Now it is *not* true, and the Physical Principle of Relativity does *not* assert, that if one observer is at rest with his instruments relatively to a certain phenomenon, and a second observer is in uniform motion with his instruments relatively to the first, the corresponding instruments of the two observers will give *the same readings*. We already know in fact that they will ascribe different time-lapses and different spatial separations to the phenomena under observation. And the same is true in general of their other measurements. Suppose, *e.g.*, that one observer with a magnetometer and a quadrant electrometer is at rest with respect to a charged particle, and the other observer, provided

with similar instruments, is in uniform rectilinear motion with respect to the first. The first observer's magnetometer will give a zero reading, whilst the second observer's will give a finite reading. What the Physical Principle of Relativity *does* assert, and what *is* true, so far as we know, is the following proposition: The equations which interconnect the readings of one observer's instruments with each other and with his measured distances and time-lapses are of precisely the same *form* as those which interconnect the readings of the other observer's instruments with each other, and with *his* measured distances and time-lapses.

To put the principle formally, let us suppose that the observers on p_1 are at rest with respect to the phenomenon in question. Let the relevant readings of their measuring instruments be P_1, Q_1, R_1. . . . Let the relevant distances and time-lapses, as measured by them, be d_1 and t_1 respectively. The velocity of the phenomenon with respect to them is o. Suppose they find that these various readings are connected with each other and with the measured distances, time-lapses, and velocity, by the equation or set of equations—

$$\phi_1 (P_1, Q_1, R_1. \ . \ . \ . ; d_1 ; t_1 ; 0) = 0.$$

Let the corresponding readings of the observers on p_2 who watch the same phenomenon be P_2, Q_2, R_2. . . . Let their measured distances and time-lapses be d_2 and t_2 respectively. With respect to them of course the phenomenon under observation has the measured velocity v_{12}. Then *their* readings will be connected with each other by the equation or set of equations—

$$\phi_2 (P_2, Q_2, R_2. \ . \ . \ . ; d_2 ; t_2 ; v_{12}) = 0.$$

Now what the physical principle states is that ϕ_2 *is the same as* ϕ_1. This may be briefly summed up in the statement that, according to the Restricted Physical Principle of Relativity, the laws of nature are co-variant

with respect to the space-time transformations of the Special or Restricted Theory of Relativity.

It is important to be quite clear as to the connexion between this principle and the invariance of the measured velocity of light with respect to all observers who move relatively to each other in straight lines with uniform velocities. This latter fact neither implies nor is implied by the physical principle, though it is of course compatible with it. It is obvious that a fact about light could not by itself logically imply a principle about all natural phenomena whatever. Conversely, the physical principle only implies that the measured velocities of light with respect to all observers will be *the same function* of their respective measurements of distance and time-lapse. It does not imply that all these measured relative velocities will have the *same numerical value*. That they do in fact have the same numerical value is an uncovenanted mercy, revealed to us by the Michelson-Morley and other experiments. This fact is of immense practical importance, because it enables us to bring the Physical Principle down from the clouds and apply it to get concrete results. For the invariance of the measured velocity of light enables us, in the way that we have described, to reach the transformations for space and time, *i.e.*, to express d_2 and t_2 in terms of d_1 and t_1. Having done this, we can see how P_2, Q_2, R_2. . . . must be related to P_1, Q_1, R_1. . . . in order that the form of the laws of any phenomenon may be the same for the observers on p_1 as for those on p_2. The result is that, if we once know the readings on the instruments of an observer who is at rest with respect to a phenomenon, we can calculate the corresponding readings of the instruments of an observer who is moving with uniform rectilinear velocity relatively to the phenomenon. This is of course an immensely important power to possess.

If we accept the Physical Principle we shall have to investigate all alleged laws of nature to see whether

they agree with it, *i.e.*, whether they be co-variant with respect to the transformations of the Special Theory of Relativity. Some alleged laws of nature, we find, are already in the right form ; Maxwell's equations are a case in point. Others are not, *e.g.*, the Conservation of Momentum, on the traditional view that mass is independent of velocity. Such examples might, at first sight, be taken as casting doubts on the principle. Here, however, there are two points to notice : (1) If the principle be true and the laws wrongly stated, it is not surprising nevertheless that the laws have seemed to be constantly verified. For the divergence would only begin to show itself when we deal with velocities which are comparable with that of light. Now of course the velocities of ordinary bits of matter are quite negligible in comparison with that of light. (2) As soon as people did come to deal with matter moving with very high velocities, as in the case of particles shot out from radio-active bodies or from the poles of vacuum tubes, it was found that the traditional laws had to be modified, and that the modification was in the same direction and of the same order as that demanded by the Physical Principle. The strong point about the principle in such cases is this : If you keep the traditional form of the laws and try to reconcile them with the facts about particles that move with velocities comparable to that of light, you have to make special *physical* hypotheses as to the nature and minute structure of matter. The other plan, of modifying the laws till they accord with the Physical Principle, has the advantage that it accounts for the experimental results, and requires no special physical hypotheses as to the nature and structure of matter.

With the further development of the Theory of Relativity, and the further modification of traditional physical concepts which this entails, I will deal in the next chapter but one.

The following works may be consulted with advantage :—

L. SILBERSTEIN, *Theory of Relativity.*
M. SCHLICK, *Space and Time in Contemporary Physics.*
E. CUNNINGHAM, *Relativity, Electron Theory, and Gravitation.*

[The reader may here be warned that *most* popular expositions of the Theory are either definitely wrong, or so loosely expressed as to be dangerously misleading; and that *all* pamphlets against it—even when issued by eminent Oxford tutors—are based on elementary misunderstandings.]

CHAPTER V

"Die Entscheidung dieser Fragen kann nur gefunden werden, indem man von der bisherigen durch die Erfahrung bewährten Auffassung der Erscheinungen, wozu Newton den Grund gelegt, ausgeht und diese durch Tatsachen, die sich aus ihr nicht erklären lassen, getrieben allmählich umarbeitet; solche Untersuchungen, welche . . . . von allgemeinen Begriffen ausgehen, können nur dazu dienen, dass diese Arbeit nicht durch die Beschränktheit der Begriffe gehindert und der Fortschritt im Erkennen des Zusammenhangs der Dinge nicht durch überlieferte Vorurteile gehemmt wird."

(RIEMANN, *Über die Hypothesen welche der Geometrie zu Grunde liegen.*)

The Traditional Kinetics, and its Gradual Modification in the Region of Physics. (1) Newton's Laws of Motion and Gravitation

I DO not propose to pass directly from the Special Theory of Relativity, explained in the last chapter, to the General Theory of Relativity. The latter is largely concerned with the laws of motion and the law of gravitation, and so it will be more profitable to begin by discussing the traditional form of these. Thus this chapter will be more closely connected with Chapter III, and the next with Chapter IV.

Newton's first law of motion states that, under the action of no forces, a body continues at rest or in uniform rectilinear motion. This statement, as it stands, is meaningless, if we do not assume the Absolute Theory, and is a mere pious opinion incapable of verification or refutation if we do assume that theory. If we assume the Relational Theory, it is an incomplete statement. If all motion be change of position of one body with respect to others it is useless to talk of rest or of motion

in a straight line until we have specified what set of bodies we are using as our axes of reference. I am at rest with respect to my room and in motion with respect to the sun. The planet Mars is describing an ellipse with respect to the sun and a very complicated curve with respect to the earth. No doubt the law, as originally stated, professed to apply to motions in Absolute Space. But, as these, even if they exist, are unobservable, the law with this interpretation is as idle as the statements in the Athanasian Creed on the internal structure of the Blessed Trinity. The first thing needed then, is to assign our axes of reference. I assume these to be the fixed stars primarily. But it follows from the form of the first two laws that any set of axes which is in uniform rectilinear motion with respect to the fixed stars will do equally well, provided we take traditional views about the measurement of Space and Time, and do not at present introduce the complications which emerged in the last chapter.

Even when the spatial axes have been fixed there remained two unexplained terms, viz., *uniformity* and *force*. Let us begin with uniformity. Uniformity of motion is meaningless unless it refers to absolute motion or states clearly what it takes as its standard measurer of time. A uniform motion means one which covers equal distances in equal lapses of time. If we take the Relational View of Time a lapse of time is a relation between two events; and, even if in theory we take the Absolute View, it is only lapses between events that can actually be observed and measured. It is therefore assumed that we have some process which recognisably repeats itself, and that the time-lapse between corresponding stages in each repetition is the same. A uniform motion is one that covers equal distances during the same number of repetitions of some standard process which is itself isochronous.

The question at once arises: How are you to tell that your standard process is isochronous, *i.e.*, that the

time-lapse between corresponding stages in it is always the same? If you determine this indirectly by mechanical arguments the first law of motion becomes a tautology, for you will first use arguments based on the law to prove that such and such a process is isochronous and will then use this process to give a meaning to the uniformity of motion, which the first law is about. This fallacy is not, of course, commonly committed in so glaring a form. But, in a rather subtler form, something very like it is committed. Our common standard of isochrony is the successive swings of a pendulum. Suppose then we define uniform motion with respect to a certain set of axes, as motion that covers equal distances with respect to these axes during successive swings of a pendulum. So far no fallacy has been committed. But if we verify the first law experimentally on this definition of uniformity, and then later on use the first law as the basis of an argument to explain that the pendulum does not take quite equal times for successive swings, and to correct its errors, we do commit a fallacy. If uniformity of motion in the first law just *means* uniformity as compared with a pendulum, anyone who afterwards says that pendula do not move quite isochronously cannot continue to use "uniformity" in the original sense in which it was used in formulating the first law. And then two difficulties will arise. (1) We must ask him what process he is now taking as his standard, since it is admitted that uniformity, if it is to be observable and measurable, must involve a comparison with some standard physical process. (2) We may remind him that, if the first law has been verified when uniformity is interpreted by reference to a pendulum, no argument resting on the law can fairly be used to prove that pendulums do not *in that sense* move isochronously. Whilst (3), if the law be *not* accurately true, when uniformity is defined in this way, it ought not to be used to prove anything until either (*a*) it has been modified so as to be accurately true on

the old definition of uniformity, or (*b*) a new meaning of uniformity has been given in which it is accurately true in its original form.

There are in fact only two alternatives open to us. Either the first law is simply a definition of uniformity, in which case it reduces to the statement that a uniform motion means one that takes place under the action of no forces. Or it is a substantial statement, in which case some standard process or set of processes must be judged immediately to be isochronous and used afterwards as the criterion of uniformity. I think it is quite certain that the first alternative is not the right one. It seems quite clear that the *meaning* of uniformity or of isochronism has nothing to do with the laws of motion. People judged certain processes, such as the swings of pendula, the burning of candles in the absence of draughts, the descent of sand in hour-glasses, etc., as isochronous long before they had thought of the question whether forces were present or absent.

We must therefore take the second alternative. This implies that, under favourable circumstances, we can directly judge equality of time-lapses, just as we can judge equality of lengths. This seems to be true. It does not of course imply that such judgments are infallible. And the question arises: Can we ever consistently correct our standard process by means of laws which are in terms originally defined by it? I think that we can and do, and that the logic of such a procedure is well worth considering.

I take it that our immediate judgment that the time-lapses between successive swings of an ordinary pendulum are equal is very approximately true, if we be at rest with respect to it. Suppose we take this as our original standard of isochrony and define uniformity by means of it, and that we find that, with this definition, the first law is verified over a wide range. This verification again will only be within the limits of experimental error. Now, suppose we apply the first

law, thus stated and thus approximately verified, to a very large number of phenomena. We may find, as we extend our observations and make our measurements more accurate, that a great number of phenomena are very approximately, but not exactly, in accordance with the first law. There are, we will suppose, small residual effects left unexplained in a number of cases. At this stage two alternatives are open to us: (1) We may keep the first law, as originally stated, and hold that small disturbing causes are operating in all the exceptional cases. We may then put forward physical hypotheses to account for these. Or (2) we may say that the first law, as originally stated, is not accurately true. Suppose we find that a single slight modification in it will account for all the slight inaccuracies in the predictions based upon it. Obviously it is more reasonable to make this one modification than to put forward different supplementary physical hypotheses in each case which the original law fails accurately to account for. Now, this modification of the first law might itself take place in two alternative ways. (*a*) We might say: "The pendulum is accurately isochronous, and under the action of no forces, bodies move with very nearly, but not quite, uniform rectilinear motions with respect to the fixed stars." Or we might say (*b*): "The swinging of a pendulum is an approximately, but not exactly isochronous process, and therefore a body that moves 'uniformly,' as judged by a pendulum, is not really moving uniformly." If we assume that the times taken by successive swings differ by a certain very small amount, we may be able to keep the form of the first law unmodified, and yet accurately explain all the facts. So, in a sense, you may say that the first law was formulated in terms of uniformity, as defined by a pendulum, and was then used to show that such "uniformity" is not quite uniform. Is there any logical objection to such a process?

Not if we clearly understand what we are doing.

We did not start by defining equality of time-lapses to *mean* the relation between the successive swings of a pendulum. We simply said that these two durations could be immediately perceived to be in fact very nearly equal. We admitted that this judgment might quite well ignore differences too small to be immediately perceived. Again, we find that, with the sense of uniformity which is based on the assumption that pendula are accurately isochronous, the first law is true within the limits of unaided observation. More extended and more delicate observations forced us either to modify the law itself, or to make a large number of supplementary physical hypotheses, or to reject the view that pendula are exactly isochronous. We preferred to take the last of these alternatives. The result is that both the law and the standard of uniformity contain a small leaven of convention and a large mass of substantial experimental fact. Uniformity is tested by a standard physical process, known to be nearly isochronous, but slightly "cooked," so as to keep the form of the first law fixed. The first law is known to be very nearly true, even when uniformity is tested by the uncorrected process; but the test for uniformity is slightly changed, so as to make the law, in its original verbal form, quite true and yet compatible with all the facts.

This mixture of convention and observation is a very common feature in scientific laws, and is unobjectionable on three conditions: (1) That, even without it, the law is verified very approximately over a very wide range; (2) that the amount of "cooking" needed is below the limits of possible direct observation; and (3) that, with it, the law keeps its original simple form, and yet now accounts accurately for all the facts without supplementary hypotheses.

The remaining ambiguous term in the first law is *Force*. Granted that the first law is not a definition of uniformity, it might still be held to be a definition of the absence of forces. If it is not to be this, but is to

be a substantial statement, the following conditions must be fulfilled. We must, in certain cases at least, be able to know whether a body is or is not acted on by forces, independently of knowing whether that body is in fact moving uniformly in a straight line in the sense defined above. For the first law says that, under the action of no forces, bodies rest or move uniformly in straight lines. If this be an experimental fact about nature it must be based on observing bodies which were known to be under the action of no forces, and finding that they always rested or moved in straight lines with respect to the fixed stars with a velocity which is uniform, as judged by some standard process, corrected, if necessary, in the way discussed above. We must therefore ask: What do we mean by *force*, and can we ever tell, apart from the laws of motion, whether forces are acting on a body or not?

To answer this question we shall need to take account of the second law of motion as well as the first. Many eminent men have held that the notion of force is needless and useless in Mechanics. Their view is that the so-called second law of motion is not the expression of an experimental fact, but is simply a definition of *force;* so that, wherever the latter word occurs in Mechanics, we can substitute for it the definition given in the second law. Now, the second law may be put in the form that the rate of change of momentum of a particle at any moment in a given direction is equal to the force which is acting on the particle at that moment in that direction. "Direction" of course involves a tacit reference to some set of axes, and "rate of change" involves a reference to some standard process for time measurement. These may be taken to be the same as those which have already been fixed upon in discussing the first law. Now, we might regard the second law in two different ways: (1) We might suppose that we already know what we mean by *force*, and already have a method of measuring its magnitude

and direction. On that view the second law is a substantial statement expressing the observed connexion in magnitude and direction between a force and the rate of change of momentum of a particle. (2) The other view is that the second law simply gives a meaning to the word "force," and defines the phrase "a force of such and such a magnitude acting in such and such a direction." The latter interpretation is, for some reason or other, considered to be tremendously hard-headed and "scientific," the former to savour of metaphysics. We shall see that, although there is a certain amount of truth underlying the second view, it is greatly exaggerated and has nothing to do with any antithesis between "science" and "metaphysics."

It seems clear to me that no one ever does *mean* or ever has *meant* by "force" rate of change of momentum. It is certain that the second law, as originally stated, was not intended for a definition of force but for a substantial statement about it. Unquestionably the sensational basis of the scientific concept of force is the feelings of strain that we experience when we drag a heavy body along, or throw a stone, or bend a bow. I do not understand that this historical fact is denied by the upholders of the "descriptive" (or better, "definitional") theory. What they would probably say is that, in this sense, force is purely human and has no relevance to the laws of Mechanics. We cannot seriously suppose, *e.g.*, that the sun feels a strain in keeping the earth in its orbit, as we do when we whirl a weight on a string. Hence it is argued that what we mean, when we say that the sun exerts a force on the earth, cannot be derived from the experiences of strain which we feel. I think there are two answers to this: (1) We must distinguish between our feeling of strain and the strains that we feel, just as we must distinguish between our feeling of movement and the movement which we feel ourselves to be making. Force is not supposed to be our *feelings* of strain; it is simply

supposed that the strains which we feel are forces, or are indications of forces. It is of course absurd to suppose that the sun feels a strain when it pulls the earth; but this is absurd, not because the sun could not be *subject to* a strain, but because—having no mind—it cannot *feel* a strain or anything else. It is thus perfectly consistent for a man to describe forces as the sort of factors in nature which reveal themselves to us directly in our feelings of strain, and to add that inanimate bodies, like the sun, are subject to forces. (2) The argument under discussion, if pressed, would make it as unreasonable to say that an inanimate body like the earth is round or rotates as to say that it is acted on by forces. For there is no kind of doubt that our concepts of roundness and rotation are founded upon sensations of sight and touch. If I had not had sensations of round or approximately round objects, I should no more know what *roundness* means than a colour-blind man knows what *red* means. The person who uses the argument about the sun not feeling strains, as an objection to the view that the feeling of strain is the sensational experience which gives a meaning to the concept of force, may be invited to consider the following parallel argument: "How can the concept of roundness be based on our sensations of sight and touch when the earth, which can neither see nor feel, is admitted to be round?" The answer of course is that the earth *has* the sort of properties which we have become acquainted with by seeing and feeling, and that it does not need to see or feel in order to have them. Similarly, there seems to be no reason why the earth should not be subject to forces which it does not feel, whilst forces are the sort of natural facts which we become acquainted with through our feelings of strain.

I think then that we may quite reasonably hold that the strains that we feel are the original sensational data on which we have based the concept of physical

force, just as coloured and shaped patches sensed by us are the original sensational data on which we have based the concepts of physical shapes and colours. The descriptive theory simply puts our sensations of sight and touch into a quite irrationally privileged position as compared with our sensations of strain. We shall see later on, what amount of *practical* justification there is for this procedure.

Now, even if we confine ourselves to the crude data of muscular sensation, we can distinguish the factors of direction and magnitude. We have to exert ourselves more to throw a heavy hammer than to throw a small stone with the same velocity. And to make a thing move in a given direction we have got to adjust our bodies so as to push, pull, or throw it in that direction. Thus force, as actually sensed in our feelings of strain, is obviously in rough general agreement with the second law, when the surface of the earth is taken as our spatial axes and any common rate measurer as our standard of time. The trouble, of course, is that felt strains are, and remain, vague both in magnitude and direction. Moreover, most of the forces with which we have to deal in science are not felt by us as strains. We cannot, then, base a satisfactory scientific measure of force on felt strains. But this is not a peculiarity of strains. It is equally true of felt temperatures. The *meaning* of temperature and of force is derived from felt hotness and felt strain respectively. A person who had no such sensations would not understand these terms at all. Again, both these felt characteristics have a perfectly noticeable though vaguely discriminated intensive magnitude. We want to define methods of measurement in each case, which shall agree in the main with our rough immediate judgments, but shall be capable of much greater accuracy, and of application to cases where the sensations cannot be got at all. This is what a thermometer does for us, in the instance of temperature; but no one "except a fool or an advanced thinker"

(to quote Mr Bradley) imagines that what we *mean* by temperature is the height of a column of mercury.

In any case, then, the second law is not a statement of what is meant by *force*. But it might still be merely a statement of how force is to be *measured* for scientific purposes. It will be so if the one and only way of measuring force is by measuring rate of change of momentum. If, however, there be any independent way of accurately determining the direction and magnitude of a *force*, the second law will be neither a definition of force nor a mere statement as to how it is to be scientifically measured. It will be a substantial statement about force. Now I think it is quite evident that, in favourable cases, we can measure force without reference to rate of change of momentum. Suppose a number of strings are attached to a body; that they then pass over pulleys; and have weights attached to them. Then the momentary directions of the strings give a clear and measurable meaning to the directions of the forces, and the weights give a clear measure of their magnitudes. And these magnitudes and directions are: (1) in fair agreement with what our sensations of strain tell us in all cases where a comparison can be made; (2) are far more accurate and definite, and can be determined in cases where we cannot get sensations of strain; and (3) are quite independent of all reference to rate of change of momentum. The second law is, therefore, neither a definition nor a statement as to how force is to be measured; but is a substantial proposition, asserting a connexion between two independently measurable sets of facts in nature. Of course, once this connexion between the magnitude and direction of a force on the one hand and the rate of change of momentum of a body on the other has been established from a study of those favourable cases where force can be measured independently, we can use the law to measure indirectly the forces which are acting in unfavourable cases, where direct measurement is impossible.

If I want to find the pull on a string which is whirling a weight, my best plan *now* is to find the angular velocity of the weight and its mass; to determine from these data its rate of change of momentum; and to equate the magnitude of the pull to this. But I now use this method, not because I *mean* rate of change of momentum by "force"; nor because this is the only possible way of *measuring force accurately;* but because, in the past and in more favourable cases, I have been able to measure force independently, and have found it to be proportional to rate of change of momentum.

So far then we have not seen anything in favour of the "descriptive" theory of force. Yet I believe that an important truth underlies it, and that it has been obscured by carelessness of statement. The typical descriptionist generally combines the two views that *force* just means rate of change of momentum and that force is not ultimately a very important conception in Mechanics. He often gives the former as a reason for the latter proposition. We have seen that the former is false. And in any case it is inconsistent to combine it with the latter. For, if *force* just means rate of change of momentum, and if force be unimportant in Mechanics, it follows inevitably that rate of change of momentum is unimportant in Mechanics. And no one in his senses would maintain this proposition. I believe the truth to be that force *is not* ultimately a very important conception in Mechanics; although this is not implied by the view that *force* means rate of change of momentum, and although that view about the meaning of *force* is mistaken.

I will now try to explain why I hold this. To know what forces are acting on a body you need to know what other bodies, near and far, are made of, what physical and chemical states they are in, and so on. For instance, when magnetic forces are under discussion, it is vital to know whether the moving body and those in its neighbourhood are made of iron or of wood, and

so on. Again, when motion is produced by impact or impeded by friction, it is vital to know the elasticities of the bodies and the state of their surfaces. Now, when we reflect on the special laws of nature which involve these special properties that vary from one bit of matter to another, we notice that force simply acts as a kind of middle term between the special laws of nature and the general laws of motion ; and that, except for convenience of expression, it might be dropped. You may regard the laws of motion as being expressed by equations, with force on one side and rate of change of momentum on the other. You may regard the special laws of nature as being expressed by equations, with forces on one side and the special configurations, electric charges, magnetic properties, etc., of the bodies that you are dealing with, on the other. Thus you might just as well express the facts by a single set of equations, directly connecting the configurations, charges, etc., with the rate of change of momentum, and drop the mention of force altogether. In practice this is what we generally do when we get the final equations for solving any particular problem. To take a very simple case, the final set of differential equations for the motion of a particle in a central orbit contains nothing that stands for force. They connect the rate of change of momentum of the particle directly with the mass and distance of the attracting central body, and with the gravitational constant.

Why then do we trouble to keep the concept of force, and why were the laws of Mechanics stated in terms of it? The main advantage of keeping it is when we want to make general statements. We want to be able to state and discuss the general laws of motion, without reference to any particular cause which produces or modifies motion. It is then convenient to lump together every such cause under the common name of force. Again, we want to be able to state the special laws of nature (*e.g.*, those of electricity or magnetism),

without referring to the particular motion of some definite body in some definite system of other bodies. It is then convenient to use the term force for the effect of any such system on a hypothetical particle of unit mass. When we pass from general statements to some definite problem the notion of force becomes useless and drops out. Now many, though by no means all, material systems which affect the motions of a body also cause feelings of strain in our own bodies. That is why force does not appear to us as a mere mathematical parameter, although this is the position that it actually comes to occupy in the treatment of concrete problems. Lastly, material systems which affect the motions of bodies do also produce other measurable effects, such as balancing weights on strings over pulleys, or stretching spring-balances. The first and second laws are really statements about the observed relations between these latter effects of material systems and their effects in modifying the motions of bodies.

We have now cleared up the notion of force, so far as it is common to the first and second of the traditional laws of motion. But the second law involves another concept, viz., that of *mass*, and this we must now discuss. The *momentum* of a body is defined as the product of its velocity by its mass. All that we need say at present about its velocity is that its magnitude and direction must be determined with reference to a suitable set of material axes, such as those given by the fixed stars, and a suitable physical time-measurer, such as an ordinary pendulum.

The factor of mass actually enters into the traditional Mechanics in two quite different ways; and it is simply a strange coincidence that the two kinds of mass are proportional to each other, so that, by a suitable choice of units, the two masses of a body have the same measure. We may call the two kinds of mass *gravitational* and *inertial* respectively. The first is the mass that is mentioned in the law of gravitation, the second

is the mass which is involved in the second law of motion. At present we shall deal with inertial mass, a factor which occurs equally in every kind of motion, whether produced by impact, gravitation, electric or magnetic attraction, or any other cause. We will start, as we did in treating force, with the crude data of sensation, and consider what feature it is in these which forms the basis of the scientific concept of inertial mass. If we take two bodies which are geometrically exactly alike, say a sphere of wood and an equal sphere of platinum, we may find that we have to exert ourselves to a markedly different extent to make them move with the same velocity relative to the same axes and the same time-measurer. We have already seen that, with a single body, *e.g.*, the wooden sphere, we have to exert ourselves more the faster we wish to make it move. We see then that the effort that we feel ourselves exerting when we try to make a body move depends on two factors. One of these is the velocity which we give to the body. The other is a factor which apparently depends simply on the material of the body itself. It is the latter which gives us the primary meaning of *inertial mass*. As usual, the crude data of sense only allow of a very crude measure of magnitude. We therefore need some method of measuring mass which shall agree *pro tanto* in its results with the rough judgments based on our experiences of effort, but shall be capable of much greater accuracy.

Experiments on the impact of bodies give us a means of accurately measuring inertial mass in favourable cases. When two bodies B_1 and B_2 hit each other, it is found that we can ascribe a numerical coefficient m_{12} to B_1 and a coefficient m_{21} to B_2, such that, if u_1 and u_2 be their respective velocities before and v_1 and v_2 their respective velocities after the collision

$$m_{12}u_1 + m_{21}u_2 = m_{12}v_1 + m_{21}v_2.$$

What we have learnt at this stage is that (1) the two

coefficients are independent of the velocities u_1 and u_2. And (2) that, for *any* pair of bodies, such a pair of coefficients can be found. But, suppose that we first try the experiments with a pair of bodies B_1 and B_2, and then with B_2 and a third body B_3. It is antecedently possible that m_{21}, the coefficient which has to be ascribed to B_2 in its transactions with B_1, might differ from m_{23}, the coefficient which has to be ascribed to B_2 in its transactions with B_3. Further experiments prove that this is *not* so, *i.e.* that the coefficient of any given body is independent, not only of its velocity, but also of the other bodies with which it is interacting. We can thus in future drop doubly-suffixed coefficients, like m_{21}, and write simply m_1, m_2, etc. We find then that to any body there can be ascribed a certain coefficient, which is independent of its velocity, and which it carries with it into all its mechanical transactions with other bodies. This coefficient is the scientific measure and meaning of inertial mass. It obviously accords in rough outline with the notion of mass which we get from our sensations of effort, but it is capable of accurate measurement. Having defined and measured the inertial mass of a body in this way, we find two further important facts about it by experiment. (1) It belongs to a body, not only in the case of motions caused by impact, but in all its motions however produced or modified. (2) Such coefficients are additive scalar magnitudes. If you do experiments with a compound body, made up of two smaller ones, to which you have already ascribed the masses m_1 and m_2, you will find that you have to ascribe to this compound body the mass $m_1 + m_2$.

We can now deal with gravitational mass. All bodies, no matter what their inertial mass may be, fall to the ground with the same acceleration *in vacuo* in the same region of the earth. Now the rate of change of momentum of a body of constant mass is equal to the product of its mass by its acceleration.

Since bodies of different inertial mass all fall with the same acceleration, it follows from the second law that they must be acted on by unequal forces, and that these forces must be proportional to the inertial masses of the bodies. Again, if we do experiments with a delicate torsion balance, we find that the attraction of a body A on a body B is proportional to the inertial mass of A. Combining these two facts we see that the gravitational attraction between any two bodies is proportional to the product of their inertial masses. It is evident then that, even if we had never done experiments with moving bodies at all, but had confined ourselves to statical experiments with balances, torsion apparatus, etc., we should have come to ascribe certain coefficients to every body. We should also have found that these coefficients were independent of the velocity, chemical or physical state, etc., of the body to which they were ascribed, and were moreover independent of the other bodies with which it was interacting. And these coefficients would have been additive. They would, in fact, be proportional to the inertial masses; and therefore, with a suitable choice of units, identical with the latter. Now, the coefficients required by the gravitational facts are what we mean by gravitational masses; and, on the traditional theory, it is just a strange coincidence that the two masses of a body are proportional to each other. The theory of gravitation which is bound up with the General Theory of Relativity suggests a reason for this identity of inertial and gravitational mass.

We must next consider the third law of motion, which says that action and reaction are equal and opposite. It involves no new concepts, but it makes a most important additional statement about force. It says, in fact, that the force on one particle is only one side of a transaction which, taken as a whole, is a stress between two particles. It is in virtue of this principle that we are able to deal with the motions

of finite rigid bodies, which rotate as well as change their places, and therefore cannot be treated as particles. The law, as stated, is indefinite both as to direction and as to time. The action and reaction between two particles might be equal and opposite, but might make any angle with the line joining them. It seems to be sometimes assumed that the law requires the direction of the two forces to be the line joining the particles. This is not so, and the law would be false if it were. Two moving electrons exert equal and opposite forces on each other, but these are not in the line joining the two electrons. In fact the question of the direction of the two opposite and equal forces belongs to the special laws of nature, such as gravitation, electricity, magnetism, etc., and not to the general laws of motion. Again, I think it is often assumed that action and reaction are always contemporary. If the law be understood to assert this, it is certainly false, unless we supplement it by assuming particles of ether and a mechanical theory about stresses among them. When a beam of light from the sun strikes upon any surface on the earth it produces a pressure on that surface. If there be any reaction from the earth it will be exerted primarily on the surface of the ether next to the earth, and will not be conveyed back to the sun in less time than light takes to travel between the two. Thus, if you confine yourself to the earth and the sun, action and reaction are not contemporary as regards light-pressure.

The first two laws of motion have been stated with respect to motions relative to the fixed stars and to a standard time-measurer, such as an ordinary pendulum. Now, it is very important to notice that, apart from the third law, this restriction to a particular set of axes and a particular physical time-measure could be removed, provided that we introduced suitable new forces with each new frame of reference. I will illustrate what I mean by two examples: (1) Suppose that a particle is

at rest on a plane with respect to a Newtonian frame of reference, *i.e.*, with respect to such axes and such a time-measurer as we have hitherto been assuming. Suppose that in this plane there lies a wheel, and that we take two mutually normal spokes of this wheel as our X and Y axes respectively. So long as the wheel is at rest, these two spokes and the line through the centre of the wheel perpendicular to the plane in which it lies, constitute a Newtonian set of axes; and the particle is at rest with respect to them. It is therefore under the action of no Newtonian forces. Now suppose that the wheel is spun with a uniform angular velocity ω in its own plane. Let us continue to take the two spokes as our axes, and the old clock as our time-measurer. The resulting frame is, of course, non-Newtonian, for it is neither at rest nor in uniform rectilinear motion with respect to the fixed stars. Relatively to this new frame the particle describes a circle in the X-Y plane with uniform angular velocity ω. It therefore has a relative acceleration of amount $r\omega^2$ towards the origin. But this can be made compatible with the first and second laws if we assume a force of this intensity per unit mass attracting the particle to the origin. The particle is acted on by no forces with respect to the Newtonian frame; it is acted upon by an attraction of amount $mr\omega^2$ towards the origin with respect to the new non-Newtonian frame. Thus the first and second laws have been rendered independent of special reference to Newtonian frames by the assumption that force (like position, velocity, etc.) is relative to the spatio-temporal frame of reference which is used for placing and dating the phenomena under consideration.

(2) Let us now take a slightly more complex case. Let us suppose that the particle in question is a frictionless ring which can slide along the particular spoke of the wheel that is chosen as the X-axis, and that the wheel rotates as before. Relative to Newtonian axes

the ring has no acceleration along the instantaneous direction of this spoke. Along the instantaneous direction of the normal to it, it has an acceleration $2\dot{x}\omega$. It is therefore acted upon by a Newtonian force (viz., the pressure of the spoke pushing it from behind) of amount $P = 2m\dot{x}\omega$. How will this appear to people who rotate with the wheel? Relatively to their axes, the particle will move along the X-axis with an acceleration $\ddot{x}$, whilst it will have no velocity or acceleration along the Y-axis. They will therefore have to say (if they want to keep the form of the first two laws of motion) that the ring is repelled from the origin with a force $m\ddot{x}$. And it is easy to show that the intensity of this must be $mx\omega^2$, *i.e.*, it will be a force varying directly with the distance of the particle from the origin. On the other hand, they will have to say that there is no resultant force acting on the ring in the direction of their Y-axis. For the ring keeps all the time to the X-axis. But, if they measured, they might be expected actually to find the pressure P acting from the spoke to the ring. How would they get over this? They would say: "The spoke attracts the ring with a force equal to P, and this just balances the pressure of the spoke on the ring." Thus by assuming a repulsive force from the origin, varying directly with the distance, and an attractive force between the ring and the spoke, varying directly with the velocity along the spoke, they could reconcile the form of the first two laws with their non-Newtonian frame of reference. This latter force would indeed be of a curious kind, for particles would be attracted by the side of the spoke that faced the direction of rotation and repelled by the other face, but they could deal with this by something like a "two-fluid theory."

In these two examples we have only partially departed from a Newtonian frame of reference. We have taken non-Newtonian axes but have kept to a Newtonian clock. It is obvious that, if we kept Newtonian axes but took a

non-Newtonian clock, we could equally preserve the form of the first two laws by introducing suitable non-Newtonian forces. Suppose a particle were moving with a uniform rectilinear velocity with respect to a Newtonian frame. Suppose that we then substituted for a pendulum clock a water-tank with a hole in it as our time-measurer, and judged equal times as those in which equal masses of water flowed from the tank. Let us keep the Newtonian spatial axes this time. As the head of water in the tank decreases the water flows out more slowly, as judged by a Newtonian clock. It follows that, at the latter part of the experiment, the particle will move further while a pound of water flows out of the tank than it did at the beginning. Hence, with respect to our new non-Newtonian clock, the particle will be moving with an accelerated rectilinear motion. If we want to keep the form of the first two laws we shall therefore have to introduce a non-Newtonian force, acting in the direction of motion of the particle.

It should now be evident that, so far as concerns the first two laws of motion, their form can be kept, irrespective of the frame of reference chosen, provided we admit the (at any rate partial) relativity of forces to frames of reference. It remains to consider more carefully the nature of the non-Newtonian forces that would have to be introduced with non-Newtonian frames of reference. In particular we want to know whether the third law can be kept too when we give up the restriction to Newtonian frames. One thing we notice at once. That is that the non-Newtonian attractionsand repulsions, which were introduced by the adoption of non-Newtonian frames of reference, are all proportional to the inertial masses of the particles on which they act. Again, they act on every particle under consideration, regardless of its physical or chemical peculiarities, of the medium in which it may happen to be, and so on. Now this reminds us irresistibly of gravitational attractions ; and suggests, as it did to Einstein, that the law of gravitation may

have some connexion with these non-Newtonian forces which are bound up with non-Newtonian frames of reference. Compare *e.g.*, the two cases of a heavy body resting on a weighing machine, and the ring in the second example. The heavy body rests in a Newtonian frame, and yet the spring of the machine is compressed, thus indicating that an upward thrust is being exerted by the spring on the heavy body. We say that this thrust must be balanced by a pull downwards on the body, and we ascribe this pull to the gravitational attraction of the earth. In exactly the same way we found that the observers who used the rotating wheel as their spatial axes would have to assume an attraction between the ring and one side of the spoke, to account for the fact that the ring did not move at right angles to the spoke in spite of the observable pressure of the latter on the former. Lastly, consider the repulsive force from the origin which the observers on the moving wheel would have to suppose to be acting on the ring. The peculiarity of this is that to all appearance it does not obey the third law. There is a *field* of force, to which every particle is subjected when referred to the axes in question; but it cannot be said that the force on one particle is balanced by an equal and opposite force on another particle. Some non-Newtonian forces then, it would seem, do not obey the third law. Thus it seems that the first two laws are more general than the third, since they can be reconciled with any frame of reference by the introduction of suitable forces, whilst it is only for Newtonian forces that the third law holds universally. This conclusion could however, in theory, be avoided by the introduction of hypothetical concealed masses; so that the non-Newtonian forces on observable masses might be regarded, as the third law requires, as one side of stresses between these observable masses and the hypothetical concealed ones. Thus all the laws of motion can be formally preserved relative to any frame of reference, provided it is assumed that new

frames imply new forces, and provided that we are allowed to assume such concealed masses as we need.

I will end this chapter by trying to make clear the difference between the laws of motion and the special laws of nature, such as those of electricity or magnetism or heat. We shall then see that, on the traditional view, the law of gravitation occupies a curious position, intermediate between the two sets of laws.

The laws of motion do not profess to tell us in detail how motions are caused or modified. What they do is to tell us the general conditions which all motions, however produced, must conform to. They take no account of the kind of matter which is moved, or of its physical or chemical state at the time; the one property of matter, other than purely geometrical properties, which appears in the laws of motion is inertial mass. The special laws of nature, on the other hand, tell us about the various causes of motion. They have to take into account all sorts of properties of bodies beside their inertial masses. They have to consider whether they be electrically charged or not, whether they be hot or cold, magnetised or unmagnetised, and what sort of medium surrounds them. Now, the law of gravitation, on the traditional view, is in one way like a special law of nature, and, in another way, more like the general laws of motion. It professes to tell us one of the causes which start and modify motions. So far it resembles a special law of nature. But the only property of matter that it has to consider is common to all matter, viz. gravitational mass. And this proves to be identical with the one property which is considered in the laws of motion, viz. inertial mass. Thus there seems to be a very much closer connexion between the laws of motion and the law of gravitation than between any of the special laws of nature and the laws of motion. Again, if we are in earnest with the Relational Theory of Motion, we must suppose that all the motions with which Mechanics deals take place with respect to

material axes. And, since all matter attracts all other matter gravitationally, on the traditional view, all bodies will be attracted more or less by the axes to which their motions are referred. It thus seems not unlikely antecedently that there should be a very close connexion between the laws of motion and the law of gravitation, and that a completely Relational system of Mechanics should contain a theory of gravitation. The details of this are reserved for the next chapter, but it is hoped that the foregoing discussion of the traditional laws of motion and gravitation may have brought the reader into a proper frame of mind for understanding and criticising the General Theory of Relativity.

The following additional works may be consulted with advantage:

B. A. W. Russell, *Principles of Mathematics*, vol. i, Part VII.
E. Mach, *Science of Mechanics*.
H. Poincaré, *La Science et l'Hypothèse*.
,, *Science et Méthode*.
,, *Le Valeur de la Science*.
P. Painlevé, *Les Axiomes de la Mécanique*. (Paris. Gauthier-Villars.)

CHAPTER VI

"What's the use of Mercator's North Poles and Equators,
Tropics, Zones, and Meridian Lines?"
So the Bellman would cry; and the crew would reply;
"They are merely conventional signs!"

"Other maps are such shapes, with their islands and capes!
But we've got our brave Captain to thank,"
(So the crew would protest), "that he's bought us the best—
A perfect and absolute blank!"

(LEWIS CARROLL, *The Hunting of the Snark*.)

Modification of the Traditional Kinetics (*continued*). (2) The General Theory of Relativity. Summary of Part I

IN the last chapter we treated the traditional laws of motion without reference to the kinematic results of the Special Theory of Relativity, outlined in Chapter IV. That is to say, we combined the traditional Kinetics with the traditional Kinematics. We must now take a step forward, and show that the traditional laws of motion are not compatible with the modified kinematics of even the Special Theory of Relativity. We shall then be able to advance to the General Theory.

There is no need for me to treat the kinetics of the Special Theory in any detail, because it is only a half-way house to the General Theory. I will therefore content myself with a single example to show that the traditional laws of motion cannot be reconciled, without modification, with the kinematics of the Special Theory and with the Restricted Physical Principle of Relativity.

Let us suppose that two sets of observers were doing

experiments to determine inertial mass by the impact of bodies, as described in the last chapter. One shall be on the platform p_1 and the other on the platform p_2 of Chapter IV. These platforms are in uniform rectilinear relative motion in a Newtonian frame. The velocity of the first with respect to the second, as measured by observers on the second, is v_{12}. Let two bodies be moving along p_1 in the direction in which p_1 is itself moving relatively to p_2. Let their velocities relative to p_1, as measured by observers on it, be U_1 and u_1 respectively, before they hit each other. After they have hit, let their velocities with respect to p_1 be W_1 and w_1 respectively. Let the observers on p_1 ascribe to these bodies the inertial masses M_1 and m_1 respectively. As we saw in the last chapter,

$$M_1U_1 + m_1u_1 = M_1W_1 + m_1w_1. \qquad (1)$$

Each body has its own coefficient, which it keeps when its velocity is altered by the collision, and which is independent of its initial velocity. There is no doubt that this is very approximately true under ordinary conditions of experiment; the question is whether it can be *exactly* true, consistently with the Physical Principle of Relativity and the kinematics of the Special Theory.

Let the whole experiment be also watched by the observers on p_2. Let the velocities which they ascribe to the bodies relatively to p_2 be U_2, u_2, W_2 and w_2 respectively. The Physical Principle of Relativity tells us that if equation (1) expresses a genuine law of nature in terms of the observations of people on p_1, the people on p_2 must be able to find an equation of precisely the same form in terms of *their* observations on the same phenomena. That is, they ought to find that their observed relative velocities are connected by an equation

$$M_2U_2 + m_2u_2 = M_2W_2 + m_2w_2. \qquad (2)$$

In this equation M_2 and m_2 will have to be independent of the velocities of the bodies; for it is obvious that

the form of the law would *not* be the same for both sets of observers, if, in the one case, the coefficients were constants, and, in the other, were functions of the velocity of the body.

Now it is easy to see that anything of the kind is inconsistent with the kinematics of the Special Theory of Relativity. If the reader will look back to equation (3) in Chapter IV he will see that

$$U_2 = \frac{U_1 - v_{21}}{1 - \frac{U_1 v_{21}}{c^2}},$$

with similar equations, *mutatis mutandis*, for u_2, W_2 and w_2. It is quite obvious that, if these values be substituted in equation (2), we shall reach a result which is inconsistent with equation (1), on the assumption that the masses are independent of the velocities. It follows that the traditional view that mass is independent of velocity cannot be reconciled with the Physical Principle that genuine laws of nature have the same form for all observers who are in uniform rectilinear relative motion, and with the kinematics of the Special Theory of Relativity. It is not difficult to see what modification is needed. Let us denote by $M_{1,U}$ the mass which has to be assigned to a body moving with a measured velocity U_1 relatively to the Newtonian frame p_1. Let us put

$$M_{1,U} = \frac{M_0}{\sqrt{1 - \frac{U_1^2}{c^2}}} = K_{1,U} M_0, \quad M_{1,W} = \frac{M_0}{\sqrt{1 - \frac{W_1^2}{c^2}}} = K_{1,W} M_0;$$

and

$$m_{1,} = \frac{m_0}{\sqrt{1 - \frac{u_1^2}{c^2}}} = k_{1,u} m_0, \qquad m_{1,w} = \frac{m_0}{\sqrt{1 - \frac{w_1^2}{c^2}}} = k_{1,w} m_0, \quad (3)$$

where M_0 and m_0 are independent of the velocity. Let us then see whether the equation

$$M_{1,U} U_1 + m_{1,u} u_1 = M_{1,W} W_1 + m_{1,w} w_1 \qquad (4)$$

expresses a possible law of nature, consistent with the Physical Principle of Relativity and the kinematics of the Special Theory. If it does, we ought to find that the measured velocities U_2, etc., which the observers on p_2 ascribe to the bodies under experiment, are interconnected by the equation

$$M_{2,U}U_2 + m_{2,u}u_2 = M_{2,W}W_2 + m_{2,w}w_2. \qquad (5)$$

By using the transformation equation for relative velocities, and doing a little tedious but quite straightforward algebra, the reader will be able to see for himself that this is so, on one condition. The condition is that the *total* mass of the system in the direction of motion is unaltered by the collision, *i.e.*, that

$$M_{1,U} + m_{1,u} = M_{1,W} + m_{1,w}. \qquad (6)$$

On the traditional view this is of course a merely analytical proposition, since it is part of that view that the mass of *each* body is an absolute constant. On the present view of mass, it is an additional assumption. The law, obtained by combining (4) and (6) with the definitions embodied in (3), is then a permissible law of nature, whilst the traditional law embodied in (1) is not. The assumption (6) is, to a very high degree of approximation, equivalent to the assumption that the total kinetic energy of the system is unaltered by the collision. For

$$M_{1,U} = \frac{M_0}{\sqrt{1 - \frac{U_1^2}{c^2}}} = M_0 + \frac{1}{2}\frac{M_0 U_1^2}{c^2} \text{ very nearly.}$$

Whence (6) practically reduces to

$$\frac{1}{2}M_0U_1^2 + \frac{1}{2}m_0u_1^2 = \frac{1}{2}M_0W_1^2 + \frac{1}{2}m_0w_1^2. \qquad (7)$$

Thus the attempt to express the laws of Mechanics in a form which is consistent with the kinematics of the Special Theory of Relativity leads to a connexion between the three principles of the Conservation of

Momentum, of Mass, and of Energy, which was not obvious on the traditional view.

The modified conception of mass, which the Special Theory of Relativity requires, differs so little in magnitude from that of the traditional view, for all ordinary velocities, that it is reasonable to suppose that the modified laws are not merely admissible in form but also true in substance. Moreover, the modified laws agree with observations on the motions of electrons, shot out with enormous velocities in vacuum tubes; whereas the traditional form of the law cannot be brought into accordance with these results, except by the help of supplementary *physical* hypotheses about the charges, shapes, etc., of the particles.

The General Theory of Relativity. Enough has now been said to show that the traditional kinetics needs modification as soon as the traditional kinematics is dropped and that of the Special Theory of Relativity is substituted for it. And, as I have tried to show in Chapter IV, the negative results of the Michelson-Morley and other experiments leave us no option about making *at least* this substitution. The question now is, not whether we shall go so far, but whether we ought not to go further still. Let us open the subject by asking: In what way is the Special Theory of Relativity *special?*

The answer to this question is obvious. In discussing the Special Theory of Relativity we explicitly confined ourselves to Newtonian frames. In the first place, our kinematic transformations assumed that the two platforms p_1 and p_2 were in *uniform rectilinear* relative motion. We did not deal at all with the case of p_2 rotating with respect to p_1 or moving with a rectilinear but accelerated motion with respect to p_1. But this is not all. If one frame be Newtonian and another moves with a uniform rectilinear motion relatively to it, the second is also Newtonian. But the converse of this is not true. Two platforms might be in uniform rectilinear

relative motion, but neither of them need, for that reason, be Newtonian. *E.g.*, if their clocks were non-Newtonian (*e.g.*, were water-tanks, as in a previous example) both these platforms would have accelerated rectilinear motions in a Newtonian frame, and therefore neither of them would be a Newtonian set of axes. Again, suppose that p_1 and p_2 were attached at different distances from the centre to the same spoke of a wheel which rotated uniformly in a Newtonian frame. There would be no relative motion between them, but neither of them would be Newtonian axes. So the "speciality" of the Special Theory is that it is wholly concerned with Newtonian frames; and this not only restricts the transformations to uniform rectilinear relative motion, but imposes a further condition, in virtue of which one at least of the set is known to be Newtonian.

How does this limitation show itself? The fundamental fact on which the kinematic transformations of the Special Theory was based was that light was found to travel with the same velocity, and in a straight line, relative to all the observers, although they were in motion relatively to each other. It is quite obvious that, if observers had chosen the spokes of a rotating wheel as their axes, they would not have found that light travelled in straight lines with respect to them. And, if they had taken as their time-measurer some process which was not isochronous as compared with a Newtonian clock, they would not have found the velocity of light to be uniform, even though they had used the fixed stars as their axes. A Newtonian frame may then be defined in one of two alternative ways: (1) It is a set of axes and a physical time-measurer with respect to which light in a homogeneous medium travels with a uniform rectilinear velocity. Or (2) it is a set of axes and a time-measurer with respect to which a particle, under the action of no resultant force, rests or moves uniformly in a straight

line. Owing to the universality of gravitation the second criterion cannot literally be applied. We shall also see, later on, that the same reason renders the first criterion not strictly true of any natural frame. Thus a Newtonian frame is an ideal limit rather than an actual fact. Still, the frame in which the fixed stars form the axes and a properly constructed and regulated clock forms the time-measurer is very nearly Newtonian for all experiments that we can do. The transformation equations of the Special Theory enable us to pass from the place and date of any event in any one such frame to its place and date in any other such frame. But they tell us nothing about its place or date in any frame which is not Newtonian; and no frame is Newtonian unless its axes either rest or move with a uniform rectilinear velocity, as judged by a Newtonian clock, relatively to Newtonian axes. Again, the Restricted Physical Principle of Relativity only says that observers on different Newtonian frames will all find laws of identical form for the same natural phenomena. It does not assert that an observer on a non-Newtonian frame will find no difference in the form of the laws which interconnect the magnitudes that he measures, when watching a certain natural phenomenon.

The question is whether, and to what extent, this restriction to a certain set of frames of reference can be removed. It is easy to state in general terms the *kind* of problem with which we are faced. On the one hand, we can get at the laws of nature only by measuring various observable magnitudes and finding out the functional correlations that hold between them. And we can do this only by referring all events in nature to a spatio-temporal frame of reference of some kind, in which each event has a certain place and date. Innumerable different frames of reference could be taken for dating and placing the events of nature. On the other hand, presumably there are laws

of nature which are absolute, and independent of any particular frame of reference. The laws discovered by observers who use a certain frame of reference will be transcriptions of these absolute relations, in terms of that particular frame. Thus, we may suppose that they will depend partly on the absolute relations of events in nature and partly on the particular frame used by these observers. It would thus be reasonable to suppose that, on comparing the laws discovered by observers who observe the same phenomenon and use all kinds of different frames of reference, we might be able to extract a kind of "kernel," which should be neutral as between them all. This kernel would be the absolute law of the phenomenon in question, and it is this which the General Theory of Relativity seeks to extract.

It may be worth while to give a few illustrations from other regions, in order to make the idea familiar to the reader. (1) Suppose the League of Nations were to lay down certain general rules about navigation, which were binding on all members of the League. They would have to be translated into English, French, Italian (and soon, one hopes, German and Russian). These various translations would look extremely different. And it would be impossible to express the rules without *some* symbolism or other until telepathy becomes commoner than it now is. Yet there would be something, viz., the content of the rules, which would be independent of *any particular* language or other system of symbols in which they happened to be expressed.

(2) Another example may be helpful to persons with an elementary knowledge of mathematics. It is a very simple intrinsic property of the triangle that the bisectors of its three angles all meet at one point. If you try to prove this by analytical geometry you will have to choose some set of co-ordinates; they may be rectangular Cartesians, or oblique Cartesians, or polars.

In any case you will get very complicated equations in terms of the co-ordinates which you assign to the three corners of the triangle. And these equations will be very different according to the system of co-ordinates that you have chosen for reference. Yet they all express the same simple fact, which is intrinsic to the triangle as such, and quite independent of any set of co-ordinates.

Now, on the traditional view, the distance between two events and the time-lapse between them are two distinct facts. It is true that, on the traditional view, the measured distances between non-contemporary events will be different for observers who are in uniform rectilinear motion with respect to each other. But it is supposed that their dates will be the same for all Newtonian frames, and that it will be independent of the distance between the events. Now, the Special Theory shows that this is not true even when we confine ourselves to Newtonian frames. We saw that observers on platforms which are in relative rectilinear uniform motion will not ascribe the same time-lapse to the same pair of events; and that, if these events be separated in space, the amount of time-lapse ascribed to them by observers who move relatively to them will depend on their distance apart. Thus, measured distance between events and measured time-lapse between events are mixed up with each other, and are partly dependent on the frame of reference, even when we confine ourselves to Newtonian frames. Is there anything connected with spatial and temporal separation which has the same measure for all Newtonian frames? There is, as can easily be seen. Suppose that two adjacent events have respectively the co-ordinates and dates x_1, y_1, z_1, t_1, and x_1+dx_1, y_1+dy_1, z_1+dz_1, and t_1+dt_1 with respect to the Newtonian frame p_1. Let them have the corresponding letters, with 2 suffixed instead of 1, with respect to the frame p_2, which moves relatively to p_1 in the x-direction with the uniform

velocity v_{21}. It follows immediately from the transformation equations of Chapter IV that

$$dx_2 = k_{21}(dx_1 - v_{21}dt_1)$$

and $$dt_2 = k_{21}\left(dt_1 - \frac{v_{21}}{c^2}dx_1\right).$$

Whence

$$dx_2^2 - c^2dt_2^2 = k_{21}^2\left\{dx_1^2\left(1 - \frac{v_{21}^2}{c^2}\right) - c^2dt_1^2\left(1 - \frac{v_{21}^2}{c^2}\right)\right\} = dx_1^2 - c^2dt_1^2,$$

since $$k_{21} = \frac{1}{\sqrt{1 - \frac{v_{21}^2}{c^2}}}$$ by definition.

Now $dy_1^2 = dy_2^2$ and $dz_1^2 = dz_2^2$, since there is no relative motion in these directions. Therefore finally,

$$dx_1^2 + dy_1^2 + dz_1^2 - c^2dt_1^2 = dx_2^2 + dy_2^2 + dz_2^2 - c^2dt_2^2. \quad (8)$$

Here then we have a magnitude, connected with a pair of events, which has the same numerical measure with respect to all Newtonian frames. We will take this magnitude with its sign reversed, for reasons which will appear later. We will call it the square of the *Spatio-Temporal Separation* of the two events, and will denote it by $d\sigma^2$. The square of the *spatial* separation is, of course, $dx_1^2 + dy_1^2 + dz_1^2$ in the one system and $dx_2^2 + dy_2^2 + dz_2^2$ in the other. The *temporal* separation is dt_1 in one system and dt_2 in the other. It is clear that the spatio-temporal separation has a claim to represent something intrinsic to the pair of events, and neutral as between different frames of reference, which claim cannot be made for either the spatial or the temporal separation. It is, at any rate, invariant and neutral as between all Newtonian frames, whilst the other two are not invariant or neutral, even with this restriction.

It will be noticed that, if the two events be the successive occupations of two adjacent places by something that travels with velocity u_1 with respect to one

frame and u_2 with respect to the other, the spatio-temporal separation takes the form

$$d\sigma^2 = (c^2 - u_1^2)dt_1^2 = (c^2 - u_2^2)dt_2^2.$$

If what is travelling be light, or any other electromagnetic disturbance, $u_1 = u_2 = c$. Whence $d\sigma^2 = 0$. That is, the spatio-temporal separation between two events which are the successive arrivals of a wave of light at two adjacent positions is 0, although of course both the spatial and the temporal separations of the two events are finite. This explains why we took the expression with its sign reversed. We want the square of the separation to be always positive for the successive events that constitute any real motion. With the present choice of sign this will be so, unless the moving thing travels faster than light. With the other choice of sign the square of the separation would always be negative for anything that travelled more slowly than light. Now we know nothing that travels faster and innumerable things that travel more slowly than light. Hence our convention as to sign is justified.

This concept of spatio-temporal separation is fundamental to the General Theory of Relativity. We take it as a hypothesis that this separation is an intrinsic relation between a pair of events, which has nothing to do with frames of reference, though, of course, we shall always meet with it and measure it in terms of the particular frame that we happen to use in order to place and date the events of nature. If it be asked what ground there is for this hypothesis, I think we must begin by distinguishing between what suggests it and what justifies it. What suggests it is the invariance of this measured magnitude as between all Newtonian frames. But, if it is to be justified, this must be done in the usual way by working out the consequences of the hypothesis and seeing whether they accord with experimental facts.

We have seen what form the spatio-temporal separa-

tion takes when expressed in terms of Newtonian co-ordinates and clock-readings. It will be worth while, however, explicitly to mention the important characteristics of this expression before going further. (1) It is homogeneous and of the second degree in the four variables which it involves. (2) The coefficients of the variables are all constants. In fact, by a suitable choice of units, they could all be reduced to unity. When distance is measured in centimetres and time-lapse in seconds, light has the velocity c, and the time-factor has to be multiplied by this constant. But, if the unit of time were taken to be, not the second, but $\frac{1}{c}$ of a second, the velocity of light would be unity. We chose our units of space and our units of time quite independently, when it was not suspected that there was a fundamental connexion between these two factors in nature. It so happens that we have chosen a very large unit of time as compared with the unit of space; and that is the only reason why the large constant c appears in the expression for the spatio-temporal separation. (3) The last important point to notice in this connexion is that the coefficient of the time-variable is of opposite sign to that of the space-variables in the expression for the spatio-temporal separation. This betrays the fact that there is ultimately a radical distinction between the space factor and the time factor in nature, in spite of their intimate interconnexion, and in spite of the fact that the two are, within certain limits, interchangeable.

Now we can quite well understand that the expression for the spatio-temporal separation, in terms of the co-ordinates and time-readings of a non-Newtonian frame, may be very different from the expression for the same fundamental fact in terms of a Newtonian frame. Let us first illustrate this by a very simple example from ordinary geometry. If we take the traditional view of Space and Time the distance between two points

is an intrinsic relation between them, and is wholly independent of the system of co-ordinates to which we refer the points. Let us first suppose that they are referred to a set of rectangular Cartesian co-ordinates C_1. Let their x-co-ordinates in this system be x_1 and $x_1 + dx_1$ respectively, with similar expressions for their y- and z-co-ordinates. Then the expression for the square of their distance apart is

$$dx_1^2 + dy_1^2 + dz_1^2.$$

Now refer them to another set of rectangular Cartesians C_2. This might consist of the original ones twisted as a rigid body about their origin. The three edges of a biscuit box with one corner fixed would be an example. Let the co-ordinates of the points with respect to this system be x_2 and $x_2 + dx_2$, etc., respectively. The expression for the square of the distance apart of the two points in the new co-ordinates is

$$dx_2^2 + dy_2^2 + dz_2^2.$$

It is of the same form and the same magnitude as before. This is exactly analogous to the invariance of the expression for the spatio-temporal separation of two *events* with respect to two Newtonian *frames*.

Suppose now that, instead of referring the two points to Cartesian co-ordinates, we were to refer them to polars. Call this system P_3. Let the co-ordinates of the two points be respectively r_3, θ_3, ϕ_3 and $r_3 + dr_3, \theta_3 + d\theta_3, \phi_3 + d\phi_3$, in this system. The distance apart will now be expressed by the formula

$$dr_3^2 + r_3^2 d\theta_3^2 + r_3^2 \sin^2\theta_3 d\phi_3^2.$$

It will be noticed that this expression has one important analogy to, and one important difference from, the expression in terms of Cartesians. It resembles the latter in that it is still a homogeneous function of the second degree in terms of the three differentials. It differs in that these differentials no longer all have constant coefficients. Their coefficients now contain functions of the co-ordinates themselves.

Now, just as the passage from Cartesian to Polar co-ordinates makes this difference in the expression for the distance between two points on the ordinary geometrical view, so we may expect that the passage from a Newtonian to a non-Newtonian frame of reference will make a similar difference to the expression for the spatio-temporal separation between two events. We may expect that the expression will still be homogeneous and of the second degree in terms of the differentials of the non-Newtonian co-ordinates and dates, but that these differentials will no longer have constant coefficients.

In order to make the next step, let us again revert to a simple example in ordinary geometry. Let us confine ourselves to points on a surface, and let us suppose, to begin with, that this surface is a sphere. We will suppose that persons confined to the surface of the sphere are trying to find an expression for the distance apart of two adjacent points, as measured on the surface of the sphere. This will of course be that part of the great circle passing through the two points, which is included between them. Now the surface of the sphere could be mapped out into a network of co-ordinates in innumerable different ways. We might fix the position of a point by parallels of latitude and meridians of longitude, as ordinary Atlases do. Or we might fix it by taking an origin on the equator and drawing a great circle from here to the point in question, and noticing the length of this arc and the angle that it makes with the equator. Again we might take the equator and some meridian of longitude as a pair of mutually normal axes and define the position of a point by the arcs of the two great circles which pass through it and are normal to the equator and the meridian respectively. The last-mentioned set of co-ordinates would be analogous to Cartesians in a plane, and the set mentioned before would be analogous to plane Polars. We should find

that two independent variables were always necessary to fix the position of a point. And we should find that the distance between any pair of adjacent points on the sphere, as measured along the sphere's surface, was always a homogeneous quadratic function of the small differences between their corresponding co-ordinates in any system. So far there is complete analogy with a plane. But we should find the following very important difference. In the plane, or in ordinary three dimensional Space, as we saw, we always can find a system of co-ordinates, viz., Cartesians, in terms of which the expression for the distance involves no coefficients other than constants (which can of course always be reduced to unity by a suitable choice of our units). On the sphere we should find that it was *impossible* to choose *any* set of co-ordinates for the whole surface, in terms of which the expression for the distance between two points involves nothing but constant coefficients. Always we shall find that one or both of the differentials is multiplied by a function of the co-ordinates.

This then is an intrinsic difference between spheres and planes. It is connected with the fact that a sphere cannot be unfolded into a plane without stretching, as, for instance, a cone can. We see then that there are at least two intrinsically different kinds of surface. With both of them the expression for the distance of two points measured along the surface will involve non-constant coefficients, when expressed in terms of *some* set of co-ordinates upon the surface. But with the one kind of surface this will be so, not merely for *some*, but for *all possible* sets of co-ordinates upon the surface. And, with the other kind, it will be possible to find a set of co-ordinates on the surface, in terms of which the expression for the distance of two adjacent points involves no coefficients but constants.

Let us now leave the points and surfaces of pure geometry, and apply our results to the events of nature and their spatio-temporal separations. Just as surfaces

may be of two intrinsically different kinds, so nature, as a sum total of events, might theoretically be of one kind or another. It might have such an intrinsic structure that it was always possible to find a frame, viz., a Newtonian one, with respect to which the spatio-temporal separation of any pair of events takes the form

$$c^2dt^2 - dx^2 - dy^2 - dz^2.$$

On the other hand nature might, like the sphere in geometry, have such an intrinsic structure that *no* possible frame could be found with respect to which the spatio-temporal separation involved only constant co-efficients. Now the most general homogeneous quadratic expression for the spatio-temporal separation of a pair of adjacent events in terms of any frame is obviously of the form

$$\begin{aligned} &g_{11}d\theta_1{}^2 + g_{22}d\theta_2{}^2 + g_{33}d\theta_3{}^2 + g_{44}dt^2 + g_{12}d\theta_1 d\theta_2 + g_{13}d\theta_1 d\theta_3 \\ &\quad + g_{14}d\theta_1 dt + g_{23}d\theta_2 d\theta_3 + g_{24}d\theta_2 dt + g_{34}d\theta_3 dt, \qquad (9) \end{aligned}$$

where θ_1, θ_2, and θ_3 are the spatial parameters, and t is the temporal parameter, which one of the events has in respect to the spatial axes and the clocks of this frame. The g's are any functions whatever of these four variables. Now, if it is to be possible to find a frame with respect to which the spatio-temporal separation takes the Newtonian form, these g's cannot be just any functions. The reducibility to the Newtonian form imposes certain very general conditions on the g's. It can be shown that it is possible to find a frame, with respect to which the spatio-temporal separation assumes the form with constant coefficients, if and only if the g's are of such a kind that a certain very complicated function of them, called the *Riemann-Christoffel Tensor*, vanishes. To say that the Riemann-Christoffel Tensor vanishes would therefore be equivalent to saying that nature, as a system of interconnected events, has a certain kind of intrinsic structure, which is formally analogous to that of the plane in Euclidean space and

formally unlike that of the surface of a sphere in Euclidean space.

The next thing that we have to consider is the dynamical meanings of the various conceptions which we have been introducing and discussing. There are now two problems to be considered. The first is independent of the view that we take as to the two alternative possible intrinsic structures of nature. This leads to a generalisation of the first law of motion, so that it becomes independent of any particular frame of reference. The second depends on which alternative the facts force us to choose as to the intrinsic structure of nature. This leads to a generalisation of the law of gravitation. We will now consider them in order.

(1) According to Newton's first law of motion a particle which is under the action of no resultant force in a Newtonian frame either rests or moves with uniform rectilinear velocity in that frame. Consider two events in the history of this particle as it moves. One is its presence at the point x_A, y_A, z_A in the axes of the frame at the date t_A as measured by the A-clock of the frame. The other is its presence at the point x_B, y_B, z_B, in the same axes when the B-clock reads t_B. Since the particle is under the action of no Newtonian forces it will have moved in a straight line between these two points with a uniform velocity. Let us consider the *total* spatio-temporal separation between these two events. By this we are going to mean the sum of all the infinitesimal spatio-temporal separations between successive closely adjacent events in the history of the particle, which are intermediate between the first and the last event under consideration. It is easy to show that, when the particle moves uniformly in a straight line, this total separation has a stationary value. This means that it would either be greater for all alternative ways of moving from the one place to the other in the given time, or that it would be less for all alternative ways. As a matter of fact the actual path is that which

makes the total spatio-temporal separation a maximum. If the particle moved in any other course, or with a non-uniform velocity, the total spatio-temporal separation would be less than it is when it moves uniformly in a straight line.

Now the fact that the total separation between remote events in the history of this particle is a maximum is an *intrinsic* fact about the history of the particle. It depends in no way on the frame of reference which is chosen for placing and dating the events. We have thus got to something about the motion of the particle which is independent of frames of reference. Now refer the particle to any other frame you like. The characteristics of the new frame are completely summed up in the ten *g's* which appear in the expression for the spatio-temporal separation of two adjacent events in terms of the spatial and temporal parameters of this frame. We have therefore simply to express the fact that the integral of the expression (9) has a stationary value for the course which the particle actually takes with respect to this frame. This can easily be done by the Calculus of Variations. As a result a set of four second-order differential equations emerges. These are the equations of motion *in any frame whatever* for a particle which is under the action of no forces in a Newtonian frame.

Now, as we saw in last chapter, the change from a Newtonian to a non-Newtonian frame of reference involves the introduction of non-Newtonian forces. These forces are completely determined by the nature of the non-Newtonian frame chosen. Again, as we have seen, the nature of the frame is completely determined by the ten *g's* which appear in the expression for the spatio-temporal separation in terms of the parameters of the frame. Thus there is complete correlation between the *g's* which characterise the frame, and the non-Newtonian forces which people who used this frame would observe to act on particles. Thus, if *all* forces

be of this type, the four differential equations which express the fact that the total spatio-temporal separation for the actual course of the particle is to be stationary will *be* the laws of motion. For they will sum up the relations between the motion of any particle with respect to any frame and the observable forces which people who use that frame find to be acting on the particle. To observers on a Newtonian frame it will appear that the other observers are using very foolish axes and very wild clocks (*e.g.*, a rotating wheel and a water-tank). For the Newtonian observers then, the *g's* will not seem to have anything to do with forces, but only to characterise the particular kind of axes and clocks which the other observers are using. But, for the observers who use the frame characterised by the *g's*, these *g's* will appear as the potentials of forces which are functions of position and time with respect to their frame. (I say as *potentials of forces*, and not as *forces*, because the *g's* do not appear as such in the equations of motion, but appear in the form of first-order differential coefficients with respect to the co-ordinates and dates which events have in the frame.) The four differential equations of motion, thus deduced for any frame whatever, degenerate, in the special case of a Newtonian frame, to the three ordinary equations which express the fact that the acceleration of the particle vanishes in three mutually rectangular directions, and to the platitude $0 = 0$.

I will illustrate the connexion between the *g's* and the potentials of the non-Newtonian forces which are introduced along with a non-Newtonian frame, by working out a little further a simple example which was used in the last chapter. It will be remembered that we there took a particle at rest on a plane in a Newtonian frame and referred it to a non-Newtonian frame, consisting of the same clock as before for the time-measurer and two mutually rectangular spokes of a rotating wheel, that lay in this plane, as the spatial

axes. We saw that the observers who use this frame will ascribe a non-Newtonian attraction from the particle to the origin of amount $mr\omega^2$. The non-Newtonian potential required to produce this force is $\frac{1}{2}mr^2\omega^2$, since $F_r = -\frac{\partial V}{\partial \nu}$ by definition, and $F_r = -mr\omega^2$ in this case. Now let us consider what will be the expression for the separation of two adjacent events in terms of the new frame. In terms of the original Newtonian frame it is, of course, $c^2dt^2 - dx^2 - dy^2$. It is easy to show that it will be $(c^2 - \omega^2 r^2)dt^2 - d\xi^2 - d\eta^2 + 2\omega\eta d\xi dt - 2\omega\xi d\eta dt$ in terms of the new frame. Thus the new frame is characterised by the following values for the *six* g's which are needed when we confine ourselves to a two dimensional space, as we are doing in this example:—
$g_{tt} = c^2 - \omega^2 r^2$; $g_{\xi\xi} = g_{\eta\eta} = -1$; $g_{\xi t} = 2\omega\eta$; $g_{\eta t} = -2\omega\xi$; $g_{\xi\eta} = 0$. If we ascribe to the non-Newtonian force a potential $-\frac{1}{2}mg_{tt}$, we shall account for the observable facts, since $-\frac{\partial}{\partial r}(-\frac{1}{2}mg_{tt}) = -m\omega^2 r$, and the observed non-Newtonian force is $-m\omega^2 r$. Thus we see that g_{tt}, which, from the point of view of observers on the Newtonian frame, is merely one of the coefficients that characterise the special non-Newtonian frame used by the other observers, is, from the point of view of the non-Newtonian observers themselves, the potential of a force which acts on all particles with respect to their frame.

So far we have confined ourselves to the case of a particle which is under the action of no Newtonian force, and we have derived the equations of motion for such a particle under the action of the non-Newtonian forces to which it will be subjected when referred to a non-Newtonian frame. But of course most particles, if not all, are, at some time at least in their history, under the action of Newtonian forces, and do not move uniformly or in straight lines with respect to Newtonian

frames. What are we to say of the equations of motion of such particles?

We have said that a particle under the action of no Newtonian force moves in such a way that the total separation between two remote events in its history is greater than it would be for any other possible way of moving. We also said that this property of the actual history of the moving particle is independent of the particular frame of reference to which it may be referred. Before we can get any further we must clear up these two statements a little further. We will begin with a geometrical analogy.

Suppose there were two remote points and we were told to find the *shortest possible* path from one to the other. The problem would not yet be perfectly determinate. Possibility is always relative to a set of conditions implied or asserted. What would be the shortest possible path, relative to one set of conditions, would not be so, relative to another set. If we were allowed to move from one point to the other on the Euclidean plane on which they both lie, the shortest possible path would of course be the Euclidean straight line joining them. But if we were told that we must keep to the surface of a certain sphere on which both points are situated, the shortest possible path would be along the great circle on this sphere which joins them. And a great circle is an intrinsically different kind of curve from a Euclidean straight line. Thus the curve which is the shortest path between two points depends on the intrinsic structure of the region in which the points are situated, and to which all paths between them are to be confined. Once this intrinsic structure is given, the property of being the shortest path between the two points is independent of all possible sets of axes which might be used for mapping out the region. But, of course, the intrinsic character of the region will impose certain restrictions on the kind of axes that are possible for mapping it out. Similarly, the nature of the move-

ment which gives the maximum possible spatio-temporal separation for two remote events in the history of a moving particle will depend on the structure of that part of the history of Nature in which the events happen, and within which all courses from one to the other are to be confined. Given the structure of this part of the history of Nature, the course with the maximum possible total spatio-temporal separation is independent of all frames of reference which can be used for placing and dating events within this region. But the intrinsic structure of this part of the history of Nature will impose certain restrictions on the kind of frames that are possible for mapping it out.

We can now deal with the case of a particle subject to Newtonian forces. We *assume* (*a*) that it is a general fact about *all* moving particles (and not merely about those which are under the action of non-Newtonian forces) that they move in such a way that the total spatio-temporal separation for two remote events in their history is greater than it would be for any other way of moving which the intrinsic structure of the part of the history of Nature in which the two events fall would allow. (*b*) That, in those parts of the history of Nature in which Newtonian forces show themselves, the intrinsic structure is *not* such that the expression for the spatio-temporal separation for two adjacent events can be reduced to the form with constant coefficients. This is equivalent to assuming that Newtonian frames are strictly applicable only to those parts of the history of Nature (if such there be) in which no Newtonian forces are acting.

On these assumptions the general equations of motion, which have just been deduced for *non-Newtonian* forces, will hold for *all* forces. These four equations are simply the analytical conditions which must be fulfilled if the actual course of a particle is to be such that the total spatio-temporal separation between two remote events in its history shall be a

maximum or minimum. And they were deduced from the most general expression possible for the spatio-temporal separation of a pair of adjacent events. For, although we were in fact dealing with cases where the expression for the separation can be reduced to the Newtonian form with constant coefficients, no use was made of this special assumption in deducing the conditions that the total separation for the actual course shall be stationary. We may say then that, if the above assumptions be true, we have got the general equations of motion in a form which is (*a*) independent of any special frame of reference, and (*b*) applies equally to Newtonian and non-Newtonian forces. If the forces be all non-Newtonian there will *in addition* be a set of equations between the *g's* of all possible frames, expressing the fact that the structure of the region under discussion is such that the separation can be reduced to the form with constant coefficients. If some of the forces be Newtonian this extra set of conditions will not of course hold, though it will still be possible that the *g's* of all possible frames are subject to some less rigid set of conditions.

On this view the one fundamental mechanical fact, which is absolute and independent of all frames of reference, is the stationary character of the actual history of a moving particle, *i.e.*, the fact that it moves with such a velocity and in such a path that the total separation between remote events in its history is a maximum or minimum. This is independent of whether it be under the action of Newtonian forces or not. But the course which in fact has the greatest or least possible separation will differ intrinsically, according to the intrinsic structure of the history of Nature in the spatio-temporal region under discussion. If this region be such that the separation between two adjacent events in it can be expressed in the form with constant coefficients, the course which has the stationary property is a Euclidean straight line traversed with a uniform

velocity as judged by a Newtonian clock. If the region be such that the separation cannot, by any choice of frame, be reduced to this specially simple form, the stationary course will be some intrinsically different kind of curve traversed with a non-uniform velocity. It is assumed that the presence of Newtonian forces in a region of the history of Nature is a sign that the intrinsic structure of that region is such that no frame can be found, with respect to which the separation of two adjacent events takes the form with constant coefficients.

How are we to verify or refute these assumptions? Obviously the only way is to see whether (*a*) they agree with known facts as well as the traditional theory, and (*b*) account for and predict facts which were not predicted or accounted for by the traditional theory. We have seen that, when the forces are purely non-Newtonian, the *g's* of any frame of reference appear to the observers who use that frame as the potentials of the non-Newtonian forces. Reversing this analogy, it is reasonable to suppose that the potentials of the Newtonian forces that are observed with respect to any frame will be the *g's* which characterise the spatio-temporal separation of two adjacent events in that part of the history of Nature in which these Newtonian forces act. In dealing with any particular field of Newtonian force we must therefore find a set of *g's* which (*a*) satisfy the general equations of motion, and (*b*) differ numerically from the potentials which the traditional theory would ascribe to this field by amounts which fall below the limits of experimental error in the experiments that have already been done with such fields. If this can be done, the resulting equations will have at least as good a claim to represent the facts of motion in this field as the traditional equations. And if, in addition, they enable us to predict small residual effects, which are not accountable for on the traditional theory but can be observed when

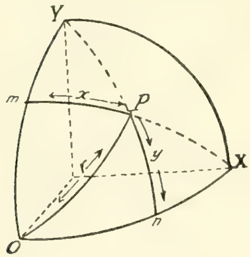
looked for, they will have better claims to truth than the traditional equations. It must be admitted, however, that this would not amount to a knock-down proof of the truth of the assumptions, since the modified equations could no doubt be deduced on traditional views of space and time, provided suitable modifications were made in the expressions for the potentials.

Evidently then we can only hope to find evidence for or against the present theory by considering definite fields of force and the observable phenomena that happen in them. And, even so, as it seems to me, no absolutely conclusive proof of the theory will ever be found, since alternative explanations which involve the traditional views of space, time, and force could always be constructed to fit the facts. If, however, these should prove to be very complicated and artificial, as compared with the explanation offered by the new theory, we shall have the same sort of grounds for preferring the latter as we had for preferring the Relational Theory of Motion, in spite of the fact that no downright refutation of the Absolute Theory is possible.

(2) We have now to raise the question whether Nature, as a sum total of events, has any one type of intrinsic structure always and everywhere, and, if so, of what type the intrinsic structure is. It is admitted that not all forces are non-Newtonian, *i.e.*, that, if we insist on trying to refer all the events in Nature to a Newtonian frame, many particles will at some time in their history be subject to observable forces with respect to it. And there is no frame that we can take which will transform away all forces always and everywhere, though it is always possible to find a sufficiently wild frame which will transform away Newtonian forces over a small enough region of space for a short enough lapse of time. Now we might deal with this fact in one of two alternative ways: (*a*) We might hold that the

intrinsic structure of Nature is such that the spatio-temporal separation of a pair of adjacent events can take the Newtonian form always and everywhere. We shall then have to hold that this fact is disguised from us by the presence of forces in Nature, which appear in every frame we choose. This is a little like Swift's view that the English Government always chose admirable bishops for Ireland, but unfortunately they were always stopped on Hounslow Heath by highwaymen, who exchanged clothes with them and travelled on in their coaches. Or (*b*) we might hold that Nature is so constituted that no frame can be found with respect to which the separation takes this simple form. We might then try to explain the forces, which are found in all frames, by reference to the intrinsic peculiarity of structure in Nature, which prevents the separation from being expressed in this simple way.

Before attempting to decide between these two alternatives for the dynamical case, I will, as usual, illustrate their precise meaning by a geometrical example. Suppose people were confined to the surface of a sphere, and that they took as axes a pair of mutually normal great circles. The coordinates of any point P on the sphere are to be the arcs of the two great circles through it which are normal to these two axes respectively. The figure above will illustrate the arrangement.



If they measured the arcs OP, P*m* and P*n*, and found their lengths to be r, x and y respectively they would find that r^2 is not equal to x^2+y^2, as it would be if the square of the spatial separation for adjacent points on a sphere were of the form dx^2+dy^2. But, if they were specially wedded to the view that the spatial separation *must* take this form, they could get over the difficulty by assuming that there are forces of suitable magni-

tudes and directions at different points on the sphere which distort their measuring rods. Conversely, they might just recognise that they were "up against" an intrinsic peculiarity of spherical surfaces, and avoid the supposition of distorting forces. Similarly, when you find that there are untransformable forces with respect to Newtonian frames, you can either leave it at that, or take up the suggestion that Nature has such an intrinsic structure that the spatio-temporal separation of two adjacent events is not accurately expressible in the Newtonian form.

The actual relation between r, the total separation, and x and y, the co-ordinates in this system is

$$\sin^2\frac{r}{k}=\sin^2\frac{x}{k}+\sin^2\frac{y}{k}$$

where k is the radius of the sphere. If the observers confined themselves to a very small region, the sines could be replaced by the angles themselves, and the relation

$$r^2=x^2+y^2,$$

which is characteristic of the Euclidean plane, would approximately hold. This is analogous to the fact, already mentioned, that it is always possible to find a frame, in terms of which particles move with uniform rectilinear velocities for a sufficiently small region of Space and for a sufficiently small lapse of time, though not for all places and all time.

We can now return from the geometrical analogy to the dynamical problem. If we consider the various kinds of Newtonian forces we find that they divide sharply into two classes, viz., gravitational attractions and the rest. We have already pointed out the peculiarities of gravitation. It acts always and everywhere, it is independent of all properties of matter except its inertial mass, it is indifferent to the surrounding medium, and so on. We saw that these peculiarities make gravitation closely analogous to the

non-Newtonian forces to which a particle, at rest or in uniform motion in a Newtonian frame, is subjected when referred to a non-Newtonian frame. Again, we saw that, in no frame composed of material axes and clocks, could a particle literally be under the action of no forces, since there would always be gravitational attractions between it and the axes themselves, though these might be negligible if it were a solitary particle referred to the fixed stars as axes. For these reasons it seems plausible to suppose that gravitation, at least, is something connected with the intrinsic structure of Nature as a sum total of events. This structure is such that no frame, in which the spatio-temporal separation takes the simple form with constant coefficients, accurately fits the whole of Nature; and the gravitational forces, which we find when we use a Newtonian frame, are an expression of the "misfit" of that frame to the structure of Nature. This is exactly analogous to the fact that the contracting and expanding forces, which observers on the sphere would have to assume to be acting on their measuring rods in the last example, would simply be an expression of the "misfit" between the intrinsic character of the surface of a sphere and the plane system of co-ordinates which they insisted on applying to it.

As regards other kinds of Newtonian forces, which depend on the special properties of bodies and of the medium, and do not show themselves always and everywhere, as gravitation does, we can hardly expect a similar explanation to work. We may illustrate this difference again from the example of people living on the surface of a sphere and trying to measure it, on the assumption that the expression for the square of the spatial separation of two adjacent points *must* be reducible to the simple form dx^2+dy^2. Let us suppose that there were big fires burning at some parts of the surface of the sphere. The measurements of the observers would then be inconsistent with their fundamental

assumption and would have to be "cooked" in two different ways: (*a*) They would be *systematically* wrong on account of the fact that *no* system of co-ordinates on the surface of a sphere can really give an expression for the separation, which shall involve only constant coefficients. This systematic error they will have to correct by ascribing contracting and expanding forces on their rods to the sphere itself. (*b*) Apart from these systematic errors, there will be *special* discrepancies when they measure near one of the fires, owing to the physical expansion of their rods in such a neighbourhood. Now we should say that it was not unreasonable of the observers to ascribe the special discrepancies in their measurements near the fires to forces acting there on their rods, for there is something visible and tangible there (viz., the fire) to account for these assumed forces. But we should think it very foolish of them to ascribe the *systematic* discrepancy between measurement and theory, which they find *everywhere* on the sphere, to forces bound up with the sphere and varying in a systematic way from place to place on its surface. We should advise them, instead of sticking obstinately to their view that the separation of adjacent points on the sphere *must* take the form with constant coefficients, and then invoking forces to account for the discrepancies between this fact and their observations, to see whether they could not account much more simply for the facts by supposing that the surface on which they live is intrinsically of such a character that no set of axes, in which the expression for the separation of two adjacent points takes this specially simple form, can exist upon it. In the same way, when you find that there is a certain kind of force, viz., gravitation, which acts always and everywhere on all particles, when referred to Newtonian frames, it becomes reasonable to suppose that this "force" is merely an expression of the inappropriateness of a Newtonian frame to the intrinsic structure

of Nature, as a sum total of events. Other Newtonian forces, which act in one place and not in others, or at one time and not at another, or on one kind of matter and not on another, are in a different situation, and may be compared to the fires at various places on the sphere in our geometrical example.

We are going to see then, whether we can account for the gravitational forces, which are present in all Newtonian frames, by the assumption that the events of Nature form an interconnected manifold of such an intrinsic structure that no frame of reference can be found, in respect to which the expression for the spatio-temporal separation of two adjacent events *accurately* takes the form (8) with constant coefficients.

Now we have so far distinguished two kinds of surfaces in ordinary space. With one of them (such as the plane, the cone, the cylinder, etc.) it was possible to find a system of co-ordinates on the surface, in terms of which the expression for the spatial separation of two adjacent points, as measured along the surface, contains only constant coefficients. The sign of this was the vanishing of the Riemann-Christoffel Tensor. The more familiar criterion is that such surfaces are either planes or can be unfolded without distortion or stretching into planes. In the other kind of surface this condition is not fulfilled. We gave the sphere as an example. We agree then that the universality and other peculiarities of gravitation suggest that the structure of Nature, as a sum total of events, is not formally analogous to that of surfaces of the first kind, *i.e.*, we shall henceforth reject the view that the intrinsic structure of Nature is such that the Riemann-Christoffel Tensor *vanishes* for all frames of reference within Nature. Does Nature then impose no general condition on possible frames of reference except this negative one?

If we return once more to elementary geometry we shall see that the surfaces for which the expression for

the spatial separation cannot take the form with constant coefficients can be further subdivided. We took the sphere as an example of such a surface. The outside of an egg would be another example. Now these two surfaces have an important intrinsic difference. A sphere is a much more special type of surface than an egg-shell, just as a plane or a cone is of a much more special type than a sphere. The sphere agrees with the plane and differs from the egg-shell in the following respect: A triangle bounded by arcs of great circles on the sphere could be slid about all over the surface, remaining everywhere in complete contact with it, and needing no stretching or distortion. In fact any figure that fits on to the sphere in one part will do so in all parts. The same is obviously true of figures in a plane. It is not true of figures on the surface of an egg-shell. A cap, which fitted the blunt end of the egg-shell, could not be made to fit exactly on to the sharp end without stretching some parts of it and folding others. Thus, granted that the Riemann-Christoffel Tensor does not vanish for Nature, and that the intrinsic interconnexions of events in Nature are therefore not formally analogous to those of points on a plane, the question can still be raised: Are the intrinsic relations of events in Nature formally analogous to those of points on a sphere or to those of points on an egg-shell? If the former alternative be fulfilled a function of the *g's*, derived from the Riemann-Christoffel Tensor, and called the *Modified Riemann-Christoffel Tensor*, will have to vanish. This imposes a limitation upon possible *g's*, and therefore upon possible natural frames of reference, but the restriction is less rigid than it would be if the unmodified Tensor were to vanish.

If then gravitation be the way in which a certain intrinsic peculiarity in the structure of Nature exhibits itself, we might suppose that the equating of the Modified Tensor to O would be the generalised expres-

sion for the law of gravitation, with respect to any admissible frame of reference. So far, however, this is merely a conjecture. It might be that gravitation is not the expression of a general intrinsic peculiarity in the structure of Nature, as a sum total of interconnected events. And it might be that, even if this were true, the structure is not of the particular kind which is expressed by the vanishing of the Modified Tensor. Here, as elsewhere, we must carefully distinguish between what *suggests* the theory and what *verifies* it. What suggests that gravitation is an expression of the general intrinsic structure of Nature is its universality and its peculiarities as compared with other forces. What suggests taking the vanishing of the Modified Tensor as the expression of this structure is that it is the next simplest assumption to make, after the facts have proved to be inconsistent with the still more special structure which would be indicated by the vanishing of the unmodified Tensor. We have now to see what verifies the theory thus suggested.

We know the traditional form of the law of gravitation, with respect to the nearest approach that we can get to Newtonian frames. For a region free from matter (approximately for the inside of an exhausted bulb) it takes the form of Laplace's Equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

where V stands for the gravitational potential at a point in the region, and x, y, and z are the Cartesian coordinates of this point with respect to a Newtonian frame. There is no doubt that this equation is true to a very high degree of approximation. It follows that any candidate for the position of the true law of gravitation must reduce to something which differs very slightly indeed from Laplace's equation, when expressed in terms of the nearest approach to a Newtonian frame that we can get.

Now the Modified Riemann-Christoffel Tensor is an expression involving second order differential coefficients of the *g's* for a frame, with respect to the co-ordinates and dates of an event as referred to this frame. So far there is a formal analogy between it and the left-hand side of Laplace's Equation, if the *g's* be regarded as analogous to Laplace's V. The right-hand side is o in both cases. Now Laplace's V is a potential, and we have already seen the close analogy between the *g's* of a frame and the potentials of the forces which act on particles when referred to that frame. The only question that remains then, is the following: Can we find a set of ten functions $g_{\mu\nu}$ of the Newtonian co-ordinates and clock-readings, which (*a*) when substituted in the expression for the Modified Tensor make it equal to o, and (*b*) differ so little from the gravitational potentials of the ordinary Newtonian theory that the difference could only have been detected by very special methods, and when there was a very special reason for looking for it? If so, we may reasonably suppose that gravitation *is* an expression of the fact that Nature has a kind of intrinsic structure formally analogous to that of the sphere, and that the formula obtained by equating the Modified Tensor to o *is* the true form of the law of gravitation. The answer to this question is in the affirmative; and so we may take it that the vanishing of the Modified Tensor is the true form of the law of gravitation for a region empty of matter.

There is one point which must be mentioned here. We are accustomed to think of the traditional law of gravitation in the form that two particles attract each other with a force proportional to their masses and inversely proportional to the square of the distance between them. And we are wont to regard Laplace's differential equation as a rather recondite mathematical deduction from this. In the Relativity theory of gravitation the order is reversed. The law obtained by

equating the Modified Tensor to o is directly analogous to Laplace's Equation. The notion of remote particles attracting each other is here a rather recondite mathematical deduction from the differential equations. In fact, material particles turn up now only as points of singularity in a gravitational field; the field itself is the fundamental thing. And, when you do make this deduction, it is found that the force between two particles is not wholly in the line joining them, if I may put it rather crudely. The remaining term, which the new form of the law involves, accounts for the slow rotation of the orbits of the planets as wholes in their own planes. This had been noticed for the planet Mercury, and was unintelligible on the traditional law of gravitation. It is accounted for both qualitatively and quantitatively by the Relativity theory.

The last point to be noticed is that, on the present theory, gravitation modifies the movements, not merely of ordinary material particles, as on the traditional view, but also of any form of energy, such as light, radiant heat, etc., which travels through space. We must now see how this comes about. In the first place some such consequence is suggested at once by the modifications which the Special Theory of Relativity entails in the traditional conception of mass. We saw at the beginning of this chapter that, if a body moves with velocity v in a straight line with respect to a Newtonian frame, it is necessary to ascribe to it a mass $\frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$ in order to get the Principle of the Conservation of Momentum into a form consistent with the Restricted Physical Principle of Relativity. We also saw that this is approximately equal to $M_0+\frac{1}{2}\frac{M_0v^2}{c^2}$. Now the second term in this is the kinetic energy of the particle divided by the square of the velocity of light. It is

thus certain that the *kinetic* energy of a particle of *matter* appears as an increase in its *inertial* mass. It is therefore plausible to suppose that any region filled with *any* form of energy, such as light or radiant heat, would thereby acquire an inertial mass equal to the total energy contained in it divided by the square of the velocity of light. It by no means follows, on the traditional theory of gravitation, that such a region would contain any *gravitational* mass. It is true that for any particle of matter the gravitational and the inertial masses are proportional, to an extremely high degree of approximation. Still, this would be compatible with the view that the gravitational effect depends wholly on the factor M_0; seeing that the second factor in the inertial mass contains the square of the velocity of light in its denominator, and is therefore excessively small unless the energy of the body be excessively great. On this view we should not expect a beam of light to have gravitational mass, in spite of its having inertial mass. On the other hand, it is of course possible that the gravitational and the inertial masses are always exactly, and not merely approximately, proportional. In that case we should expect the course of a beam of light to be modified when it passes through a gravitational field, just as the path of a material particle is known to be modified under like conditions. Now experiments with pendulums had already suggested very strongly that the gravitational mass of a piece of matter is accurately, and not merely approximately, proportional to its *whole* inertial mass, and not only to the first factor in this. Thus, the Special Theory of Relativity had already made it extremely likely that the course of a beam of light or any other kind of radiant energy would be modified when it passed through a gravitational field.

Now what is thus merely a plausible suggestion on the traditional theory of gravitation, combined with the modified dynamics of the Special Theory of Relativity,

is a necessary consequence of the General Theory of Relativity. We know that light would not travel uniformly or in a straight line with respect to non-Newtonian frames. The people, *e.g.*, who used the spokes of the rotating wheel as their axes would not find that light travelled in a straight line with respect to their axes, or with a uniform velocity with respect to their clocks. And the actual course that a beam of light would follow in their system would be determined by the *g's* which characterise that system. Now it is a fundamental assumption of the General Theory that the analogy between the *g's* of a non-Newtonian frame and the potentials of the non-Newtonian forces which act on particles with respect to that frame is to be extended to the potentials of Newtonian forces.

Suppose then that we have found the equations for the path of a beam of light with respect to any frame, in terms of the *g's* of that frame, on the assumption that it would move accurately in a straight line with a uniform velocity relative to a Newtonian frame in the absence of gravitation. To find its actual path with respect to a Newtonian frame in a gravitational field we must just substitute in these equations those values of the *g's* which (*a*) satisfy the condition that they make the Modified Riemann-Christoffel Tensor vanish, and (*b*) account for the observed strength and distribution of the field. These equations will not in general represent a motion with a uniform velocity in a straight line with respect to the axes defined by the fixed stars. The divergence, which is excessively small even in the intense gravitational field which surrounds a huge body like the sun, can be calculated and has been experimentally detected.

I have now sketched to the best of my ability the gradual modifications which experimental facts and reflection upon them have forced upon physicists. There are two dangers to be avoided here by plain men. One is to think that the Theory of Relativity is

essentially unintelligible to all but profound mathematicians, and that therefore it is useless to try to understand it. The other, and much more serious danger, is to suppose that it can be made intelligible in popular expositions of a few pages to men who have never had occasion to consider the subjects with which it deals. Like every other conceptual scheme it grew up, by a kind of inner necessity, against a whole background, of interconnected concepts, principles, and experimental facts. Presented in the absence of this background it is and must be as unintelligible as the orthodox doctrine of the Trinity is to persons who know nothing of the theological controversies which preceded the formulation of the Athanasian Creed. In the course of my exposition I have constantly enlivened the discussion by geometrical anecdotes about men living on spheres, and dynamical parables about persons with an unintelligible fondness for rotating wheels as axes of reference. I think this course was inevitable, in order to illustrate the conceptions which I was expounding. But it has the grave disadvantage of breaking the train of argument and obscuring that distinction between inference and illustration which it is so important to keep clear. I shall therefore end by summarising the whole matter in a connected form.

Summary of Arguments and Conclusions of Part I. (1) Nature is a sum total of interconnected events; and every actual event lasts for some time, has some extension, and is in spatio-temporal relations to the other events in Nature. (2) But the extensions, durations, and spatio-temporal relations of events are of such a kind that we can apply the Principle of Extensive Abstraction to them, and thus define "instantaneous point-events" and their exact spatio-temporal relations. We can then give a clear meaning to the statement that the actual extended and enduring events of Nature are "composed of" instantaneous point-events, and that the crude relations of such actual

events are "compounded out of" the exact relations of the instantaneous point-events which compose them. (3) This being so, we can henceforth safely state our theory in terms of instantaneous point-events and their exact relations, which are *notiora nobis*, though not *notiora Naturæ*. For we know how to translate propositions about instantaneous point-events and their merely *conceivable* relations into propositions about actual extended and enduring events and their *perceptible* relations.

(4) It is impossible to state general laws about the events in Nature till we have fixed on some way of assigning a date and a position to every instantaneous point-event in Nature. For the laws of Nature express universal types of connexion between events of one kind happening in one place at one date and events of the same (or some other) kind happening at the same (or some other) place at the same (or some other) date. If the places and dates be different, the laws of Nature will in general involve the difference between the spatial co-ordinates and the difference between the dates of the events. (5) There are infinitely many different ways of assigning places and dates to all the instantaneous point-events in Nature; but each will involve the choice of certain observable events and processes in Nature as spatial axes and time-measurer. All other events will be placed and dated by their spatio-temporal relations to these chosen ones. Any such chosen set of events may be called a *Frame of Reference*. (6) It is reasonable to suppose that the expression for the laws of Nature in terms of any frame will depend partly on the particular frame chosen for placing and dating the events of Nature and partly on the intrinsic structure of Nature. The aim of science should be to find general formulæ for the laws of Nature, which will immediately give the special expression of the law in terms of any particular frame, as soon as the defining characteristics of the frame are

known. This is as near as anyone but God can get to the absolute laws of Nature. (7) There are two intrinsic peculiarities of Nature which reveal themselves at once. (*a*) No matter what frame we choose, we shall need *four* independent pieces of information to place and date any instantaneous point-event. This fact is expressed by saying that Nature is a four-dimensional manifold; and nothing further is expressed thereby. (*b*) In whatever frame we choose we shall find that our four pieces of information divide into two groups; three of them are spatial and one is temporal. Thus we must be careful not to talk, or listen to, nonsense about "Time being a fourth dimension of Space."

(8) There is one frame which has been tacitly used in the past for placing and dating the events of Nature for scientific purposes, and therefore the laws of Nature have been expressed in terms of this frame. The axes of it are defined by the fixed stars, the dating is done by pendulum clocks set in agreement with each other by means of light signals. (9) The choice of this frame is not altogether arbitrary. With it, the supposed laws of Nature can be expressed in a comparatively simple form, and yet are verified to a high degree of approximation. With it, again, distances and time-lapses which we should immediately judge to be unequal, when we are favourably situated for making such comparisons, *are* unequal, whilst those that we should immediately judge to be equal, under similar conditions, *are* either exactly or approximately so. In many frames this approximate agreement with our immediate judgments of equality and inequality would not hold. (10) With respect to such a frame, light *in vacuo* travels, to an extremely high degree of approximation, in straight lines and with a constant velocity; and the laws of motion, in the traditional Newtonian form, are very approximately true. Until quite recent years there was no motive for adding these qualifying phrases.

(11) Suppose now that we take a set of frames, whose clocks are set in the same way as those of the fundamental frame just described, and which only differ from it and from each other in that they move with various uniform velocities in the same straight line with respect to the fundamental frame. On traditional views about the measurement of space and time the measured time-lapse between any pair of events should be the same with respect to all these frames, and should be independent of their spatial separation and of the relative velocities of the two frames. The spatial separations should have different measured values in terms of any two frames of the set, and they should depend on the time-lapse and the relative velocities; but they should depend on nothing else, and the connexion between them should be of a very simple form. If this be so, the measured velocity of anything that moves with respect to the various frames should be different for each frame. (12) But very accurate experiments, which would be quite capable of detecting these expected differences in the measured velocity of light with respect to a pair of such frames, fail to show any sign of difference. Hence the traditional views about the measurement of time and space must be revised, or some purely physical explanation must be found for this discrepancy between theory and observation. (13) No plausible physical explanation can be found, which does not conflict with other well-established physical results. Hence the traditional views about the measurement of space and time *must* be revised. (14) The transformation equations of the Special Theory of Relativity express the relations which must hold between the measured distances and the measured time-lapses of a pair of events with respect to any two frames of this set, if the measured velocity of light with respect to all these frames is to be the same. They must therefore be accepted. (15) According to these transformation equations the measured time-lapses between

the same pair of events will not be the same with respect to all frames of the set. They will depend on the measured distances of the events and on the relative velocities of the frames. And the measured distances will not be connected with the measured time-lapses in the simple way in which they are connected on the traditional theory. The differences between the traditional transformation equations and those of the Special Theory of Relativity are, however, so extremely small, when the relative velocities of the frames are small as compared with that of light, that it is not surprising that the defects of the traditional view should have remained unnoticed until recent years. (16) It follows that, although (as stated in (7)) the distinction between time and space will appear in *every* frame, time-separation and space-separation are not *independent* facts in Nature. Events that are separated in time but coincident in space for one of these frames will always be separated in space for another of them. And events which are separated in space but coincident in time for one frame will be separated in time for another. But, if a pair of events be coincident both in time and in space for one frame, they will be so for all.

(17) Newton's laws of motion are in such a form that they are co-variant with respect to this set of frames for the traditional transformation-equations, but are not co-variant for the transformations of the Special Theory of Relativity. On the other hand, Maxwell's equations for the electro-magnetic field are co-variant for the latter and not for the former. This means that Maxwell's equations are already in a form which remains unchanged with change of frame, so long as we confine ourselves to the particular group of frames at present under discussion and use the transformation equations which the facts about light have shown to be necessary. Since this is not true for Newton's laws, unless we use a set of transformation equations which the facts about light have proved to be slightly inaccurate (viz.,

those of the traditional kinematics), we must conclude that Maxwell's equations are a nearer approach to "absolute" laws of Nature than the laws of motion in their traditional form. (18) It is, however, easy to make quantitatively small modifications in the traditional laws of motion, which will render them co-variant for all frames of the present set when the true transformation equations are used. The modified laws will then be as near an approximation to absolute laws of Nature as Maxwell's equations. (19) The necessary modifications require us to drop the notion that inertial mass is an absolute constant. The measured inertial mass of a particle with respect to a frame of the set depends on its velocity in that frame, and very approximately splits up into two factors, one of which is a constant and the other is its traditional kinetic energy divided by the square of the velocity of light. (20) Delicate experiments with pendula strongly suggest that the gravitational mass of a body is accurately proportional to its *total inertial mass*, and not merely to the part of this which is independent of the energy. (21) The frame whose axes are determined by the fixed stars and whose clocks are regulated by light-signals, and all other frames whose clocks are regulated in the same way and whose axes move with a uniform rectilinear velocity with respect to the former, together make up the set of empirically *Newtonian frames*. With respect to all frames of this set it is certain that light travels very approximately in straight lines with the same constant velocity, and it is certain that Newton's laws of motion—as modified by the Special Theory of Relativity—very approximately hold. So close is the approximation in both cases that nothing but theoretical considerations would induce us to look for any exception to it. We have now to remove our previous restriction to Newtonian frames, and to try to generalise the laws of Nature for frames that are not Newtonian.

(22) It is possible to keep the form of Newton's first

two laws of motion for non-Newtonian frames, provided we will introduce appropriate non-Newtonian forces with each non-Newtonian frame. These forces will be peculiar in that (*a*) they act on all particles referred to the frame, and are in general functions of the position and date of an event in the frame ; (*b*) they depend on no property of the particle except its inertial mass ; and (*c*) they do not in general obey Newton's third law, unless concealed particles be assumed *ad hoc* to carry the reaction. In the first two of these respects they resemble the Newtonian force of gravitation. (23) According to Newton's first law a particle under the action of no force rests or moves uniformly in a straight line with respect to a Newtonian frame. This is equivalent to saying that the total spatio-temporal separation between any two remote events in its history is either greater or less than it would be for all other possible ways in which the history of the particle might unfold itself between these two events. (24) The spatio-temporal separation between two adjacent events (unlike the spatial and the temporal separations) is independent of the frame of reference, though it depends on the intrinsic structure of the region in which the events happen, and this in turn determines the set of frames which can be used for mapping out this part of the history of Nature. Its particular expression, in terms of co-ordinates and dates, of course varies with the particular member of the whole set of admissible frames which is used. Thus, the fact that the total spatio-temporal separation between a pair of remote events is a maximum or minimum is independent of frames, though the particular course for which the total separation is in fact stationary differs according to the intrinsic structure of the region in which the events are contained. (25) The particular conditions which must hold if the total spatio-temporal separation is to be a maximum or minimum can be stated in a form which applies equally to all structures and all frames. The four equations which sum

up these conditions constitute the equations of motion of a particle, at least under the action of non-Newtonian forces. On comparing them with the traditional Newtonian equations, we see that the *g's* which characterise any non-Newtonian frame are of the nature of potentials of the non-Newtonian forces introduced by that frame. (26) We now make two assumptions, which are only justified in so far as they work. (*a*) We assume that it is a *universal* law of Nature that a particle moves in such a way that the total separation of remote events in its history is stationary, as compared with that of all other possible ways of moving. This is to hold equally whether it be subject only to non-Newtonian or also to Newtonian forces. In that case the equations deduced for the non-Newtonian case become *the* equations of motion. (*b*) We assume that in those regions of Nature, regarded as a sum total of events, in which Newtonian forces show themselves, the structure of Nature is not such that the separation can be reduced to the form with constant coefficients. If that be so, the course with the maximum total spatio-temporal separation is not a Euclidean straight line traversed with a constant velocity, as judged by a Newtonian clock. We treat the traditional potentials of the Newtonian forces in any field as first approximations to a set of *g's*, which satisfy the general equations of motion thus deduced. And we treat the result as the true law of the field.

(27) Continuous manifolds of several dimensions, such as Nature has proved itself to be, can be of various intrinsically different kinds. As we might put it, they can be "plane-like," "sphere-like," "egg-like," and so on. Whatever intrinsic spatio-temporal structure Nature may have, there will be an infinite number of different possible frames to be found for placing and dating the events of Nature. Nevertheless, the intrinsic structure of Nature will impose certain conditions on all possible natural frames of reference. These re-

strictions will take the form of certain very general equations connecting the *g's* of any possible natural frame. If the structure of Nature be plane-like, the condition is that the unmodified Riemann-Christoffel Tensor shall vanish for the *g's* of all possible natural frames. If its structure be sphere-like, the condition is that the Modified Riemann-Christoffel Tensor shall vanish for the *g's* of all possible frames. The latter is a less rigid condition than the former. (28) If the intrinsic structure of Nature be plane-like, an accurately Newtonian frame will be fitted for dating and placing all the events of Nature; otherwise it will not. (29) If we try to map out a manifold by a frame which is unsuited to its intrinsic structure, we shall only be able to square our measurements with our theory by the assumption of forces which distort our measuring instruments and upset their readings.

(30) We cannot find any frame that will transform away gravitational forces always and everywhere, though we can find non-Newtonian frames which will transform them away over sufficiently small regions of space and time. With respect to Newtonian frames all particles are always acted on by gravitational forces, though these may sometimes be negligibly small for practical purposes. It is therefore plausible to suppose that the universality of gravitation with respect to Newtonian frames is a mark of the misfit between this type of frame and the intrinsic structure of Nature. (31) On the other hand (*a*) gravitation has many analogies to non-Newtonian forces; (*b*) the traditional law of gravitation, which is certainly very nearly true, can be expressed as a differential equation of the second order, involving the gravitational potential at a place and the co-ordinates of the place with respect to Newtonian axes; and (*c*) we have already assumed that potentials and the *g's* of frames are mutually equivalent. (32) The facts mentioned in (31) strongly suggest that the law of gravitation must be some

general condition imposed on the *g's* of all possible natural frames, and expressed as a differential equation of the second order involving these *g's*. The facts mentioned in (30) suggest that this condition is *not* that the unmodified Tensor vanishes. For, if this were so, the intrinsic structure of Nature would be such that a Newtonian frame is suited to it, and the necessity of assuming gravitational forces always and everywhere with Newtonian frames strongly suggests that this is not so. (33) It is obvious that the next suggestion to try is to suppose that the law of gravitation is expressed by the vanishing of the Modified Tensor, *i.e.*, that gravitation is the sign of an intrinsically sphere-like structure in Nature. (34) It is found that, if this be the true law of gravitation, the observable effects will in most cases differ so little from those predicted by the traditional law that the difference could not be detected. Hence the very full verification which the traditional law has received is no obstacle to accepting the amended law. (35) On the other hand, there are certain very special cases in which a small observable effect might be expected on the new form of the law and not on the old. In such cases (notably the movement of the perihelion of Mercury and the bending of a ray of light in passing near a very massive body like the sun) the predicted effects have been verified both qualitatively and quantitatively.

The following additional works may be consulted with advantage:

A. S. EDDINGTON, *Report on the Relativity Theory of Gravitation.*

" " *Space, Time, and Gravitation.*

E. CUNNINGHAM, *Relativity, Electron Theory, and Gravitation.*

B. RIEMANN, *Über die Hypothesen welche der Geometrie zu Grunde liegen*. (Julius Springer. Berlin.)

H. WEYL, *Space, Time, and Matter.*

PART II

THE SENSATIONAL AND PERCEPTUAL BASIS OF OUR SCIENTIFIC CONCEPTS

Contents of Part II

CHAPTER VII

> "Fallunt nos oculi, vagique sensus
> Oppressa ratione mentiuntur.
> Nam turris, prope quae quadrata surgit,
> Detritis procul angulis rotatur."
>
> (*Petronius Arbiter.*)

Matter and its Appearances; Preliminary Definitions

In the First Part we have been dealing with the gradual development and modification of the traditional scientific concepts of Space, Time, and Motion, within the region of Physics. These concepts were taken over by science from educated common-sense, and we have been tracing the process of clarification and definition which they have undergone at the hands of scientists in pursuit of their own business. At two places only have we deliberately gone outside the range of ordinary scientific reflection. The first was where we explained the Principle of Extensive Abstraction, and tried to justify by its means what mathematical physicists take for granted, viz., the application of geometry and mechanics, stated in terms of points, instants, and particles, to a world of extended objects and non-instantaneous events. The second was where we dealt with the general problem of Time and Change, and tried to defend their reality against the very plausible objections which have been made to them by certain philosophers.

Now the careful reader will have been struck by two points in Part I. (1) He will have noticed that the "raw material," which science took over from common-sense and elaborated, was really anything but "raw."

It was already highly complex and sophisticated. The common-sense notions of a single Space, a single Time, and persistent bits of Matter which exist, move, and change within them, are by no means primitive. They must be the results of a long and complex process of reflection and synthesis, carried out by countless generations of men on the crude deliveries of their senses, embodied in everyday speech, and thus handed down from father to son for further elaboration. The main outlines of this conceptual scheme have been accepted without question by scientists, and we have so far merely been tracing those modifications of detail within the scheme, which a more accurate knowledge of the facts of nature has shown to be necessary. In Part II, I want to dig below the foundations of Part I, and to try to connect the concepts of science and common-sense with their roots in crude sensation and perception. If we should find, as I think we shall, that recent modifications in the traditional concepts, which have been made on purely scientific grounds, bring the general scheme into closer connexion with its sensible and perceptual basis, this will be an additional argument in favour of such modifications, and should tend to neutralise the impression of paradox which these later developments produce on men who have been brought up on the traditional scheme.

(2) The second point which will have struck the reader is that practically nothing has been said so far about the concept of Matter. This is true. There is a much wider divergence between the common-sense and the scientific concepts of Matter than between the two concepts of Space or of Time. The scientific concepts of Space and Time are fairly straightforward developments and clarifications of the concepts of common-sense. But common-sense thinks of Matter as having many intrinsic qualities, such as colour, temperature, etc., besides its merely spatio-temporal characteristics. Science, on the other hand, tends to think of Matter

as being simply "the movable in space," and to ascribe to it no intrinsic non-spatio-temporal qualities except mass. Now the treatment of Matter and our knowledge of it will bring us in the most direct way to the heart of the problem of Part II. Matter is admitted to be, or to be specially closely connected with, what we perceive with our senses. And again, it would be admitted by most people that we should never have known of spatial attributes, like shape, size, and position, if we had not perceived bits of matter of various shapes and sizes in various places. Lastly, we learn about Motion by watching bits of Matter moving about, and by moving about ourselves. Thus, in trying to clear up the relations between Matter, as conceived by science, and what we perceive with our senses, we shall at the same time be dealing with the sensible and perceptual bases of the concepts of Space, Time, and Motion. So, in one sense, this Part will be wholly about the concept of Matter. But this will involve a reconsideration of the concepts of Space, Time, and Motion. I shall begin by stating the problem in its most general form, and shall gradually go into greater detail.

The Traditional Notion of a bit of Matter.—When we ask what is meant by a bit of Matter the question is itself ambiguous. In one sense a complete answer to it would be a complete theory of Matter, and this could only be made, if at all, at the very end of our discussion. This, however, is not the sense in which I am asking the question here. All that I am asking is: "What is the irreducible minimum of properties which practically everybody would agree that an object must possess if it is to be called a bit of Matter?" I think that science and common-sense would agree that at least the following conditions must be fulfilled: (i) Its existence and properties must be independent of the minds that happen to observe it, and it must be capable of being observed by many minds. This

characteristic may be summed up by saying that Matter is neutral as between various observers, or is "public"—to use a convenient word of Mr Russell's. This distinguishes Matter sharply from any ordinary conscious state of mind. The latter is in a unique way private to the person whose state it is. My belief that $2+2=4$ is different from yours, though the two beliefs refer to the same fact. My belief cannot literally wander out of my mind and turn up in yours. It is true that I may convert you from your erroneous belief that $2+2=5$, and replace it by my true belief that $2+2=4$. This does not, however, mean that my belief has become yours, in the sense that it has left my mind and taken up its abode in yours. Were this so, I could never persuade you of anything without losing my own belief in it, and schoolmasters would presumably be distinguished from other men by an ultra-Humian scepticism as to all the subjects that they teach. This is not, in fact, found to be the case. All that really happens when A converts B to his own belief is that A's arguments, or the amount of A's bank balance, produce in B's mind a state of belief which refers to the same fact as B's belief, and has the same relation of concordance or discordance to this fact. My belief and yours are only called the *same* belief in the derivative sense that they are two different acts of believing which are related in the same way to the same fact.

Exactly the same is true of desires. We do sometimes say that you and I have the same desire; but what we mean is that your desire and mine, though two states of mind, have a single object. Now, if there be such things as bits of Matter at all, they are not private in this way to each mind, but are common to all the minds that observe them. We talk of *my* beliefs and *your* wishes; we do not talk of my hydrogen atom or of your electron. We just speak of *the* or *this* atom or electron. It is, of course, true that a hat or

an umbrella is regarded as a bit of Matter, and that we do talk of my hat and of your umbrella. But this, which at first sight seems an objection, is seen on further reflection to support what we have been saying. The sense in which my umbrella is mine is different from that in which my beliefs are mine. My umbrella is mine only in the sense that it is legally my property; my beliefs are mine in the sense that they could not exist out of my mind or pass into yours. You cannot take my beliefs; it is only too fatally easy for you to take my umbrella. So that even those bits of Matter to which we apply possessive adjectives are public in a way in which no state of mind is public.

(ii) A bit of Matter is supposed to be neutral, not only between different observers, but also to be in a certain way neutral as between several senses of the same observer. We are said to see, hear, and feel a bell. This sort of neutrality is not supposed to be complete. The shape and size of the bell are indeed supposed to be in some way common to sight and touch. As regards its sensible qualities the view of common-sense is that any bit of Matter combines a number of these, and that different senses are needed to reveal different sensible qualities. Thus sight, and it alone, makes us aware of the colours of bodies; touch, and it alone, makes us aware of their temperatures; and so on. But it is part of the ordinary view of a piece of Matter that all these various sensible qualities co-exist in it, whether the requisite senses be in action to reveal them all or not. If we first only look at a body, and then shut our eyes and go up to it and feel it, it is not supposed that it had no temperature on the first occasion and no colour on the second.

(iii) These two properties of publicity, as between different observers, and neutrality, as between the various senses of a single observer, are closely connected with a third feature which is held to be

characteristic of Matter. Bits of Matter are supposed to persist with very little change, whether anyone happens to observe them or not, and to pursue their own affairs and interact with each other, regardless of our presence and absence.

(iv) This brings us to the fourth characteristic of Matter. It is commonly held to be part of what we mean by a bit of Matter that it shall have a more or less permanent shape and size, and that it shall have a position in Space, and be capable of moving from one position to another. It is admitted that bits of Matter are constantly changing their shapes, sizes, and positions; but it is held that they do this through their interactions with each other and not through any change in our acts of observation, and that in all their changes they continue to have *some* shape, size and position. If it could be shown that nothing in the world actually has such properties as these, it would commonly be held that the existence of Matter had been disproved, even though there were public, independent, and persistent objects.

Berkeley, *e.g.*, is commonly held to have denied the existence of Matter, and he certainly thought himself that he had done so. Yet Berkeley's theory undoubtedly involves the existence of certain entities, viz., the volitions (and perhaps the sensations) of God, which are independent of the mind of any finite observer and are neutral as between my mind and yours. The reason why we say that, if Berkeley be right, there is no Matter, is because the volitions of God, though neutral and independent of us observers, have nothing corresponding to shape, size, and position; whilst the only entities which Berkeley allows to have these attributes, viz., our sensations, are private to each of us, and exist only so long as we have them. Very few philosophers have denied that there are entities answering to the first three conditions, but a great many have denied that there are any answering both

to these and to the fourth condition. Such philosophers are held by themselves and by common-sense to have denied the existence of Matter. Now we shall have plenty of opportunity for seeing that there is a real difficulty in holding that the entities which have shapes, sizes, and positions are neutral and independent, and that those which are neutral and independent have shapes, sizes, and positions.

Before we consider these points in detail at all we must mention an additional complication which, though partly verbal, is sure to puzzle us if we do not resolutely drag it into the light. No doubt it is part of what we mean by a bit of Matter that it shall, *in some sense*, have shape, size, and position. But in how literal a sense must this be true? We have already seen that, in some sense, an extension or a duration is composed of points or of instants respectively. But this sense is highly complicated and sophisticated, or, to use a happy phrase of Dr G. E. Moore's, "Pickwickian." Now we shall doubtless be able to find Pickwickian senses in which there are entities that are at once public and extended. The question is: How Pickwickian may the terms in our statement become before it ceases to be useful, and becomes merely misleading, to say that we accept the existence of matter? Our theological friends have much the same difficulties in their interpretations of the terms that are used in the Creeds. It could obviously only be true in a highly Pickwickian sense that the Second Person of the Trinity is the son of the First. No one supposes it to be true in the literal sense in which George V is the son of Edward VII; and the only substantial point at issue is whether the sense in which it might be true (assuming, for the sake of argument, that the Persons exist) is not so extremely Pickwickian that the statement is more likely to mislead than to enlighten. Fortunately for us the terminology of our problem is not surrounded with the same emotional fringe as surrounds the terms used in

Theology. It is no part of our duty to pay compliments to Matter, and so long as we state clearly what we do mean, it is of little importance whether our terms be used in a literal or in a highly Pickwickian sense. It will be a question of taste whether it shall be said that the theory that we finally adopt amounts to the acceptance or the denial of Matter. If we should be accused of saying that "Matter is not Matter," we shall at least be better off than *Dr F. R. Tennant, who labours under the dreadful imputation of teaching that "Sin is not Sin."

The Notion of Sensible Appearance.—I have now tried to point out what is the irreducible minimum of properties which ordinary people consider must be possessed by anything if it is to count as a piece of Matter. I have also pointed out, by anticipation, that the history of philosophy shows there to be a great difficulty in holding that there are any entities which fulfil all these conditions in a literal sense. Lastly, we have noticed that the question of the reality or unreality of Matter, thus defined, is not perfectly clear-cut, because of the practical certainty that many of our terms will have to be interpreted in a more or less Pickwickian manner, and the doubt whether it is worth while to go on using familiar phrases after their literal meaning has been departed from beyond a certain point. We must now consider what facts make it hard to believe that anything obeys all four conditions in at all a literal sense.

The difficulty arises because of the group of facts which we sum up by saying that it is necessary to distinguish between things as they are and things as they seem to us, or between physical reality and sensible appearance. Difficulties always arise when two sets of properties apparently belong to the same object, and yet are apparently incompatible with each other. Now the difficulty here is to reconcile the supposed neutrality, persistence, and independence of a physical object with

* See his *Origin of Sin.*

the obvious differences between its various sensible appearances to different observers at the same moment, and to the same observer at different moments between which it is held not to have undergone any physical change. We know, *e.g.*, that when we lay a penny down on a table and view it from different positions it generally looks more or less elliptical in shape. The eccentricity of these various appearances varies as we move about, and so does the direction of their major axes. Now we hold that the penny, at which we say that we were looking all the time, has not changed; and that it is round, and not elliptical, in shape. This is, of course, only one example out of millions. It would be easy to offer much wilder ones; but it is simple and obvious, and involves no complications about a transmitting medium; so we will start with it as a typical case to discuss.

Now there is nothing in the mere ellipticity or the mere variation, taken by itself, to worry us. The difficulty arises because of the incompatibility between the apparent shapes and the supposed real shape, and between the change in the appearances and the supposed constancy of the physical object. We need not at present ask *why* we believe that there is a single physical object with these characteristics, which appears to us in all these different ways. It is a fact that we do believe it. It is an equally certain fact that the penny does look different as we move about. The difficulty is to reconcile the different appearances with the supposed constancy of the penny, and the ellipticity of most of the appearances with the supposed roundness of the penny. It is probable that at first sight the reader will not see much difficulty in this. He will be inclined to say that we can explain these various visual appearances by the laws of perspective, and so on. This is not a relevant answer. It is quite true that we can *predict what particular appearance* an object will present to an observer, when we know the

shape of the object and its position with respect to the observer. But this is not the question that is troubling us at present. Our question is as to the compatibility of these changing elliptical appearances, however they may be correlated with other facts in the world, with the supposed constancy and roundness of the physical object.

Now what I call *Sensible Appearance* is just a general name for such facts as I have been describing. It is important, here as always, to state the *facts* in a form to which everyone will agree, before attempting any particular *analysis* of them, with which it is certain that many people will violently disagree. The fundamental fact is that we constantly make such judgments as: "This *seems to me* elliptical, or red, or hot," as the case may be, and that about the truth of these judgments we do not feel the least doubt. We may, however, at the same time doubt or positively disbelieve that this *is* elliptical, or red, or hot. I may be perfectly certain at one and the same time that I have the peculiar experience expressed by the judgment: "This looks elliptical to me," and that in fact the object is not elliptical but is round.

I do not suppose that anyone, on reflection, will quarrel with this statement of fact. The next question is as to the right way to analyse such facts; and it is most important not to confuse the facts themselves with any particular theory as to how they ought to be analysed. We may start with a negative remark, which seems to me to be true, and is certainly of the utmost importance if it be true. Appearance is *not* merely mistaken *judgment* about physical objects. When I judge that a penny looks elliptical I am not mistakenly ascribing elliptical shape to what is in fact round. Sensible appearances *may* lead me to make a mistaken judgment about physical objects, but they *need* not, and, so far as we know, commonly do not. My certainty that the penny looks elliptical exists comfortably along-

side of my conviction that it is round. But a mistaken judgment that the penny *is* elliptical would not continue to exist after I knew that the penny was really round. The plain fact is then that "looking elliptical to me" stands for a peculiar experience, which, whatever the right analysis of it may be, is not just a mistaken judgment about the shape of the penny.

Appearance then cannot be described as mistaken judgment about the properties of some physical object. How are we to describe it, and can we analyse it? Two different types of theory seem to be possible, which I will call respectively the *Multiple Relation Theory*, and the *Object Theory* of sensible appearance. The Multiple Relation Theory takes the view that "appearing to be so and so" is a unique kind of relation between an object, a mind, and a characteristic. (This is a rough statement, but it will suffice for the present.) On this type of theory to say that the penny looks elliptical to me is to say that a unique and not further analysable relation of "appearing" holds between the penny, my mind, and the general characteristic of ellipticity. The essential point for us to notice at present about theories of this kind is that they do not imply that we are aware of *anything* that *really is* elliptical when we have the experience which we express by saying that the penny looks elliptical to us. Theories of this type have been suggested lately by Professor Dawes Hicks and by Dr G. E. Moore. So far, they have not been worked out in any great detail, but they undoubtedly deserve careful attention.

Theories of the Object type are quite different. They do not involve a unique and unanalysable multiple relation of "appear*ing*," but a peculiar kind of object—an "appear*ance*." Such objects, it is held, actually *do have* the characteristics which the physical object *seems to have*. Thus the Object Theory analyses the statement that the penny looks to me elliptical into a statement which involves the actual existence of an

elliptical object, which stands in a certain cognitive relation to me on the one hand, and in another relation, yet to be determined, to the round penny. This type of theory, though it has been much mixed up with irrelevant matter, and has never been clearly stated and worked out till our own day, is of respectable antiquity. The doctrine of "representative ideas" is the traditional and highly muddled form of it. It lies at the basis of such works as Russell's *Lowell Lectures on the External World*. In this book I shall deliberately confine myself to this type of theory, and shall try to state it clearly, and work it out in detail.

The following additional works may be consulted with advantage:

G. E. MOORE, *Philosophical Studies*, V. and VII.
G. D. HICKS, *Proceedings of the Aristotelian Society*, 1913, 1916.
G. F. STOUT, *Manual of Psychology*, Bk. III., Part II. Cap. I.
" " *Proceedings of the Aristotelian Society*, 1913.

CHAPTER VIII

"JACK.—That, my dear Algy, is the whole truth, pure and simple.

"ALGERNON.—The truth is rarely pure and never simple. Modern life would be very tedious if it were either, and modern literature a complete impossibility."

(WILDE, *Importance of being Earnest.*)

The Theory of Sensa, and the Critical Scientific Theory

I PROPOSE now to state more fully the theory that appearances are a peculiar kind of objects, and to consider what sort of objects they must be. The reader will bear in mind throughout the whole of the long story which follows that there is a totally different view of sensible appearance, viz., the Multiple Relation Theory, and that this may quite possibly be true. In this book I shall leave it wholly aside. On the theory that we are now going to discuss, whenever a penny looks to me elliptical, what really happens is that I am aware of an object which is, in fact elliptical. This object is connected in some specially intimate way with the round physical penny, and for this reason is called an appearance *of* the penny. It really is elliptical, and for this reason the penny is said to look *elliptical*. We may generalise this theory of sensible appearance as follows: Whenever I truly judge that x appears to me to have the sensible quality q, what happens is that I am directly aware of a certain object y, which (a) really does have the quality q, and (b) stands in some peculiarly intimate relation, yet to be determined, to x. (At the present stage, for all that we know, y might sometimes be identical with x, or

might be literally a part of x.) Such objects as y I am going to call *Sensa*. Thus, when I look at a penny from the side, what happens, on the present theory, is at least this: I have a sensation, whose object is an elliptical, brown sensum; and this sensum is related in some specially intimate way to a certain round physical object, viz., the penny.

Now I think it must at least be admitted that the sensum theory is highly plausible. When I look at a penny from the side I am certainly aware of *something;* and it is certainly plausible to hold that this something is elliptical in the same plain sense in which a suitably bent piece of wire, looked at from straight above, is elliptical. If, in fact, nothing elliptical is before my mind, it is very hard to understand why the penny should seem *elliptical* rather than of any other shape. I do not now regard this argument as absolutely conclusive, because I am inclined to think that the Multiple Relation theory can explain these facts also. But it is at least a good enough argument to make the sensum theory well worth further consideration.

Assuming that when I look at a penny from the side I am directly aware of something which is in fact elliptical, it is clear that this something cannot be identified with the penny, if the latter really has the characteristics that it is commonly supposed to have. The penny is supposed to be round, whilst the sensum is elliptical. Again, the penny is supposed to keep the same shape and size as we move about, whilst the sensa alter in shape and size. Now one and the same thing cannot, at the same time and in the same sense, be round and elliptical. Nor can one and the same thing at once change its shape and keep its shape unaltered, if "shape" be used in the same sense in both statements. Thus it is certain that, if there be sensa, they cannot in general be identified with the physical objects of which they are the appearances, if these literally have the properties commonly assigned to them. On the other hand, all that I ever

come to know about physical objects and their qualities seems to be based upon the qualities of the sensa that I become aware of in sense-perception. If the visual sensa were not elliptical and did not vary in certain ways as I move about, I should not judge that I was seeing a round penny.

The distinction between sensum and physical object can perhaps be made still clearer by taking some wilder examples. Consider, *e.g.*, the case of looking at a stick which is half in water and half in air. We say that it looks bent. And we certainly do not mean by this that we mistakenly judge it to be bent; we generally make no such mistake. We are aware of an object which is very much like what we should be aware of if we were looking at a stick with a physical kink in it, immersed wholly in air. The most obvious analysis of the facts is that, when we judge that a straight stick *looks* bent, we are aware of an object which really *is* bent, and which is related in a peculiarly intimate way to the physically straight stick. The relation cannot be that of identity; since the same thing cannot at once be bent and straight, in the same sense of these words. If there be *nothing* with a kink in it before our minds at the moment, why should we think then of kinks at all, as we do when we say that the stick looks bent? No doubt we can quite well mistakenly *believe* a property to be present which is really absent, when we are dealing with something that is only known to us indirectly, like Julius Cæsar or the North Pole. But in our example we are dealing with a concrete visible object, which is bodily present to our senses; and it is very hard to understand how we could seem to ourselves to *see* the property of bentness exhibited in a concrete instance, if in fact *nothing* was present to our minds that possessed that property.

As I want to make the grounds for the sensum theory as clear as possible, I will take one more example. Scientists often assert that physical objects are not

"really" red or hot. We are not at present concerned with the truth or falsehood of this strange opinion, but only with its application to our present problem. Let us suppose then, for the sake of argument, that it is true. When a scientist looks at a penny stamp or burns his mouth with a potato he has exactly the same sort of experience as men of baser clay, who know nothing of the scientific theories of light and heat. The visual experience seems to be adequately described by saying that each of them is aware of a red patch of approximately square shape. If such patches be not in fact red, and if people be not in fact aware of such patches, where could the notion of red or of any other colour have come from? The scientific theory of colour would have nothing to explain, unless people really are aware of patches under various circumstances which really do have different colours. The scientists would be in the position of Mr Munro's duchess, who congratulated herself that unbelief had become impossible, as the Liberal Theologians had left us nothing to disbelieve in. Thus we seem forced to the view that there are at least hot and coloured sensa; and, if we accept the scientific view that physical objects are neither hot nor coloured, it will follow that sensa cannot be identified with physical objects.

The reader may be inclined to say, "After all, these sensa are not real; they are mere appearances, so why trouble about them?" The answer is that you do not get rid of anything by labelling it "appearance." Appearances are as real in their own way as anything else. If an appearance were nothing at all, nothing would appear, and if nothing appeared, there would be nothing for scientific theories to account for. To put the matter in another way: Words like *real* and *reality* are ambiguous. A round penny and an elliptical visual sensum are not real in precisely the same sense. But both are real in the most general sense that a complete inventory of the universe must mention the one as

much as the other. No doubt the kind of reality which is to be ascribed to appearances will vary with the particular type of theory as to the nature of sensible appearance that we adopt. On the present theory an appearance is a sensum, and a sensum is a particular existent, though it may be a short-lived one. On the Multiple Relation theory appearances have a very different type of reality. But *all* possible theories have to admit the reality, *in some sense*, of appearances; and therefore it is no objection to any particular theory that it ascribes a sort of reality to appearances.

I hope that I have now made fairly clear the grounds on which the sensum theory of sensible appearance has been put forward. Closely connected with it is a theory about the perception of physical objects, and we may sum up the whole view under discussion as follows: Under certain conditions I have states of mind called sensations. These sensations have objects, which are always concrete particular existents, like coloured or hot patches, noises, smells, etc. Such objects are called sensa. Sensa have properties, such as shape, size, hardness, colour, loudness, coldness, and so on. The existence of such sensa, and their presence to our minds in sensation, lead us to judge that a physical object exists and is present to our senses. To this physical object we ascribe various properties. These properties are not in general identical with those of the sensum which is before our minds at the moment. For instance, the *elliptical* sensum makes us believe in the existence of a *round* physical penny. Nevertheless, all the properties that we do ascribe to physical objects are based upon and correlated with the properties that actually characterise our sensa. The sensa that are connected with a physical object x in a certain specially intimate way are called the appearances of that object to those observers who sense these sensa. The properties which x is said to *appear to* have are the properties which those sensa that are

x's appearances *really do* have. Of course, the two properties may happen to be the same, *e.g.*, when I look straight down on a penny, both the physical object and the visual appearance are round. Generally, however, there is only a correlation between the two.

It follows from this theory that sensa cannot appear to have properties which they do not really have, though there is no reason why they should not have more properties than we do or can notice in them. This point perhaps needs a little more elaboration, since a good deal of nonsense has been talked by opponents of the sensum theory in this connexion. We must distinguish between failing to notice what is present in an object and "noticing" what is not present in an object. The former presents no special difficulty. There may well be in any object much which is too minute and obscure for us to recognise distinctly. Again, it is obvious that we may sense an object without necessarily being aware of all its relations even to another object that we sense at the same time. Still more certain is it that we may sense an object without being aware of all its relations to some other object which we are not sensing at the time. Consequently, there is no difficulty whatever in supposing that sensa may be much more differentiated than we think them to be, and that two sensa may really differ in quality when we think that they are exactly alike. Arguments such as Stumpf's render it practically certain that the latter possibility is in fact realised.

The real difficulty is when we seem to be directly aware of some property in an object, and this property is not really present and is perhaps incompatible with others which are present. This is the kind of difficulty that the sensum theory is put forward to meet. We seem to recognise elliptical shape in the penny, when the penny really has the incompatible quality of roundness. The solution which the sensum theory offers is to "change the subject." *Something*, it admits, is elliptical,

and something is round ; but they are not the same something. What is round is the penny, what is elliptical is the sensum. Now clearly, this would be no solution, if the same sort of difficulty were to break out in sensa themselves. In that case we should need to postulate appearances of appearances, and so on indefinitely.

We must hold, as regards positive sensible qualities which characterise a sensum as a whole and do not involve relations to other sensa, that a sensum is at least all that it appears to be. Now, so far as I know, there is no evidence to the contrary. Some people have thought that arguments like Stumpf's raised this difficulty ; but that is simply a mistake. Stumpf's argument deals merely with the relation of qualitative likeness and difference between different sensa, and shows that we may think that two of them are exactly alike when there is really a slight qualitative or quantitative difference between them. This has no tendency to prove that we ever find a positive non-relational quality in a sensum, which is not really there.

Next, we must remember that attributes which involve a negative factor often have positive names. A man might quite well think, on inspecting one of his sensa, that it was exactly round and uniformly red. And he might well be mistaken. But then, "exactly round" means "with no variation of curvature," and "uniformly red" means "with no variation of shade from one part to another." Now universal negative judgments like these can never be guaranteed by mere inspection ; and so, in such cases, the man is not "seeing properties that are not there" in the sense in which he would be doing so if a round sensum appeared to him to be elliptical. To sum up, it is no objection to the sensum theory that a sensum may seem to be *less* differentiated than it is ; it would be a fatal objection if a sensum ever seemed *more* differentiated than it is ; but we have no evidence that the latter ever happens.

Before going further we must remove a baseless prejudice which is sometimes felt against the sensum theory. It is often objected that we are not aware of sensa and their properties, as a rule, unless we specially look for them. It is a fact that it often needs a good deal of persuasion to make a man believe that, when he looks at a penny from the side, it seems elliptical to him. And I am afraid that very often, when he is persuaded, it is not by his own direct inspection (which is the only relevant evidence in such a matter), but by some absurd and irrelevant argument that the area of his retina affected by the light from the penny, is an oblique projection of a circle, and is therefore an ellipse. Accordingly, it is argued that we have no right to believe that such a man is directly sensing an object which is, in fact, elliptical. To this objection a partial answer has already been given, by implication. It is only when we are looking at a penny almost normally that any doubt is felt of the ellipticity of the sensum ; and, in that case, the sensum is, in fact, very nearly round. Now we have seen that it is no objection to our theory that a sensum which is not quite round should be thought to be exactly round, though it would be an objection if an exactly round sensum seemed to be elliptical. The reason, of course, is that an ellipse, with its variable curvature, is a more differentiated figure than a circle, with its uniform curvature. There is no difficulty in the fact that we overlook minute differentiations that are really present in our sensa ; difficulties would only arise if we seemed to notice distinctions that are not really present.

Apart, however, from this special answer, a more general reply can be made to the type of objection under discussion. The whole argument rests on a misunderstanding of the view about perception which the sensum theory holds. If the theory were that, in perceiving a penny, a man first becomes aware of a sensum, then notices that it is elliptical, and then infers from this

fact and the laws of perspective that he is looking at a round physical object, the argument would be fatal to the theory. But this is quite obviously not what happens. Perceptual judgments are indeed *based upon* sensa and their properties to this extent, that if we were not aware of a sensum we should not now judge that any physical object is present to our senses, and that if this sensum had different properties we should ascribe different properties to the physical object. But the relation between the sensum and its properties, on the one hand, and the perceptual judgment about the physical object, on the other, is not that of *inference*. The best analogy that we can offer to the relation between our sensing of a sensum and our perceiving a physical object, is to be found in the case of reading a book in a familiar language. What interests us as a rule is the meaning of the printed words, and not the peculiarities of the print. We do not explicitly notice the latter, unless there be something markedly wrong with it, such as a letter upside down. Nevertheless, if there were no print we should cognise no meaning, and if the print were different in certain specific ways we should cognise a different meaning. We *can* attend to the print itself if we choose, as in proof-reading. In exactly the same way, we are not as a rule interested in sensa, as such, but only in what we think they can tell us about physical objects, which alone can help or hurt us. Sensa themselves "cut no ice." We therefore pass automatically from the sensum and its properties to judgments about the physical object and its properties. If it should happen that the sensum is queer, as when we see double, we notice the sensum, as we notice an inverted letter. And, even in normal cases, we generally can detect the properties of sensa, and contrast them with those which they are leading us to ascribe to the physical object, provided that we make a special effort of attention.

From what has just been said, it will not appear

strange that, even though there be sensa, they should have been overlooked by most plain men and by many philosophers. Of course, everyone is constantly sensing them, and, in specially abnormal cases, has noted the difference between them and physical objects. But sensa have never been objects of special interest, and therefore have never been given a name in common speech. A result of this is that all words like "seeing," "hearing," etc., are ambiguous. They stand sometimes for acts of sensing, whose objects are of course sensa, and sometimes for acts of perceiving, whose objects are supposed to be bits of matter and their sensible qualities. This is especially clear about hearing. We talk of "hearing a noise" and of "hearing a bell." In the first case we mean that we are sensing an auditory sensum, with certain attributes of pitch, loudness, quality, etc. In the second case we mean that, in consequence of sensing such a sensum, we judge that a certain physical object exists and is present to our senses. Here the word "hearing" stands for an act of perceiving. Exactly the same remarks apply to sight. In one sense we see a penny; in a somewhat stricter sense we see only one side of the penny; in another sense we see only a brown elliptical sensum. The first two uses refer to acts of perceiving, the last to an act of sensing. It is best on the whole to confine words like "seeing" and "hearing" to acts of perceiving. This is, of course, their ordinary use. I shall therefore talk of seeing a penny, but not of seeing a brown elliptical sensum. I shall speak of the latter kind of cognition as "visually sensing," or merely as "sensing," when no misunderstanding is to be feared by dropping the adjective. This distinction will be found important when we come to deal with illusory perceptions.

I have now tried to clear up certain ambiguities in the sensum theory, and to remove certain mistaken objections which many folk feel against it. If it be

admitted that there *may* be such things as sensa, and that the sensum theory at least provides a possible and even plausible way of analysing sensible appearance, we can pass to the question of the nature of sensa and their status in the universe. This splits into two questions, viz., (i) the relation of sensa to minds; and (ii) their relation to physical objects. Neither of these can be completely answered at the present stage, but we can say a good deal here that is relevant, and will be useful, about them.

(i) **Are Sensa in any way Mental?**—Sensa have been supposed by many philosophers to be in some way mental. This opinion is based partly on sheer verbal confusions, and partly on genuine facts. The verbal confusion is that the word "sensation" has often been used ambiguously, and that, in one of its meanings, it does undoubtedly stand for something that is mental. When a man talks of a "sensation of red," he is sometimes referring to a red patch which he senses, sometimes to his act of sensing the patch, and sometimes to the whole complex state of affairs which, on the sensum theory, is analysable into (act of sensing)—directed on to—(red patch). In the second meaning, "sensation" is obviously mental; in the third it is undoubtedly a complex whole which involves a mental factor. In the first meaning it is by no means obvious or even plausible to say that a sensation is mental. I shall always use "sensation" in the third meaning. Now, as the same name is thus often used, both for the patch and for something which undoubtedly is mental, or is a complex, involving a mental factor, it is not surprising that some people should have been inclined to think that the red patch is itself mental. For is it not a "sensation"? And is not a sensation a mental state? This is, of course, mere verbal confusion, and need not trouble us further. But philosophers who have not fallen into this confusion between sensum, sensation, and act of

sensing, have yet held that sensa are mental. The most important living holder of this view is Professor Stout (at any rate he held it at the time when he wrote the last edition of his *Manual of Psychology*).

Before we can profitably carry the discussion of this point further, we must clear up the various meanings which can be attached to the statement "x is mental." (1) The first distinction that we must draw is between being "a state of mind" and being "mind-dependent." It is commonly held (and I do not here propose to question it) that whatever is a state of mind is mind-dependent, *i.e.*, that it could not exist except as a constituent of *a* mind, and, in fact, that it could only exist as a constituent of *that particular* mind, whose state it is said to be. An example would be my belief that $2+2=4$ or my desire for my tea. But it seems perfectly possible that a term might be mind-dependent without being a state of anyone's mind. What would this mean? I think it would mean that such a term can only exist as a constituent of a *state of mind*, but that it is not itself a constituent of a *mind*. Take some admitted state of mind, such as my perception of my table. There is clearly an important sense which we can all recognise, even though none of us can define it, in which it is true to say that this perception is a constituent of my mind, whilst the table is not. I should say that there was also an important (though very different) sense in which it is true to say that the table is a constituent of my perception of it, so long as that perception lasts. It is thus quite common for a term to be a constituent of one of my states of mind without being a constituent (and therefore without being a state) of my mind. Now, if chairs are anything like what they are commonly supposed to be, they do not *only* exist as constituents of states of mind, since it is commonly believed that such things go on existing with little or no change of quality when we cease to perceive them. But, just as states of mind can only

exist as constituents of minds, so there *might* be terms which can *only* exist as constituents of states of mind. Such terms would be mind-dependent without being states of mind. If Berkeley's famous saying that "the essence of a sensible object is to be perceived" be taken quite literally, it implies that such objects are mind-dependent, whilst it does not *imply* (though it is, of course, *consistent with*) the view that they are states of mind.

(2) Even when this distinction has been drawn, there is a possibility of confusion. We must distinguish a more and a less radical sense of "mind-dependence." The sense just discussed is the more radical, and may be termed "existential mind-dependence." A term that is existentially mind-dependent, though not a state of mind, can only exist as a constituent of a certain state of mind. But a term which was not existentially mind-dependent, might be to a certain extent "qualitatively mind-dependent." By this I mean that, although it can exist and have qualities when it is not a constituent of any state of mind, it might acquire some new qualities or alter some of its old qualities on becoming a constituent of a state of mind. It is certain that everything that at some period in its history becomes a constituent of any state of mind thereby acquires at least one new quality, viz., that it is now cognised, or desired, or shunned, or so on, by that mind. And I do not see any reason in principle why these changes of relation should not produce changes in the non-relational qualities of the object. If wax melts when brought into the relation of proximity to a fire, I know no reason why some qualities of an object should not be added or modified when it comes into the relation of being sensed by a mind.

(3) Some psychologists, of whom Stout is one, draw a fundamental distinction between two sorts of states of mind. They divide them into acts and non-acts. And a state of mind which is not an act they call a *presentation*.

I propose to state this distinction in a different way, for reasons which I will now explain. A little while ago I took my perception of my table as an undoubted example of a state of mind. And I said that there was no doubt that the table is a constituent of it. That is, I took the whole complex situation (my perceiving)—of—(table) as a state of mind. What Stout calls an "act" is "my perceiving." He calls this a "state of mind," I call it a "constituent of a state of mind." The table is not a constituent of the state of mind, in Stout's sense of the word, whilst it is a constituent of the state of mind, in my sense of the word. In my terminology the act may be described as the non-objective constituent in a state of mind whose other constituent is its object. An act is something which cannot exist by itself, but can only exist as a constituent in a complex, whose other constituent is its object. And it is, of course, the characteristically mental factor in such a complex, since the other constituent may (though it need not) be non-mental. My reason for calling the whole complex fact, and not the act itself, a state of mind, is the following: Practically everyone agrees that there are such things as states of mind. And practically everyone agrees that the phrase "my perception of the table" describes something real. But people differ greatly as to the right analysis of this fact, and the notion of "act" is connected with one special mode of analysis which would not be accepted by everyone. It therefore seems better to give the name "state of mind" to the *fact* which everyone admits to exist, and not to a supposed constituent, which some people deny to be present in it.

It is quite easy to restate the distinction which Stout has in mind in terms of my phraseology. Some mental states can be analysed into an act directed on an object. These are non-presentational states of mind. Others cannot be analysed into act and object. These are presentations. A non-presentational state may contain a

presentation as object. For instance, a feeling of toothache would be a presentation on Stout's view. For, according to him, it *is* mental and is *not* analysable into an act of sensing and a "toothachy" object; it is just a "toothachy" state of mind. Now, if I were to introspect my toothache, in order to describe it to my dentist, my introspection would be a non-presentational mental state whose object is a presentation; for it is a complex containing an act of introspecting directed on to a toothachy feeling. The perception of a chair would be an example of a non-presentational mental state, whose object is not a presentation, because not mental.

We are now in a better position to deal with the question: "Are sensa mental?" This might mean (1) Are they acts? (2) Are they states of mind analysable into act and object? (3) Are they presentations? (4) Are they existentially mind-dependent, though not states of mind? (5) Are they to some extent qualitatively mind-dependent, though not existentially mind-dependent?

No one has ever suggested that sensa are acts or that they are states of mind analysable into act and object. A red patch sensed by me when I look at a pillar-box is an example of a sensum. It is plausible to hold that the whole fact known as "my sensation of the red patch" is a state of mind, analysable into act of sensing and red patch sensed. But there would be no plausibility in holding that the red patch itself was an act, or that it was itself divisible into act and object. Thus, if sensa be states of mind at all, they must be presentations. Now, there are two very different views included under the statement that sensa are presentations. The first would deny the analysis of "my sensation of red patch" into act of sensing and red sensum. It would treat the whole thing as an unanalysable state of mind, and therefore as a presentation. This view would hold that there is no real distinction between sensa and sensations. It would say that "sensation of red patch" = "red patch sensed," and

is a presentation.* The second view would admit that in my sensation of red we can distinguish my act of sensing and the red patch sensed; but it would hold that the red patch is itself a state of mind, and, being indivisible into act and object, is a presentation. I do not think that most philosophers have very clearly distinguished these two varieties of the presentational theory of sensa. Moreover, those philosophers who have accepted the analysis of sensations into acts of sensing and sensa, and have asserted that sensa are mental, have seldom clearly distinguished the alternatives that sensa are presentations and that sensa are mind-dependent without being states of mind. And lastly, the distinction between existential and qualitative mind-dependence has not always been clearly seen. So that there is a very pretty mess for us to wipe up as well as we can.

(1) *Are Sensations analysable into Act of Sensing and Sensum?* The most plausible argument against this analysis would seem to be the following: If we consider the various experiences called "sensations," we seem to be able to arrange them in an order, starting with those of sight, passing through those of taste and smell, and ending with bodily sensations, like headache. Now, as regards the top members of the series, the analysis into act of sensing and object sensed seems pretty clear. A sensation of red seems clearly to mean a state of mind with a red object, and not to mean a red state of mind.

If we now pass to the other end of the series the opposite seems to be true. It is by no means obvious that a sensation of headache involves an act of sensing and a "headachy" object; on the contrary, it seems on the whole more plausible to describe the whole experience as a "headachy" state of mind. In fact the distinction of act and object seems here to have

* This seems to be Stout's view in the *Manual of Psychology*, but I may be misinterpreting him.

vanished; and, as there is clearly *something* mental in feeling a headache, just as there is in sensing a red patch, it seems plausible to hold that a sensation of headache is an unanalysable mental fact, within which no distinction of act and object can be found.

Now this contrast between the top and the bottom members of the series would not greatly matter, were it not for the fact that the two kinds of sensation seem to melt insensibly into each other at the middle of the series. It is about equally plausible to analyse a sensation of a sweet taste into an act of sensing and a sweet sensum, or to treat it as an unanalysable mental fact, having no object, but possessing the property of sweetness. Common speech recognises these distinctions. We talk of a sensation of red, but never of a feeling of red or of a red feeling. On the other hand, we talk indifferently of a sensation of headache, a feeling of headache, a headachy sensation, and a headachy feeling. The English talk of a sensation of smell, whereas the Scots more usually speak of "feeling" a smell. Now sensations of smell are just on the borderline between the two kinds of sensation. The rule is that, when a sensuous experience seems clearly to involve act and object, it is called a sensation and never a feeling; when it is doubtful whether any such analysis can be applied, it is called indifferently a feeling or a sensation.

Now the fact that all these experiences are classed together as sensations, and that the two kinds melt into each other at the middle of the series, naturally tempts men to treat them all alike. If we do this, we must hold either (α) that it is a mistake to think that a sensation of red *can* be analysed into an act of sensing and a red sensum; or (β) that it is a mistake to think that a sensation of headache *cannot* be analysed into an act of sensing and a headachy sensum. The former alternative makes sensation and sensum fall together into a single peculiar state, even in the case of sight;

and, since the experience as a whole certainly is mental, we have to say that a sensation of red = a red sensum = a feeling or presentation which is red. The second alternative is that which is taken by Realists, like Professors Laird and Alexander.

Now it is evident that, if you insist on treating all experiences which are called "sensations" in the same way, it is antecedently as reasonable to take the Laird-Alexander alternative as the Presentationist alternative. You might argue: "It is obvious that a sensation of red involves an act of sensing and a red sensum, so a sensation of headache must involve an act of sensing and a headachy sensum." Thus the mere fact that sensations can be arranged in a series, such as I have described, does not *specially* favour the presentationist view; since exactly the same type of argument, starting from the other end of the series, would lead to exactly the opposite conclusion. There are just two remarks that seem to me worth making at this point.

(*a*) I do not find either the realist or the presentationist view very satisfactory as a complete account of all the experiences which are called "sensations." But, if I were forced to take one alternative or the other, I should prefer the former. It seems to me much more certain that, in a sensation of red, I *can* distinguish the red patch and the act of sensing it, than that, in a sensation of headache, I *cannot* distinguish a headachy object and an act of sensing it. (*b*) I think, however, that there is no need to insist on the realist analysis of bodily feelings in order to deal with the question whether sensations be analysable into act of sensing and sensum. It seems to me that the simplest and least doubtful way of treating the whole question raised by the series of sensations is the following: The word "sensation," as commonly used, is defined, not by direct inspection, but by causation. We say that we are having a sensation, if our state of mind is the immediate response to the stimulation of a nerve. Now, since sensations are not defined

psychologically through their intrinsic properties, but physiologically through their bodily antecedents, it is surely very likely that they may include two very different kinds of experience, one of which can and the other cannot be analysed into act of sensing and sensum. These might be called respectively "true sensations" and "bodily feelings." The mere fact that both are often *called* "sensations" is surely a very poor reason for insisting that the structure of both must be the same. It is true indeed that there are marginal cases of which it is very difficult to say into which class they fall. But this ought not to make us slur over the plain introspective difference between the top and the bottom members of the series. The top ones at least do seem quite clearly to involve acts of sensing and sensa on which these acts are directed. It does seem clear that, when I have a sensation of a red triangular patch, some things are true of the patch itself (*e.g.*, that it is red and triangular) which it is very difficult to believe to be true of my sensation of the red patch. If so, it seems necessary to hold that the sensation and the sensum are not identical; that the sensum is an objective constituent of the sensation; and that there is another constituent which is not objective and may be called "the act of sensing." Into the question whether this latter factor is capable of further analysis, and, if so, what the right analysis of it may be, it is fortunately not necessary to go for our present purposes.

I conclude, then, that some sensations at least are analysable into act of sensing and sensum, and therefore that we cannot argue that sensum = sensation = a presentation.

(2) *Are Sensa, though distinct from Sensations, themselves Presentations?* Though sensations are not presentations but contain objects, which are sensa, it is perfectly possible that these objects might themselves be presentations. To prove that sensa are presentations, it would be necessary to prove that they are states of

mind. And this involves proving (*a*) that they are existentially mind-dependent, and (*b*) that they are constituents of minds and not merely of certain states of mind. Obviously it might be possible to prove the first, even if it were not possible to prove the second, of these propositions. I do not know of any reasonably plausible argument to prove that sensa are not merely mind-dependent, but are also states of mind, once you accept the view that sensa must be distinguished from sensations. Indeed, the assertion would be open to the same kind of objection which we made to the view that sensa and sensations can be identified. On either view something is said to be a state of mind, though it possesses properties which it is very difficult to ascribe to states of mind. If a sensum be a state of mind, then there are states of mind which are literally red or round or hot or loud or triangular, and so on. I have no difficulty in believing that many states of mind *contain* such terms as objects, but I do find it very difficult to believe that any state of mind actually *is* a term of this sort. Yet the latter is implied by the statement that sensa are presentations, just as much as by the statement that sensations are presentations. In fact, the reasons which forced us to distinguish sensations from sensa, and to regard the latter as objects contained in the former, equally forbid us to treat sensa themselves as states of mind. This objection *may*, of course, be a mere prejudice; but it *is* worth while to point out that the view that sensa are presentations does logically imply the very paradoxical propositions that some states of mind are literally hot or red or round, for most philosophers who have held the view under discussion have successfully concealed this consequence from themselves and their readers. I shall therefore reject the view that sensa are states of mind, until someone produces much better reasons than anyone has yet done for believing such an extremely paradoxical proposition.

There are, however, quite plausible arguments to

prove that sensa are existentially mind-dependent, though not states of mind. That is to say, that, although sensations are analysable into act and sensum, and the sensum must therefore be distinguished both from the sensation and from the act of sensing, which is the other factor in the sensation, yet these two factors are not capable of existing separately from each other. No act of sensing without some sensum on which it is directed, and no sensum without an act of sensing directed upon it. The arguments for this view are three : (*a*) The privacy and variability of sensa ; (*b*) the analogy between sensa and bodily feelings ; and (*c*) the analogy between sensa and so-called "mental images."

(*a*) We notice at once that sensa have some of the characteristics of physical objects and some of those of mental states. On the one hand, they are extended, and have shapes, sizes, colours, temperatures, etc. On the other hand, they do seem to be private to each observer ; and this, it will be remembered, is one of the chief marks of the mental as distinct from the physical. It is at least doubtful whether two people, who say that they are perceiving the same object, are ever sensing the same sensum or even two precisely similar sensa. This does suggest that sensa are mental—at any rate in the sense of being mind-dependent.

If, however, we look more closely, we see that this conclusion does not necessarily follow. The facts are on the whole much better explained by supposing that the sensa which a man senses are partly dependent on the position, internal states, and structure of his body. Since no two men's bodies can be in precisely the same place at precisely the same time, it is not surprising that the sensa of the two men should differ. And, since the internal states and the minute structure of no two living bodies are exactly alike, it is still less surprising. Now this explanation not only accounts *as* well for most of the facts as the view that sensa are mind-dependent ; it accounts a great deal better for some of the most striking

of the facts. The orderly variation in the shapes of visual sensa, as we move about, is intelligible if we suppose that the sensa which we sense are partly conditioned by the positions of our bodies. The assumption that they depend on our minds gives no explanation whatever of such facts.

There is, however, a better form of this argument, which has, I think, been somewhat neglected by people who want to hold that sensa are *never* mind-dependent to any degree. It does seem to me undeniable that in certain cases, and to a certain extent, our past experiences and our present expectations affect the actual properties of the sensa that we sense, and do not merely affect the judgments about physical objects which we base upon sensa. We shall go into this point in some detail in a later chapter; at present I will just illustrate my meaning by two examples.

When I look at the "staircase figure," which is given in most psychology text-books as an instance of ambiguous figures, it seems to me that it actually looks sensibly different from time to time. Its sensible appearance changes "with a click," as I look at it, from that of a staircase to that of an overhanging cornice. This change tends to take place as I concentrate my mind on the idea of the one or on that of the other. Now, on the present analysis of sensible appearance, such a change as this involves an actual qualitative change in the sensum. So far is it from being a mere change in the judgments which I happen to base on one and the same sensum, that the direction of my thoughts changes first and is the condition of the change in the sensible appearance.

Again, when I turn my head, the visual sensa are not as a rule affected with any sensible movement. If, however, I put my glasses a little out of focus or look through a window made of irregularly thick glass, and then turn my head, the sensa do sensibly move. Whether they move or keep still seems to depend on

my past experiences and my present expectations about physical objects. The whole psychology of vision is full of such cases, some of them of a highly complex kind.

Now, of course, these examples do not suggest for a moment that sensa are existentially mind-dependent, but they do strongly suggest that they are to some extent qualitatively mind-dependent. And it cannot be said here, as in the previous examples, that reference to the mind gives no help in explaining the facts. Here the boot is rather on the other foot. No doubt the facts just mentioned could *in theory* be accounted for by referring to the past history of the *body*, in addition to its present state and position. *I.e.*, we could talk learnedly about the traces left on our brains and nervous systems by the past experiences, and could say that they are among the conditions of our sensa. But this would not help us to explain any concrete characteristic of our sensa in any particular case. For the plain fact is, that we do often know what relevant experiences we or others have had, whilst we know nothing whatever in detail about traces in the brain and nervous system. So here a reference to *mental* conditions really does explain concrete facts, whilst a reference to *bodily* conditions does not. We shall have to return to this point at a much later stage.

(*b*) We have already noticed the arrangement of "sensations" in a scale from sensations of colour and sound to bodily feelings. We saw that this might be used as an argument to prove that even sensations of colour and sound are presentations, or equally as an argument to prove that even sensations of headache are divisible into act and object. Suppose we take the latter alternative, which, as I have said, seems to me to be the more plausible of the two, though I do not think that the facts compel us to adopt either. It is then possible to produce a fairly plausible argument for the view that sensa are existentially mind-dependent.

The argument would run as follows: "Granted that a sensation of headache can be analysed into act of sensing and headachy sensum, it is surely obvious that the latter, from its very nature, could not exist without the former. An unfelt headache is surely a mere *Unding*. Now, if this be true of headachy sensa, does not the very continuity of the series of sensations on which you have been insisting make it likely to be true of red sensa, and indeed of all sensa? If so, sensa will be from their very nature existentially mind-dependent and incapable of existing save as objective constituents of sensations."

I think that this is quite a plausible argument, but I do not think it conclusive. Two questions could be asked about it. (α) Supposing it to be true that an unfelt headache is inconceivable, does the continuity of the series of experiences called "sensations," justify us in extending this conclusion to all sensa, and, in particular, to those of sight and hearing? Secondly (β), is it really true that an unfelt headache is inconceivable? (α) To the first question I answer that, as a matter of fact, I do not find the slightest intrinsic difficulty in conceiving the existence of unsensed red patches or unsensed noises, whilst I do find a considerable difficulty in conceiving the existence of unfelt headaches. I do not think that it is safe to reject this plain difference on the grounds of a mere argument from continuity.

(β) Moreover, I think I can see why it seems so difficult to conceive of the existence of unfelt headaches, and can see that this difficulty is not really conclusive. Our main interest in bodily feelings is that they are pleasant or painful; sensations of sight are, as a rule, intrinsically neutral, or nearly so. Now I am quite prepared to believe that an object has to be cognised by us in order to be pleasant or painful to us. For it seems to me that the pleasantness or painfulness of anything is (or, at any rate, depends upon) my recognising it and taking up a certain attitude of liking or

disliking to it. It might, therefore, be perfectly true that an unfelt headache would not be *a pain*, just as an unmarried woman is not a wife. Since we are mainly interested in headaches as pains, we are inclined to think that an unfelt headache would be *nothing*, when the truth merely is that it would not be a *pain*. This would be comparable to the mistake which a fanatical admirer of matrimony would make if he ignored the existence of all spinsters because they were not wives. I, therefore, am not convinced that, if a feeling of headache be a genuine sensation and not a mere presentation, the headachy sensum which it contains could not exist unsensed. Still less could I extend this view to sight and sound sensa.

(*c*) The third argument for thinking that sensa are incapable of existing unsensed is founded on their resemblance to "mental images," whose very name implies that they are commonly supposed to be existentially mind-dependent, if not actually states of mind. The resemblances must be admitted, though in favourable cases there seems to be some intrinsic difference which it is easy to recognise but hard to describe. But it seems to me doubtful whether images are existentially mind-dependent. I do not see any very obvious reason why there should not be "unimaged" images. It is, of course, perfectly true that images are to a much greater extent qualitatively mind-dependent than are sensa. Most, if not all, of them depend on our past experiences; and many of them depend in part on our present volitions. Voluntary images do, no doubt, depend on our minds, in the sense that they would not be imaged here and now, if we did not *will* them. But exactly the same is true of many things, which no one would think of calling existentially mind-dependent. Most chemical reactions that take place in a laboratory would never have happened if someone had not deliberately mixed the reagents in a flask and heated the latter over a flame. No one supposes that this renders

such reactions in any important sense mind-dependent. Thus the fact that some images are voluntary seems irrelevant to the present subject.

The other point, that all images that we can now image are in part determined in their characteristics by our past experiences, is more important. It must be counted along with the fact, already admitted, that many sensa are to some extent qualitatively mind-dependent. Here, as before, we can, if we like, substitute a reference to traces in our brains and nervous systems. But here, too, the doubt remains whether this kind of explanation is ultimately of much philosophic importance, in view of the fact that we often know directly what our relevant past experiences are, whilst the traces, etc., of the physiologist are purely hypothetical bodily correlates of these. Further treatment of this subject must be deferred till we face the problem of the part played by our own bodies in sensation and imagination.

I will now try to sum up the results of this rather long and complex discussion on the relation of sensa to minds and their states. The sensum theory is bound up with a special view as to the right analysis of the kind of fact which is described by such phrases as "my sensation of x." It holds that this is complex, and that within it there can be distinguished two factors —x itself, which is the sensum and is an object, and a subjective factor, which is called the "act of sensing." The latter may, of course, be capable of further analysis, such, *e.g.*, as Russell attempts in his *Analysis of Mind;* or it may be (or contain) a peculiar unanalysable relation. Now, there is also a theory which refuses to analyse "my sensation of x" in this way. It holds that the whole thing is unanalysable into act and object. On such a view the distinction between sensum and sensation vanishes; and the experience, which may be called indifferently by either name, is a mental state of the kind called *presentations*. This view is supported by reference to bodily feelings, and by an argument

from the continuity between them and the higher sensations. As against this we pointed out (*a*) that there is just as good reason to use the argument from continuity in the opposite direction ; and (*b*) that very possibly, in spite of the continuity, there is a real difference in nature between genuine sensations and bodily feelings. In favour of the view that genuine sensations are analysable into act and object, we pointed out that there seems to be a plain difference between a red patch sensed by me and the total fact described as "my sensation of a red patch." And we suggested that those who refuse to make this analysis are forced to the very paradoxical conclusion that there are states of mind which are literally red, round, hot, loud, etc.

The next point was this. Assuming that sensations are analysable into act of sensing and sensum, we raised the question whether sensa are states of mind, or, if not, whether they are existentially mind-dependent. We agreed that, if they are states of mind at all, they must be presentations. But we found no positive reason *for* thinking that they are states of mind, and much the same reasons *against* that view as led us to hold that sensations are analysable into act and sensum.

We then discussed three more or less plausible arguments to show that sensa are existentially mind-dependent, *i.e.*, that they cannot exist except as objective constituents of sensations. We saw no intrinsic reason why coloured patches or noises should not be capable of existing unsensed. And we refused to be moved from this view by an argument from continuity with bodily feelings. For we were far from sure whether bodily feelings really are analysable into act of sensing and sensum ; and we suggested that, even if they be, it is by no means certain that their sensa could not exist unsensed. We tried to show why this was *thought* to be obvious, and to show that it is not really so.

The two remaining arguments seemed to us to show that sensa are partly dependent on the position, etc.,

of the *body*, but they did not have any tendency to show that they are *existentially* dependent on the *mind*. Still, some of the facts adduced did rather strongly suggest that sensa and, *a fortiori*, images, are to some extent *qualitatively* mind-dependent. We thought that this reference to the mind *might* be removed by extending the bodily conditions, so as to include physiological traces and dispositions. But, in view of the wholly hypothetical character of these, we were not prepared at this stage to deny that sensa and images might be to some extent qualitatively mind-dependent. And there we leave the matter, till we deal more fully with the part played by the human body in sense-perception.

We have seen that the whole question is highly complex, and that the arguments for the view that sensa are mental are by no means lacking in plausibility. We shall not therefore be tempted to think that everyone who has been persuaded by them must be either a knave or a fool. Some of those who call themselves *New Realists* have been too much inclined to take this attitude; and, on one reader at least, they have produced the impression of being rather offensively "at ease in Zion."

(ii) **How are Sensa related to Physical Objects?**—We can now turn to the second question which we raised about sensa. The plain man does not clearly distinguish between physical objects and sensa, and therefore feels no particular difficulty about their mutual relations. We first come to recognise sensa as distinct from physical objects by reflecting on the fact of sensible appearance, and the contrast between it and the supposed properties of physical reality. But once the existence of sensa has been clearly recognised, the problem of their relation to the physical world becomes pressing. We all believe in a world of physical objects, and profess to have a great deal of detailed knowledge about it. Now this world of physical objects makes

its existence and its detailed nature known to us by the sensible appearances which it presents to us. And, on the sensum theory, these appearances are sensa. Sensa are therefore in some way the *ratio cognoscendi* of the physical world, whilst the physical world is in some way the *ratio essendi* of sensa. Our problem therefore divides into an epistemological and an ontological one. The two problems are not ultimately independent, but it is useful to state them separately.

(1) How far is it true that our beliefs about the physical world depend on our sensa? Before we can answer this, we must draw some distinctions among our beliefs. First, there is our belief that there is a physical world of *some* kind. This, as we have seen, involves at least the belief that there are things which are relatively permanent, which combine many qualities, and which persist and interact at times when they are not appearing to our senses. These we may call *constitutive* properties of the physical world, since they are part of what we mean by "physical." Then there is the belief that these objects have spatial or quasi-spatial characteristics. This may almost be called constitutive, but it is a shade less fundamental than the first set of properties. Lastly, there are what might be called *empirical* beliefs about the physical world. These are beliefs about points of detail, *e.g.*, that some things are red, and that there is now a red fluted lamp-shade in my rooms.

Now I have already asserted that it is false psychologically to say that we, in fact, reach our perceptual judgments about the existence and properties of physical objects by a process of inference from our sensa and their properties. Further, it is false logically to suppose that the *existence* of a physical world in general could be inferred from the existence of our sensa, or from anything that we know about their intrinsic properties or their mutual relations. I suppose that the existence of sensa is a necessary condition, but it is certainly not

a sufficient condition, of my belief in the existence of the physical world. If there were no sensible appearances to me, I suppose that I should not judge there to be any physical reality. But, on the other hand, there is nothing in my sensa to force me logically to the conclusion that there must be something beyond them, having the constitutive properties of physical objects. The belief that our sensa are appearances of something more permanent and complex than themselves seems to be primitive, and to arise inevitably in us with the sensing of the sensa. It is not reached by inference, and could not logically be justified by inference. On the other hand, there is no possibility of either refuting it logically, or of getting rid of it, or—so far as I can see—of co-ordinating the facts without it.

There are groupings among my own sensa and correlations between my sensa and those of others which fit in extremely well with the belief in a physical world of which all the sensa are so many appearances. It might be held that this at least forms the basis of a logical argument in inverse probability, to show that the belief in the physical world is highly probable. But the snag here is that all such arguments only serve to multiply the antecedent probability of a proposition, and, unless we have reason to suppose that this probability starts with a finite magnitude, they lead us nowhere. Now, although I do not know of any reason antecedently against the existence of a physical world, I also know of no antecedent reason for it. So its antecedent probability seems quite indeterminate, unless we are prepared to hold that the fact that everybody does in practice believe it, is a ground for ascribing a finite antecedent probability to it. It seems to me that the belief that there is a physical world is logically in much the same position as those assumptions about the constitution of the existent on which all inductive proofs of special laws of nature rest. If these assumptions start with a finite antecedent probability,

their success justifies us in ascribing a high final probability to them. But do they have a finite antecedent probability? We can say of them, as of the belief in a physical world, that we all do believe them in practice, that there is no positive reason against them, and that we cannot get on without assuming them. But, having said so much, we shall do wisely to change the subject and talk about the weather.

We shall not then attempt to *prove* the existence of a world of entities having the constitutive properties of physical objects; for, if this can be done, I at any rate do not know how to do it. But we shall point out those facts about our sensa and their groupings which specially fit in with the view that sensa are various partial and fleeting appearances of relatively permanent and independent things. That is, we shall try to indicate those facts about our sensa which *would* give a high final probability to the belief in a physical world, *provided* it had a finite antecedent probability. This will be our main task in the next two chapters, which deal with the spatial and temporal characteristics of sensa and of physical objects and events. The first of these chapters will be concerned with the facts about our sensa which fit in with the view that they are appearances of objects which combine many properties, and which can be perceived by many different observers at the same time. The second will be concerned with the facts about our sensa which fit in with the view that they are relatively fleeting appearances of more permanent things and processes.

Now, assuming that there is a world of enduring and independent things, there is still room for wide differences of opinion as to the kind of whole that it forms, the way in which it is divided into parts, and the various empirical qualities which these parts possess. Common-sense and science are agreed that it is in some sense a spatial whole, whose parts have various shapes, sizes and positions, and are capable of moving about

within the whole. This alleged spatial character of the physical world may be called "semi-constitutive"; for, as I have said, we hardly admit that a world of non-spatial entities would deserve to be called "physical," even though it were persistent, independent of us, and many-qualitied. Now, it is clear that all the spatial characteristics which we ascribe to the physical world are based, both in general outline and in detail, on the spatial characteristics of our sensa. Moreover, I think it can be rendered highly probable that, if there be a physical world at all, and our sensa be appearances of it, then that world is quasi-spatial. The importance and complexity of this subject seem to justify the length of the next chapter, in which I have treated it to the best of my ability.

When we come to the purely empirical qualities of the physical world there is a sharp difference of opinion between science and common-sense. The latter ascribes qualities, like colour, temperature, etc., to physical objects, whilst the former refuses to do so. In discussing this matter the partial dependence of sensa on what goes on inside the body of the observer becomes of great importance, and the concluding chapter has been devoted to this problem.

(2) This last question leads in the most natural way to the ontological problem as to the status of sensa in the existent world. There is a world of physical objects and a world of sensa. In some way the latter seems to be dependent on the former. But both are parts of the whole of existent reality. How are the two related? This is a problem which common-sense ignores, because it does not definitely distinguish between sensa and physical objects. Science also ignores it, because, although in theory it makes an equivalent distinction, it uses it simply as an excuse for ignoring sensa and concentrating on physical objects and processes. This is a perfectly legitimate procedure for the special purpose which natural science has in view, but it is not

permissible to the philosopher. His whole business is to drag such skeletons from the cupboards in which it has been found convenient to shelve them, and to give them their right place in the whole scheme of things.

Now the epistemological and the ontological problems about sensa and their relations to physical objects are connected in the following way. Our primitive belief in the existence of a world of relatively permanent, independent things is extremely vague. It is little more than a general scheme, in terms of which the actual groupings which we find among our sensa are stated. Even when we go a step further, and say that the spatial character and the special groupings of sensa practically force us to think of the physical world as a quasi-spatial whole, containing parts with fairly definite shapes, sizes, and positions, we still have only a very general, though much more definite scheme. Within this general quasi-spatial scheme all kinds of alternative specifications are possible. We are not tied down to any special view as to the number of its dimensions. Again, we are not tied down to any special view as to the "geometry" of it, when the number of its dimensions is settled. Lastly, we might put forward dozens of different theories as to the nature of physical objects, all compatible with the general scheme and with the special facts about our sensa and their groupings. It is this extreme variety of alternative theories, left open to us by the general concept of a physical world and the special facts about our sensa, which gives a legitimate hope for indefinite progress with the problem under discussion, provided the scientists and the patriots between them do not destroy civilisation, and with it all disinterested thinking. With traditional views about the nature of Space, Time, and Matter, it is extremely difficult to fit the world of sensa and the world of physical objects together into a coherent whole. But, once the immense number of possible alternatives within the scheme is grasped, the devising of theories of the physical object which shall

give sensa a *locus standi* in the physical world will be a winter evening's pastime for symbolic logicians. This task we shall leave to those better fitted than ourselves to accomplish it; we shall be concerned rather with those facts about our sensa with which any theory of physical objects must deal.

The Critical Scientific Theory.—I propose now to try to state clearly, in terms of the Sensum theory, what appears to be involved in the common scientific view of physical objects and their sensible appearances. As scientists never state their own position on this point clearly, it is necessary for us to do so for them. We can then see how far the view can be accepted, and how far its plausibility has depended on its modest obscurity.

Let us take the old example of a boy looking at a penny. He believes that it is quite literally round and just as literally brown. He believes that the brown (and, as he thinks, round) patch which he is sensing is quite literally a part (viz., the upper side) of the penny. And he believes that this, which he now sees, is the same as what he can feel if he puts out his hand. As he grows up he is probably told, on the authority of "science," that the penny is not "really" brown, though it is "really" round. The sort of reason which he is given for this startling statement is (so far as I can remember) that things appear to have different colours in different lights. If he should study heat and light, he will be told that the colour which he sees depends on vibrations which strike his eye, and that the temperature that he feels depends on molecular movements which are going on in the penny. He still thinks of the penny as literally round, and thinks now of all sorts of movements going on within its contour, and sending disturbances to his eye and his hand. But he no longer thinks of the penny as literally brown or cold. The brownness and coldness

are thought to be *effects* which the processes in the penny produce by transmission. The round shape is "in" the penny; the brownness and coldness are not. They are *effects* which the penny produces "in" his eye or his hand or his brain or his mind. He still thinks that he literally senses the same round upper side of the penny, both with his eyes and with his hand, but he no longer thinks that there is a brown colour or a cold temperature literally spread over this round surface.

This, I think, is a fair account of what the average person with a scientific training believes on these matters; so far as anything so incoherent can be said to be believed by anyone. It is perfectly obvious that such a view as this cannot stand criticism. It is an inconsistent mixture of two utterly different theories of perception. As regards spatial attributes, it keeps to the naïvely realistic view of unsophisticated common-sense. According to it, the seen and felt shape is not an effect produced in us by something else. It is out there, whether we see it or feel it or not. Processes in it simply make us see it or feel it under suitable circumstances. But, as regards colour and temperature, the scientific theory takes quite a different view. It is a causal theory. The processes in the penny do not make us see a colour or feel a temperature which is already there to be seen or felt. They *produce* the colour or temperature "in us," to use a discretely vague phrase, which may cover our minds, our brains, and our special sense-organs.

Now this muddled mixture of theories is not consistent with itself or with the facts. It is inconsistent with itself for the following reason. When I look at a penny, the brown colour that I see is seen spread out over the round contour. Similarly with the cold temperature that I feel. We are asked to believe that there is brownness without shape "in me," and round shape without colour out there where the penny is,

and yet that in some mysterious way, the shapeless brownness "in me" is projected into the round contour of the penny "out there." If this be not nonsense I do not know what nonsense is. We can all *say* this kind of thing, but can we attach any clear meaning to what we are saying?

Moreover, as Berkeley long ago pointed out, the theory only takes account of half the facts. Certainly colours vary with the illumination, the state of our eyes, and so on. But it only needs a little careful inspection to see that visible shapes also vary with changes in the medium, and with the position of the observer. If the former fact proves that colours and temperatures are not "in the object" but "in us," the latter should prove the same thing for visible shapes. It is impossible to reconcile the view that the penny is round, in the literal straightforward sense, with the view that, when we look at it, we literally sense visually the upper surface of it. For we sense all sorts of elliptical patches from various positions. It is clear that none of these can be *identical* with the round upper surface of the penny, and it is equally clear that they are not *parts* of it in the literal sense in which the King's head is a part of it.

If we want to be consistent then, we must treat visual shape in the same way as colour and temperature. What we sense visually is a sensum, and the shape and the brownness both belong to it. If anything be produced "in us" by an external object when we look at it, it is not just the colour, but is the whole patch with its colour and its shape. And, as we have seen, this patch cannot be regarded as being the upper surface of the external object, or as being literally a part of that surface. Nor can we any longer hold that what we sense by touch is literally identical with what we sense by sight, and that sight and touch merely reveal two different qualities of this one object. For what we sense tactually is round and of constant size.

What we sense visually is not round, except when we are in that very special set of positions from which we are said to be "looking straight down on" the penny. And, even if we confine ourselves to this series of positions, the sizes of the various round patches which we sense are not the same for different positions in the series. It is therefore clear that the scientific view needs to be completely restated in terms of the sensum theory. And this is not easy, because the scientific theory assumed that we really were sensing the contour of the actual physical object out in space, and that our sensations were due to what was going on within that contour.

As we move about and continue, as we say, to "look at the same object," we are aware of a series of sensa, each having shape and colour, and all very much alike in these respects. But there are certain variations which we commonly overlook. These strike us in exaggerated cases, and can be noticed by careful inspection in all cases. Moreover, they are as a rule reversed when we retrace our steps. If we are going to attempt a causal theory of perception we must try to explain this conjunction of predominant agreement throughout the series with slight, regular, and reversible variations between its different members. The explanation that naturally strikes us is that the series of sensa depends on two sets of conditions. One of these is relatively permanent, and accounts for the predominant agreement of the members of the series. The other is variable, and accounts for their minor variations.

Again, if we feel an object, such as a penny, and meanwhile look at it from various points of view, the series of predominantly similar, but slightly variant, visual sensa is correlated with an invariant tactual sensum. The shape of the latter is very much, but not exactly, like those of most of the former. It is exactly like that of the visual sensa which are sensed from a certain series of positions. As regards other qualities, there

is complete difference between the visual and the tactual sensa. The former have colour, but no temperature or hardness; the latter have coldness and hardness, but no colour. Now we have to explain this predominant agreement, combined with minor differences, between the shapes of the many visual sensa and the shape of the one tactual sensum. And we have to remember that, as regards other sensible qualities, the difference is complete. Here, again, it seems natural to suppose that there is something common and relatively permanent, which accounts for the predominant agreement in shape between the visual and the tactual sensa, and something variable that accounts for their minor differences in shape. This other factor seems clearly to be connected with the position of the sense-organ. As the eye moves about, the shape of the visual sensa varies. The shape of the tactual sensum does not change: but then we cannot move the hand to a distance and continue to sense the tactual sensum at all, as we can change the place of the eye and still continue to see. We may further suppose that different factors are needed to determine such very different sensible qualities as colour and temperature; but it is reasonable to suppose that, whatever these factors may be, they are subject to some common condition which determines the very similar shape of both visual and tactual sensa.

Lastly, when we compare notes with other people who say that they are looking at the same thing as we are, we find again a predominant agreement between their sensa and ours, combined with minor variations. It seems reasonable to suppose that there is a set of conditions, common to their sensa and ours, which accounts for the predominant agreement between the two. In addition, there must be variable factors, one specially connected with one observer and another with another observer. These are responsible for the minor variations. It seems, then, that we have good grounds for supposing that there are physical objects in the sense of conditions

which (*a*) are common to us and to others; (*b*) are relatively permanent, and, at any rate, do not *ipso facto* change when we move about; and (*c*) determine in some way the attributes of our sensa, in conjunction with other conditions which do vary from person to person at the same time and for the same person at different times.

It might be asked at this point by a sceptical reader, "Why go outside the series of correlated sensa at all? Why not be content to take them as a fact? Why make them all depend on conditions outside the series of sensa itself?" As I have said, this is a step which everyone does take, but which no one can be logically compelled to take. At present we may say that what induces us to do this is the fact that we have reason to think that physical objects change and act on each other when we do not happen to be sensing any sensa from them. We can drop such series of sensa as I have been describing (*e.g.*, by turning our heads or going out of the room), and then by making suitable movements we can pick it up again either where we left it, or in a form that is obviously a later development of a course of change whose earlier stages we noticed before we turned away. It is facts of this kind which (rightly or wrongly) make us look beyond such series of correlated sensa to relatively permanent conditions, which lie outside the series and can develop on their own account when the series is interrupted.

Now these common and relatively permanent conditions might, for all that we have seen up to the present, be so utterly unlike the sensa that they condition that it would be misleading to call them physical objects. The question therefore at once arises: "Can we determine anything further about their properties, either with certainty or with reasonably high probability?" I do not think that we could determine anything further with certainty, but I do think that we might determine something further with

high probability. It is, of course, perfectly true that a set of conditions—and, moreover, a set which is only one *part* of the total conditions—of a sensum, must not be *assumed* to resemble in its properties the sensum which it partially determines. On the other hand, it were equally unreasonable to assume that the two *cannot* resemble each other. There can be no inner contradiction in the qualities of shape and size, since *sensa*, at least, certainly have shape and size and certainly exist. If such qualities involved any kind of internal contradiction, no existent whatever could possess them. Hence it is perfectly legitimate to postulate hypothetically any amount of resemblance that we choose between sensa and the permanent part of their total conditions. If now we find that, by postulating certain qualities in these permanent conditions, we can account for the most striking facts about our sensa, and that without making this hypothesis we cannot do so, the hypothesis in question may reach a very high degree of probability.

Now we find that the visual sensa of a group which we ascribe to a single physical object are related projectively to each other and to the tactual sensum which we ascribe to the same object. If we regard their common permanent condition as having something analogous to shape, we can explain the shapes of the various sensa in the group as projections of the shape of their common permanent condition. If we refuse to attribute anything like shape to the permanent conditions, we cannot explain the variations in shape of the visual sensa as the observer moves into different positions. This does not, of course, *prove* that the common and relatively permanent conditions of a group of sensa do have shape, but it does render the hypothesis highly plausible. We have already seen that it is a legitimate one, that there is no reason why these common conditions should not have shape; we now see that it is also a plausible one, since with it we

can, and without it we cannot, account for the variations in the shapes of the sensa of the group.

What about the so-called "secondary qualities," like colour and temperature? We know that Descartes, Locke, and the orthodox natural scientists, hold that we have no right to ascribe them literally to physical objects, whilst Berkeley and many other philosophers have argued that primaries and secondaries must stand or fall (and that they, in fact, fall) together. What is the truth about this matter? The first need is to state the doctrine of primary and secondary qualities in a clear and intelligible form. Unquestionably, colour and temperature belong to *our sensa*, at any rate, in the same literal way in which shape and size belong to them. What I am immediately aware of when I look at a penny stamp is as indubitably red as it is indubitably more or less square. Similarly, when I hold a round piece of ice in my hand, what I am aware of is as certainly cold as it is certainly round. Thus, to say that colours and temperatures are "unreal," or "do not really exist," is patently false, if this means that there is *nothing* in the Universe of which it is true to say: "This is literally red," or "This is literally cold." Such statements are true of many sensa, at any rate, and sensa are parts of the existing Universe.

The only substantial question is: "Do colours and temperatures ever literally belong to physical objects, or do they belong literally *only* to sensa?" What the scientist is trying in an extremely muddled way to do is to assert the *physical* reality of shapes and sizes, and to deny the *physical* reality of colours, temperatures, noises, etc. Now this view, when clearly stated, comes to the following: "Shapes and sizes belong to physical objects in the same literal way in which they belong to sensa, and from the shapes and sizes of sensa we can generally infer with reasonable certainty those of that physical object of which these sensa are appearances. Colours, temperatures, etc., belong literally to sensa,

but they belong to physical objects only in a derivative and Pickwickian sense. There must, of course, be *something* in the permanent conditions of a group of sensa which wholly or partly determines the colour or temperature of the latter. But this something is not colour or temperature." We have seen what sort of ground there is for the positive part of this view: is there any good reason to believe the negative part of it?

It is sometimes thought that the physical theories of light and heat positively *disprove* the common-sense view that physical objects are literally coloured or hot. This is a sheer logical blunder. The physical theory of light, *e.g.*, asserts that, whenever we sense a red sensum, vibrations of a certain period are striking our retina. This does not prove that bodies which emit vibrations of that period are not literally red, for it might well be that only bodies which are literally red can emit just these vibrations. The vibrations might simply be the means of stimulating us to sense the red colour, which is literally in the body, whether we happen to sense it or not. (I am quite certain that this simple-minded theory cannot be made to fit the extremely complicated facts; but it *is* compatible with the fact that we only become aware of colours when vibrations of a certain kind affect our eyes; and therefore *this fact* does not, as is often supposed, refute the common-sense view that bodies are literally coloured and that we actually sense the colours which are on their surfaces.)

I think that the negative part of the scientific view does express an important fact, but that it needs to be stated in a much more guarded way. (1) It is *certain* that, if physical objects possess shape and size at all, they must have *some* other quality, related to shape and size in the same general kind of way in which colour and temperature are related to the shape and size of sensa. You cannot have extension *et prœterea nihil;* you must have something that can be spread out and

cover an area or fill a volume. (2) There is no reason why these "extensible" qualities, which *must* be present in physical objects, if they be extended at all, should not actually be colour and temperature. Since sensa certainly exist, and are certainly coloured, there can be no internal contradiction in the notion of an existent colour. (3) On the other hand, of course, the extensible qualities of physical objects *need* not be colour or temperature. So long as they are qualities that can cover areas and fill volumes, as colour and temperature do, they might differ from any quality that is ever present in our sensa. (4) Whilst we found that the assumption that the permanent conditions of groups of sensa have shape, and that they and our bodies have position, does help us to predict the shapes of various sensa in the group, we do not find that the ascription of colours or temperatures to these permanent conditions helps us to predict the colours or temperatures of the sensa in the group. It is found more profitable to correlate the colours and temperatures of sensa with the hypothetical movements of hypothetical parts of their permanent conditions. This does not prove, as has often been thought, that physical objects *cannot* literally have colours or temperatures. Of course, if the sensa that we sense cannot literally be parts of the surfaces of physical objects, it follows that the colours and temperatures of these sensa cannot literally *be identical with* the colours and temperatures of physical objects, even if the latter have such qualities. The facts under discussion do show that the hypothesis that physical objects literally have colours and temperatures, though legitimate enough, is not capable of empirical verification, and therefore cannot be asserted with any high probability.

The view which I have been trying to state may be called the *Critical Scientific Theory*. It is simply an attempt to formulate clearly, in terms of the Sensum Theory of sensible appearance, the view about the ex-

ternal world which has been at the back of the scientific mind since the time of Descartes and Locke. In its original form this view was a mass of inconsistencies, since it was naïvely realistic for our perception of shape, size, and position, and held a causal theory for our perception of colour, temperature, etc. This combination of theories proved to be inconsistent with the inextricable entanglement of the two kinds of qualities, which we actually find. Moreover, the naïvely realistic part of it proved untenable in face of the variations of visual shape and size, which are obvious when we view what is regarded as a single unchanged physical object from various positions.

Thus the only hope for the scientific view was to restate it in a completely causal form. A serious difficulty at once arose. The causal part of the old view presupposed the naïvely realistic part. When we were told that motions within a circular contour at a certain place in space caused sensations of colour and temperature "in us," we understood this, because we thought that we literally saw and felt this contour in this place. But, as soon as the theory is made *completely* causal, *both* spatial and non-spatial attributes belong primarily to the effect produced "in us" by something else. It then becomes difficult to see that we have any better right to regard this cause as literally endowed with shape, size, and position, than as literally endowed with colour and temperature. Yet the scientific theories about the causation of our sensations of colour, temperature, etc., are stated in terms which seem to lose all meaning unless the causes of these sensations literally have shapes, sizes, and positions. The Critical Scientific Theory, as stated by us, has been an attempt to meet these difficulties, to reformulate the distinction between primary and secondary qualities, and to estimate the amount of value which this distinction can justly claim.

I think that the Critical Scientific Theory is internally consistent, so far as it goes; but I certainly do not believe that it is ultimately satisfactory. In the first

place, it continues to use a number of phrases whose meanings are no longer obvious when we have given up the notion that we literally sense parts of the surfaces of physical objects. It still talks of pennies being "round," of a number of different people at "the same time" and the same person at "different times" all perceiving "the same penny" from "different places." We must reinterpret all these phrases in terms of our sensa and their relations before we can hope to get a consistent theory. I shall try my hand at this very difficult job in the next three chapters.

Secondly, our theory uses the phrase that processes in external physical objects and our bodies "jointly produce in us" the sensa by which we become aware of them. The phrase in inverted commas covers a multitude of problems. Do physical processes create sensa out of nothing? Or do they just cause us to sense now one and now another selection out of a mass of already existing sensa? And, on either alternative, what is the status of sensa once they have come into existence? Do they just exist alongside of physical objects? Do they ever interact with each other or produce effects on the physical world? Or are they, in some Pickwickian sense, parts of physical objects? With some of these problems I shall try to deal in my last chapter.

The following additional works may be consulted with advantage:

B. A. W. Russell, *Lectures on the External World*, Lects. III. and IV.
,, ,, *Analysis of Mind*, Lects. V. and VII.
G. F. Stout, *Manual of Psychology*, Bk. III. Part II. Cap. I., and Bk. II. Cap. I.
,, ,, *Proceedings of the Aristotelian Society*, 1913.
J. Laird, *Problems of the Self*, Cap. III.
S. Alexander, *Space, Time, and Deity*, Vol. II. p. 124, *et seq.*; p. 170, *et seq.*
G. E. Moore, *Philosophical Studies*.
Berkeley, *Principles of Human Knowledge*.
Descartes, *Meditations*.

CHAPTER IX

"Nam si colores et soni in ipso Objecto essent, separari ab illis non possent. Separantur autem, ut manifestum in reflexionibus visibilium per specula, et audibilium per loca montana. Scimus autem corpus quod videmus in uno tantum loco esse, sed apparentias in plurimis."

(HOBBES, *Leviathan*, Part I. Cap. I.)

The Positions and Shapes of Sensa and of Physical Objects

WE have now to dig beneath the assumptions that are tacitly made by the Critical Scientific Theory, and to discover their precise meaning and value. In expounding it we talked of a number of people all "looking at the same penny." We assumed that there is a certain place "seen" by all the observers, and that in this place there is a round physical object. We have now to ask what is meant by a common place; what is meant by a physical object occupying that place; and what is meant by calling that object round. We shall find that all these questions, which seem so childishly simple, present great difficulties, and can only be answered in highly Pickwickian senses. They seem easy, because we habitually confine ourselves to cases, which are indeed of frequent occurrence, and are of practical interest, but which really owe their simplicity to the existence of specially simple conditions. These conditions are not always fulfilled, and then difficulties arise. This happens, for instance, with mirror images which turn up in places where nothing relevant is going on. As a rule, we simply ignore these "wild" sensa; but we shall find that the only way to deal fairly with all the facts is to base our theory on them, and to

regard "tame" sensa as owing their tameness to the fulfilment of certain special simplifying conditions.

In dealing with our present problem we shall not only be learning something more about the concept of Matter and its appearances; we shall also be carrying the theory of Space a step further. In Chapter I we simply took the common-sense notion of a single all-containing Space for granted; we have now to consider the exact cash value of that conception.

If we want to discover the meaning of the statement that we all see a certain physical object in a certain place, we must start from the spatial characteristics of our visual sensa. Unfortunately, there is a good deal of disagreement as to what these actually are. Thus we are often told that we do not "see" distance or solidity; and this is undoubtedly meant to mean that distance and solidity are not characteristics of visual sensa, as shape and size are. This seems to me to be a mistake, and the whole matter has become so much confused that our first duty is to try to clear it up. This will be rather a long process.

Spatial Characteristics of the Visual Field.—Whenever I open my eyes I am aware of a coloured field of view, which I will call a "visual field." It is admitted that this is spread out and internally differentiated into patches of various shapes and colours. These are at once joined and separated by a background, which also has colour. The middle part of this field is the most distinct. If I turn my head a little, the field changes slightly. What is now in the middle and most distinct differs from what was in the middle of my former field. But it is extremely like something that was slightly to one side of the former field and was slightly indistinct. Conversely, what is slightly to one side of the present field is very much like what was in the middle of the former field and had there maximum distinctness. The process of turning one's head is, of course, associated

with certain kinæsthetic sensations, which last longer and grow more intense the more the head is turned.

(*a*) *Visual Motion.*—So much, I suppose, is admitted by everyone. I now want to call attention to certain facts that have an important bearing on our present problem, and are not so commonly noticed. As a rule, we see objects through a practically homogeneous medium, viz., air, in which they and we are immersed. Under these conditions the slight turning of the head only produces those changes in centrality and distinctness that we have noticed, combined, of course, with the loss of certain features which were on the extreme edge of the first field and the gain of others on the opposite extreme edge of the second. So long as the medium is homogeneous, the turning of the head does not affect the visual sensa with sensible movement. If, on the other hand, we are looking through a bad bit of window glass, or through any optical instrument imperfectly focused, the sensa in the field do visibly move as we turn our heads. What I call "sensible movement" is as distinct and irreducible a character of certain sensa at certain times as colour or shape. We notice then that, under normal conditions of sight, the sensa in our visual field may be unaffected with sensible movement, though we turn our heads; but, as soon as the conditions become unusual, a turn of the head affects all the sensa of the field with sensible movement.

Again, some of the sensa in a field may be affected with sensible movement though I keep my head still. As I write, I am sitting at an open window in Trinity, and looking out at the opposite side of Nevile's Court. All the points that I have mentioned are illustrated in my present visual field. I can turn my head without the visual appearances of the opposite windows being affected with sensible movement. If I look through the shut window, which is at the side of my open one, and is made of rather irregular glass, I find that I cannot turn my head without the visual appearance of the

opposite side of the Court jumping about. Lastly, there are certain features in the field, viz., the visual appearances of bedmakers and washerwomen—for it is a Saturday—which sensibly move, even though I keep my head still. To these cases we must add one more, which is the least common in ordinary experience. Sometimes we find the whole field affected with sensible movement, though we keep our heads still. This happens if my open window swings to in the breeze.

The position, then, is this: There is no doubt that sensible motion and rest are genuine unanalysable properties of visual sensa. I am aware of them as directly as I am aware of the redness of a red patch, and I could no more describe them to anyone who had never sensed them than I could describe the colour of a pillar-box to a man born blind. Now, there are three entirely distinct, but constantly confused, questions that can be asked about a quality of a sensum. (1) Do sensa really have this quality? (2) What conditions must be fulfilled in order that sensa with this quality may occur? and (3) What right have I to base on this quality of my sensa those judgments about physical objects and their properties which I do in fact base on it? The first question is absolutely independent of the other two. The only way to find out whether a sensum does or does not have a certain quality is to inspect the sensum itself as carefully as possible. The second question belongs partly to physics, partly to physiology, and partly perhaps to psychology (if sensa be to any extent mind-dependent). The third is a question for Critical Philosophy. Naturally, the answer to it will determine the interpretation which we put on the answers given by scientists to (2). Conversely, the answer to (3) will have to be such as to allow for any well-established facts that the scientists have discovered in answering (2).

Now it is a very common mistake to suppose that if (2) has to be answered in a certain way it follows that sensa cannot have the quality in question. This fallacy

seems to me to have been committed by those persons who deny that visual sensa have sensible solidity and position. They argue that those qualities could only have been acquired through certain past experiences, and conclude from this that the qualities in question cannot now belong to visual sensa. This is, of course, a sheer fallacy; but before discussing it in detail for position and solidity, I propose to deal with the case of sensible motion. For exactly similar arguments could be used to prove that visual sensa do not have sensible motion; and it must surely be obvious, even to the most advanced thinker, that some visual sensa do have this quality.

When I look through a homogeneous medium and turn my head, the stimulus of light from various objects moves over my retina; nevertheless, my sensa are not affected with sensible motion. When I look through a non-homogeneous medium, and turn my head, the stimulus moves across my retina; and this time my sensa *are* affected with sensible motion. Thus the movement of the stimulus over the retina may be a necessary, but is certainly not a sufficient, condition of the sensible movement of my visual sensa. When I believe that the object that I am looking at is the sort of object that will not move (*e.g.*, the opposite side of the Court), and when I am seeing it under normal conditions (*i.e.*, through a homogeneous medium) the sensa keep still, in spite of the movement of the stimulus, provided this movement is caused by the voluntary turning of my head. Thus it seems to me to be clear that one condition which partly determines the present motion or rest of my visual sensa is my beliefs as to the motion and rest of the objects of which these sensa are appearances. These beliefs must be due to past experiences, not wholly visual, in connexion with similar sensa. They are presumably present in the form of traces. Under normal circumstances these traces neutralise the sensible movement which the motion of

the stimulus over the retina would itself produce. But, as soon as the conditions become abnormal, this neutralisation (which is merely associative and instinctive, not deliberate and rational) fails to fit the unusual conditions, and the sensa visibly move.

If the above theory be true, the present motion or rest of a sensum is not entirely determined by anything in the nature of the present stimulus. The traces left by past experiences, some of which were not wholly visual, also co-operate; and we have what Mr Russell calls a case of "mnemic causation." Yet it is clear that this makes no difference to the fact that here and now visual motion and rest are properties of visual sensa, which are "seen," as truly as shapes and colours, and which would be inexplicable to a blind man.

These facts are typical of visual perception, and render the situation with which we have to deal highly complex and confusing. On the one hand, we *now* pass from the visible motion or rest of our sensa to perceptual judgments about the behaviour of our bodies, of the medium, and of the object at which we say that we are looking. We could not get so much out of so little if it were not that many past experiences of ourselves and others co-operate with the present visual sensum to form the basis of our perceptual judgments. But they do not only co-operate to form *judgments*. The actual present qualities and movements of our sensa are modified by the traces left by these past experiences. We have thus to deal with a double process. The experiences of many people (conveyed to us from our earliest years by speech and corporate action) and many past experiences of our own, have helped to produce our present beliefs in the places, shapes, movements, etc., of physical objects, and have helped to produce our present classification of these into medium, observer's body, object looked at, etc. *Pari passu* with this, the traces left by these past experiences (which express themselves in consciousness, if

they do so at all, as expectations and beliefs about physical objects) co-operate with present stimuli, and modify the qualities of our sensa. And our present judgments about physical objects are, of course, based on our sensa as thus modified.

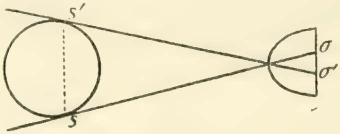
(*b*) *Visual Solidity*.—Let us now apply these general principles to the debated case of visual solidity and distance; and let us begin with solidity. It seems to me perfectly clear that, whatever may have been true of my infancy or of my remote ancestors, solidity is *now* as genuine a quality of some of my visual sensa as flat shape or red colour. A sphere does *look* different from a circle, just as a circle looks different from an ellipse. That this is due to past experiences of touch and past kinæsthetic sensations may very well be true in one sense, though I think that it is certainly false in another. We must distinguish between a general quality, capable of various specific modifications, and the particular form of it possessed by a certain particular sensum. Thus visual solidity, on my view, is a general quality of visual sensa, whilst sphericity is a particular form of it, which belongs to some sensa and not to others. Now I can quite well believe that the particular form of solidity possessed by a certain sensum may be in part due to traces of past experiences of touch and movement. I can believe, for instance, that the particular distribution of light and shade over my present sensum resembles that of a past sensum which was associated with the experience of passing my hand over a spherical surface. And I can believe that the resemblance of the stimulus excites the traces left by that experience, and that these co-operate with the present stimulus on my retina to produce a sensum which is visibly spherical. But I find it very hard to believe that experiences of touch or movement could *create* a third dimension in visual sensa which originally had only two.

Now it does seem to me clear that visual solidity is in

itself as purely visual as visual shape and size. It does not consist of visual flatness, together with *judgments* about past or future tactual sensations. Nor does it consist of visual flatness, together with associated *images* of past or future tactual sensa. It is a matter of plain inspection that the experience of visual solidity is as unitary an experience as that of visual shape in two dimensions, and that it is impossible to distinguish it into a visual and a tactual part. We are therefore forced to suppose, either that the experiences of one sense can create an additional dimension in the sensa of another sense, or that visual sensa are of their own nature three dimensional. I should not be prepared to accept the former alternative unless very strong arguments could be produced against the second. We shall see in a moment that the arguments are feeble in the extreme. I shall therefore suppose that visual solidity is a primitive characteristic of visual sensa, and that the traces left by past visual and tactual experiences merely help to determine *what particular form* of visual solidity a particular sensum shall have.

If this be the genuine result of careful inspection, no argument from the physical and physiological conditions of visual sensation can possibly have anything to say against it. On the contrary, it will be one of the facts with which any theory as to the conditions of visual sensation will have to reckon. All arguments which attempt to prove that solidity is not a primitive property of visual sensa are of the following type. Whenever we see an object, a certain *area* of the retina is stimulated by the light from this object. This area is a projection of the object on to the surface of the retina, and such an area could equally well be the projection of a solid or of a plane figure of suitable shape. Consequently, it is argued, there is nothing in the retinal stimulus to distinguish between light from a solid and light from a plane figure of suitable contour. Therefore sight cannot give us an awareness of solidity.

This may be illustrated in the following way: Take a sphere, and suppose that we are looking at it with one eye. The light from it affects a circle on the retina, of diameter, $\sigma\sigma'$.



If we were to cut away all the sphere in front of SS′ and all the sphere behind it, leaving merely the circular disc of diameter SS′, the area of the retina affected by the light from this disc would be exactly the same as that affected by the light from the whole sphere, viz., the circular area of diameter $\sigma\sigma'$. Hence, it is argued, the visual sensum must be the same in both cases. No doubt there will be a difference in light and shade in the sensum connected with the sphere, but this is the only difference. And this effect could be reproduced by using a suitably shaded flat disc instead of an uniformly illuminated one, as is in fact done when painters want to represent spheres on flat canvases. Conversely, arrangements of lines which are really in one plane may "look solid." It is concluded (*a*) that solidity is not a primitive property of visual sensa; and (*b*) that, even now, "to look solid," *means* simply to evoke certain images, memories, or expectations of tactual and kinæsthetic experiences.

This argument, which *must* be mistaken if it is a fact that visual solidity is a unitary and unanalysable property of sensa, does rest on tacit assumptions; and, when these are laid bare, it loses its plausibility. It assumes (*a*) that, because the *retinal stimulus* for visual sensation is two-dimensional, therefore, the corresponding *visual sensum* cannot have more than two dimensions. It is this assumption that makes it so plausible to hold that the visual sensum *must* itself be a mere surface, and therefore that visual solidity needs

to be explained. But there is not the least reason to accept the assumption. There is no reason, whatever, why a sensum should not have a greater number of dimensions than the physiological stimulus on which it depends. Hence, even if it be true that the necessary and *sufficient* condition of a visual sensation is an excited *area* on the retina, this is no reason why some or all visual sensa should not be voluminous. (*b*) The argument in question does make the further assumption that the complete conditions of a visual sensum must be present in the retinal stimulus with which it is connected. If anything else, such as the trace of a past tactual or kinæsthetic experience, co-operates, it is assumed that it can only produce associated *tactual images* and not modifications of *visual* sensa. This again is a sheer assumption, and one that is not even antecedently probable. In any case, the visual sensation does not arise till the stimulus has passed from the retina, through the optic nerve, to the brain. It is the wildest dogmatism to assert that what happens in the brain corresponds point for point to what happened on the retina, and that no additional factors come into operation there, which may be constant when the retinal stimuli vary, or variable when the retinal stimuli are the same. Now if *every* visual sensation is partly dependent on what happens in the brain as well as on what has happened on the retina, it is surely mere pedantry to assert that the solid shape of a certain visual sensum cannot be a genuine property of *it*, because one of its conditions was a trace left on the brain by a past *tactual* experience. We must judge sensa, like O.B.E.'s, by their present properties and not by their ancestry.

The truth seems to me to be as follows: (1) Visual sensa, as such, are capable of being solid. There is such a quality as visual solidity, and it belongs to some sensa as much as the shape of a flat sensum belongs to it. (2) The complete conditions of any visual sensum include (*a*) a stimulated area of the retina (or what

corresponds point to point with this in the brain by transmission through the optic nerve); and (*b*) certain conditions in the brain which are independent of the present stimulus on the retina. (3) Among these independent conditions are traces left on the brain by past experiences of sight, touch, and movement. These do not generally show themselves in consciousness at all. If they happen to do so, they express themselves as memories and expectations about physical objects. (4) Generally these traces merely co-operate with the brain states which are due to the retinal stimulus, to produce a visual sensation whose sensum is of such and such a kind. It is, therefore, reasonable to expect that the visual solidity of two sensa may be different, though the stimulated retinal area is the same. Let us illustrate this by the case of the disc and the sphere. In both cases the same circular area of the retina is stimulated and the disturbance is transmitted from it to a correlated part of the brain. In neither case is this sufficient to determine completely the nature of the visual sensum which shall be sensed at the moment. The other necessary conditions include factors in the brain which are independent of the present stimulus and existed before it took place. Among these are traces left by past experiences. Now the distribution of the light in the case of the sphere excites certain traces, t_s, whilst the different distribution of the light in the case of the uniformly illuminated disc excites certain other traces, t_d. Calling σ_s and σ_d the visual appearances of sphere and disc respectively, and r the common area of the retina stimulated, we have

$$\sigma_s = \phi(r, t_s) \text{ and } \sigma_d = \phi(r, t_d);$$

and the sensible shape of the two sensa takes different forms, viz., the solid spherical form and the flat round form. Conversely, suppose we are looking at a perspective drawing of a cube on a flat bit of paper. If we happen to be thinking mainly of solids, as we

generally are, a trace, t_c, left by past experiences of touching cubes, will tend to be excited; if we are thinking mainly of the flat bit of paper a different trace, t_f, will tend to be excited. The two visual sensa,

$$s_c = \phi(r, t_c) \text{ and } s = \phi(r, t_f),$$

will then differ in the specific form that their sensible shape takes.

(*c*) *Visual Distance.*—We can now pass to the question of visual distance, which is more important for our present purpose, and about which almost exactly the same controversy has arisen. We have been told *ad nauseam* since the days of Berkeley that we do not *see* objects at a distance from ourselves, but that the perception of distance by sight is simply associated images of tactual and kinæsthetic sensations. I take this to mean that distance is not an intrinsic property of our visual fields, as colour, size, and shape are. Now it is perfectly obvious to me that I do sense different patches of colour at different visual distances. When it is said that we do not *see* distances out from the body, the only sense in which it is true is that, in monocular vision, there is nothing in the retinal stimulus which is *uniquely* correlated with the distance of the source of light from my eye. In binocular vision there is, I suppose, parallax between the two retinal impressions. To make the case that I am arguing against as strong as possible, I will confine myself to monocular vision.

It is true that, if I fix a stick 6 inches long at 6 feet from my eye, its projection on my retina is the same as that of a stick 1 foot long held at 12 feet from my eye and parallel to the first. The *one* factor of length in the retinal impression has to represent the *two* factors of length and distance in the physical object. This is, of course, still clearer if we keep one end of the stick fixed and move the other end about in various directions in Space. The various projections on the retina are

of many different lengths; but all these various projections could equally have been produced by sticks of suitable lengths, with their directions all confined to the plane parallel to the observer's body. Hence there is nothing in the retinal impression to distinguish between a number of sticks of various lengths put in various directions in a plane parallel to the body, and a single stick with one end at a fixed distance and the other turned in various directions in Space. The conclusion drawn is that distance out from the body is not an attribute of visual sensa as such, like length and breadth; the distance that is apparently "seen" consists of associated images of kinæsthetic and tactual experiences that have been enjoyed in the past.

We must make much the same criticisms on this argument as we have already made on the argument to prove that there is no such quality as visual solidity. (1) Whatever may be the history of the process, it is *now* a fact that one visual sensum is visibly remoter than another, and that a stick 6 inches long and 6 feet away *looks* different from a parallel stick 1 foot long and 12 feet away. (2) This sensible distance is not now analysable into a sensum of a certain size and no distance, together with revived images of past kinæsthetic and tactual sensations. Visual distance is as simple and unitary a quality in itself (whatever may be true of *its conditions*) as visual length or breadth. (3) It is extremely difficult to believe that visual sensa started with no such quality as distance, and then acquired an extra quality, perfectly interchangeable with their former qualities of length and breadth, through association with experiences of another sense. (4) The fact that there is nothing in the retinal stimulus which is uniquely correlated with distance in no way proves that visual sensa do not, from the very first, have some form of visual distance. It is equally true that there is nothing in the retinal stimulus that *uniquely* corresponds to the length or breadth of the

object at which we are looking; yet the present theory does not hesitate to hold that length and breadth are genuine qualities of visual sensa. In fact, nothing but prejudice can make us suppose that, because a physiological stimulus has only *n* dimensions, the sensum which is correlated with it cannot have more than *n* dimensions. It is, therefore, perfectly open to us to hold that all visual sensa have, of their very nature, some visual distance or other. The only problem is to account for the fact that here and now one visual sensum has one sensible distance and another visual sensum has another. (5) To account for this we have to remember that, on any view, it is not the retinal stimulus itself, but a process in the brain, which is the last link in the train of events which ends with a visual sensation. This being so, it is not unreasonable to suppose that the total physiological conditions of any visual sensation include (*a*) a set of brain-states which correspond by transmission to the events in an excited area of the retina; and (*b*) certain brain-states which are independent of the present retinal stimulus. Among the latter are traces left by past experiences of sight, touch, movement, etc.; and these play an important part in determining the particular visual distance that a given visual sensum shall have. It is thus perfectly intelligible that the sensible length and distance of two sensa should differ when the retinal stimulus is of the same size and shape, and conversely. This is simply another instance of the same general principle which we have already seen at work in the case of sensible motion and rest and in that of visual solidity.

A special difficulty with which we must now deal, has been felt about ascribing distance to visual sensa. It is argued that distance is essentially a relation between two terms, and that a relation cannot literally be sensed unless both its terms are also sensed. Thus we do not visually sense a given line, unless we visually sense both ends of it. Now we certainly do not visually sense

our own retinæ, and therefore it is impossible that we should visually sense the distance of visual sensa from them. This is a perfectly sound argument, and to meet it we must draw certain distinctions.

(1) The first thing to recognise is that the awareness of visual distance is primarily an awareness of the distance between two visual sensa, and is not an awareness of the distance of either of them from our retina. It is perfectly true that the distance of sensa from our retina is not sensed by sight. Indeed, it is only possible to give a meaning to the notion of distance between a visual sensum and something, like the retina, which is not a sensum at all, in a highly Pickwickian sense. All I am asserting is that, when I open my eyes, I am aware of a visual field in which different parts have different depths. What I sense as visual distance is the *difference of depth* between two sensa in this field.

(2) We must therefore distinguish between visual *depth* and visual *distance*. Depth is a sensible quality, not a sensible relation. Visual distance is a sensible relation between two visual sensa, *founded upon* the difference of their respective visual depths. When we sense two sensa with different visual depths we *ipso facto* sense the relation of visual distance between them. If we only sense a single visual sensum (say a luminous flash on a perfectly dark night) we do not sense distance, but we do sense depth. It is, of course, quite true that it is extremely difficult to estimate depth accurately apart from distance. But there is nothing odd in this. It is extremely difficult to estimate length accurately except by comparing an object with some other. Nevertheless, objects do have lengths of their own, and the relations between them which we notice when we compare and measure, are founded on the lengths of each of them.

(3) Sensa are at no distance from our retina, not in the sense that they are at zero distance from it, as the points of contact of two billiard balls are from each other

when they hit, but in the sense that the concept of visual distance does not apply at all to anything but pairs of visual sensa. They are at no distance apart in the kind of way in which it is true that my belief that $2 \times 2 = 4$ is at no distance from my desire for my tea. A Pickwickian sense of distance can be defined in which it is true generally to say that visual sensa of less depth are nearer to my eye than visual sensa of greater depth. But this Pickwickian sense involves a reference to movement and other things which we have yet to consider. The interpretation of the depth of a single visual sensum in terms of distance between it and the eye is, of course, greatly helped by the fact that, when *two* sensa of different depth are both sensed, the correlated relation of visual distance between them is also immediately cognised.

I have spoken at some length about visual motion and rest, solidity, and distance, for three reasons: (i) They illustrate the extreme complexity of the relations between sensa (if there be such things, as we are assuming throughout this book) and physical objects and processes, and show that the past history and present expectations of the percipient must be supposed to be partial conditions of some of the qualities and relations of sensa. This cuts out at once any of those cheap and easy forms of naïve realism which are produced in mass and exported in bulk from the other side of the Atlantic. (ii) The problem of the perception of distance and solidity by sight is an intrinsically interesting and very complex one, and we have at least shown that many venerable arguments on these subjects rest on assumptions which are not convincing when clearly stated. (iii) The conclusions which we have reached about visual distance and solidity are of the utmost importance for our immediate purpose, viz., a discussion of the concepts of position and shape, as applied to sensa on the one hand and to physical objects on the other.

My view is that nearly all the *general concepts* that we

use in dealing with Space, *e.g.*, distance, direction, place, shape, etc., come from sight, whilst the notion of *one* Space and the *particular quantitative values* which these general concepts assume in special cases are due mainly to touch and to movement. Series of kinæsthetic sensations are not, as such, experiences of distance, direction, etc.; and I do not see how they could ever be interpreted in such terms unless the necessary concepts had already been supplied by sight. Before going further, I will sum up our conclusions and sketch the general outline of the view that I take.

(*a*) The physical world is conceived as comprising at any moment a number of co-existing objects of various shapes and sizes in various spatial relations to each other. (*b*) The concepts, in terms of which this view is stated, come mainly from sight, and could hardly have arisen apart from it. Sight supplies each of us at each moment with an extended visual field in which there are outstanding coloured patches of various shapes and sizes. These co-exist; are in many cases sensibly solid; and have various spatial relations to each other in three dimensions, which relations are directly sensed. (*c*) These visual experiences, however, need much supplementation before they can give rise to the traditional concept of physical Space. In the first place, visual shape, size, distance, etc., are not quantitatively very definite. Again, Space is not thought of as either momentary or private. It, and the objects in it, are thought of as public property which all observers can perceive. And it is thought of as the permanent container in which physical objects exist, persist, change, and move. Thus it is necessary to connect up with each other (i) the successive visual fields of the same observer, and (ii) the contemporary visual fields of different observers. This fact may well make us anticipate that the traditional separation of Space and Time is not an ultimate fact, but is a convenient fiction, which works as well as it does because

of certain simplifying conditions which are generally fulfilled in everyday life. (*d*) The connecting link between various visual fields I believe to be mainly experiences of bodily movement and of touch. These also enable us to give quantitative definiteness to the mainly qualitative concepts which we derive from sight. (*e*) These series of movement-sensations are not themselves sensations of spatial relations. They are series in Time, whereas spatial relations are conceived to link contemporary terms. They are *interpreted* spatially, in terms of the concepts which sight alone can supply, through their association with visual experience. (*f*) The accurate quantitative detail, and the unity of physical Space, as conceived by us, are thus due to the intimate association of sight with touch and movement-sensations. But the traces of the latter do not work simply by calling up judgments or images of past or possible movements and touch experiences. They also continually modify the actual properties of our visual sensa; so that the sensa connected with a given retinal disturbance may come to acquire different visual shape, size, and depth, from that which they at first had. (*g*) I do not, of course, mean that the spatial attributes of visual sensa can be *indefinitely* modified by association with other experiences, or that such association does not often express itself by mere judgment, without modification of the qualities of the sensa. For instance, it is true that if I look at what I believe to be a round object in a considerably oblique direction, the visual sensum is not rendered round by the traces of past experiences, but remains visibly elliptical. What the traces do here is not to modify the sensum, but merely to produce the judgment that I am in fact dealing with a round physical object. The *meaning* of roundness is mainly based on visual experiences; the fact that I *apply* the concept of roundness and not that of ellipticity to the perceived object is mainly due to the associated traces of past tactual and

motor experiences; but the latter only modify my judgment about a physical object in this case, and do not actually render the visual sensum round. This may be contrasted with the case of looking through a homogeneous medium at an object which is believed to be still, and turning my head. Here the traces left by tactual and kinæsthetic experiences, which I have had in the past in connexion with similar retinal stimuli, do prevent the sensum from having any sensible movement. If the medium be not in fact homogeneous, these traces will automatically supply an "over-correction," and the sensa will visibly move. (*h*) On the whole, we may say that traces of past experiences do *tend* to modify the qualities of visual sensa in such a direction that they approximate more closely to those which we believe the object at which we are looking possesses. Often the approximation is very imperfect; but, as a rule, this makes little difference to the judgments that we make about physical objects on the basis of our sensa. (*i*) In any case, the spatial attributes that we ascribe to a physical object, on the basis of a present stimulus and the traces of past experiences, gain their whole *meaning* from sensa and their properties, and in the main from the properties of visual sensa. I may judge that I am looking at a round penny because I am sensing an elliptical sensum; but what I mean by calling it "round," is that it has the same sort of shape as certain visual sensa that I have sensed in the past (*e.g.* when I look straight down on pennies). (*j*) We must further remember that, in ninety-nine cases out of a hundred, the result of association, whether it modifies the present sensum or not, is not to produce an explicit *judgment* about a physical object and its properties, but to guide us to appropriate *actions*. When we say that an elliptical sensum, together with traces of past experiences, leads us to judge that we are looking at a round physical object, this is generally an over-intellectual statement of the facts. The peculiar

experience of judging or believing may not arise in our minds at all, and probably will not, if we are at the time more interested in action than in reflection—as the present state of the world proves most people to be at most times. What really happens is that we act as we might reasonably have been expected to act *if* we had made such and such a judgment.

The Concept of Place: (*a*) *Sensible Place.*—Let us now deal in detail with the concept of place, as applied to sensa and to physical objects. We will start with visual sensa. The fundamental meaning of "place" for visual sensa is their place in the visual field of the observer who senses them. This I shall call *Sensible Visual Place*. We shall also find it convenient to say that such and such a coloured patch is *sensibly present* at a certain place in a visual field. Sensible presence is (*a*) directly experienced by sight; (*b*) is literal and unanalysable, not Pickwickian; and (*c*) is private to a single observer, in the sense that it only applies to the sensa of his field. It is a relation between a sensum, which is part of a field, and the rest of the field. Two different men have different visual fields, and the same man has different fields at different times. A given field may be said to last as long as the specious present of the observer whose field it is. We shall have to go fully into this matter when we deal with the concepts of date and duration, as applied to sensa and to physical objects. In the present chapter I shall make the simplifying assumption that our successive fields are literally momentary. This is certainly not true, for a momentary field is something that can only be defined by Extensive Abstraction; but it is best to deal with one difficulty at a time.

I have already said that it seems to me that the visual field, with its various coloured patches standing out at different depths and in different directions against a more neutral background, is the sensible basis which

alone gives *meaning* to the concept of Space. The concept of Space is that of a perfectly unique kind of whole of co-existing parts, and, if we had never been sensibly acquainted with a concrete individual instance of such a whole, we could never have formed the concept. The visual field seems to me to be *an* instance, and *the only* instance, of a space-like whole with which we are directly acquainted. Now, of course, once a concept has been *acquired* through sensible acquaintance with a particular instance of it, it can be *applied* by thought to wholes which are never sensed as such, but are only conceived by reflection on experiences which come to us piecemeal. In order to apply the concept to such wholes, many modifications in detail may be necessary, and these will be suggested by the characteristics of the various experiences which we are synthesising under the concept of a quasi-spatial whole. For example, if you ask a scientist what he understands by the statement that an atom consists of a number of electrons arranged in a characteristic pattern in Space, he will not be able to answer you by *defining* his meaning in terms of other *concepts*. But he will be able to answer you by *exemplifying* what he means. He can ask you to look up at the sky on a clear night. He can then say that he thinks of the electrons as analogous to the little twinkling dots in your visual field, and that he thinks of them as forming a pattern in Space, in the sense in which those little dots form a pattern in your visual field. In fact, a bit of matter is to physical Space as a visual sensum is to a visual field. This is the fundamental, non-Pickwickian sense in which things are conceived to occupy places in Space. What we have now to consider is the facts about our sensa and the other experiences which encourage us to extend the application of this concept beyond the visual field and its sensa.

(*b*) *Compresence of Visual Sensa from different Fields.*—If I look at a penny, and either stand still or walk

about, I sense a successive series of visual fields. In each of these there is a sensum which is an appearance of the penny. Again, if a number of observers look at the penny together, there are as many different visual fields at any moment as there are observers. Each contains a sensum which is an appearance of the penny. We say that the appearances in the successive fields of each observer, and the appearances in the contemporary fields of the various observers, are in a certain sense all "in the same place," and we say that this is the "place where the penny is." It is evident that facts such as I have just been describing are the sensible basis of such statements as that I "go on seeing the same penny," and that other people and myself "see the same penny together." If there were no such correlations between the successive fields of myself and between the contemporary fields of several observers, there would be no ground for making assertions of this kind.

Now it is quite clear that when I say that a number of sensa from different fields are in the same place, I cannot be talking of "sensible place," as described above, for that concept refers essentially to the relation between a sensum and its own field. We must, therefore, try to find the exact cash-value, in terms of sensible experience, of the statements (*a*) that the various visual sensa are in the same place; and (*b*) that this is the place where the physical penny is. By considering abnormal cases, like mirror images, we shall see that sometimes the first is true when the second is false. But we will begin with more ordinary cases.

Very often the successive visual fields of an observer are largely similar. In particular, there may be a series of sensa $s_1 \ldots\ldots s_n$ in his successive fields $f_1 \ldots\ldots f_n$, which are very much alike. Let us take the case of a man who would be said to be looking directly at some resting luminous object through a homogeneous medium. What sort of visual sensa will he sense? To start with, a certain sensum s_0 in the field f_0 may attract his atten-

tion. This may be somewhere to the side of the field. Suppose he turns his head so that, as we say, he is now looking at the object of which this sensum is an appearance. What happens is that he turns his head until he is aware of a field f_1, in the middle of which is a sensum s_1, which in colour, shape, etc., very much resembles the sensum s_0, which originally attracted his attention. This will have a certain sensible depth. Suppose that he now begins to walk, "following his nose." He will sense a series of visual fields, of which the following propositions will generally be true. (i) In any one of these f_r there will be a sensum s_r in the middle, closely resembling s_1 in shape and colour. (ii) The sensible depths of the successive sensa $s_1 \ldots\ldots s_n$ will steadily diminish, whilst their brightness, distinctness, and size will increase. (iii) This increase in distinctness and size will go on up to a maximum, say in the sensum s_n of the field f_n. (iv) If he now goes further, various new and startling things will begin to happen. He will often find that, if he stretches out his hand in front of him, he will sense tactual sensa, correlated in shape with the visual sensum. He may also burn his fingers badly. He will generally find that his path is blocked. (v) If he manages to get past the obstacle he will find that his field f_{n+1} contains no sensum s_{n+1}, like those of the series $s_1 \ldots\ldots s_n$. (vi) Very often he will be able to sense a field f'_{n+1}, which does contain a sensum s'_{n+1} of the right kind, provided that he turns right round. The essence of the process, then, is a succession of visual fields, each containing at its centre one of a series of qualitatively similar sensa of steadily diminishing depth and increasing brightness and clearness, followed by a great discontinuity and the beginning of new, though often correlated, sensations.

Next, let us suppose that on another occasion the man does not try to turn his head so as to sense a visual field with a sensum like s_0 in the middle of it. Let him, instead, walk in some other direction, and let him stop

at some point in this course. Call his visual field at that time ϕ_n. ϕ_n may or may not contain a sensum like s_0. If it does, the sensum will certainly not be in the middle of the field, and will probably be a very distorted projection of s_0. But, on either alternative, he will generally be able, by suitably turning his head, to sense a field f'_1, in the middle of which there is a sensum s'_1, which is a good deal like s_0, though not as a rule so much like it as the sensa of the series $s_1 \ldots\ldots s_n$ are like each other. (As we say, he is seeing a different side of the object.) If he now follows his nose, he will in general sense a series of visual fields $f'_1 \ldots\ldots f'_n$, in the middle of each of which is a sensum of a series $s'_1 \ldots\ldots s'_n$. This series will have the same sort of internal relations as the series $s_1 \ldots\ldots s_n$, and will end up in the same catastrophic way. Now our solitary observer will often find that, wherever he starts, he can, by suitable head-turning, sense such a series of sensa. He thus comes to recognise a central region of discontinuity, to which he can walk from any position, and to which he passes through series of similar visual sensa of decreasing depth and increasing brightness.

Now he will find this notion of a central volume reinforced by some of his other senses. The two other senses that act at a distance are hearing and the feeling of radiant heat. They have interesting differences from each other and from sight, which will be worth mentioning. Let us begin with sound. There is an auditory continuum from which particular noises stand out, as particular coloured patches stand out from the sight continuum. But, whilst patches of colour have definite shapes and sizes, noises do not. It is extremely hard to state the vague spatial characteristics of a field of sound. Differences of direction in it can certainly be sensed, but each sound seems to fill the whole sound-field, though one is more intensely present in one part of it and another in another part. Coloured patches

in the same visual field do not interpenetrate. Two different colours cannot be sensibly present in the same place in the same visual field. A colour is either sensibly present in a place or it is not. There is no question of degree. But each sound seems to be present everywhere in the auditory field, though it is "more" present in some parts than in others. This difference between the sensible presence of sounds and of colours leads to a difference in the way in which common-sense supposes them to be present in physical Space. Common-sense says that the colours that it sees are spread out over the surfaces which it can touch. It refuses to say that they are present in the medium between this and the observer's body. But common-sense does not hold that the noise of a bell is spread out over the surface of the bell, or even that it is confined to the volume of the bell. I think it would prefer to say that the noise is present throughout the whole surrounding air, and that there is merely "more of it per unit volume" as we approach the bell.

Apart from this very important difference, to which we shall have to return, there are striking likenesses between sight and hearing. If we sense a sound s_0 (*e.g.* the auditory appearance of a tolling bell) we can turn our heads in such a way that a similar sensum s_1 "occupies the middle of the auditory field." If we then follow our noses we shall, as a rule, sense a succession of auditory fields $f_1 \ldots\ldots f_n$, each of which contains at its centre one member of a series of auditory sensa $s_1 \ldots\ldots s_n$. These are qualitatively alike and of increasing loudness, though I do not think we can say that there is anything corresponding to the continual decrease in sensible depth which we should find in a series of visual sensa. After you have reached a certain stage in this series you will generally find that, on stretching out your hands in front of you, you get tactual sensa, and that, as you do so, the sound ceases or is modified. Exactly parallel results to those

described in the case of sight are found, when we approach from different starting-points, or pass the obstacle in which such series generally end. Thus auditory sensa equally lead us to the notion of "centres." Now in very many cases, whether you move under the guidance of your visual sensa or under that of your auditory sensa, you will end up with similar tactual sensations after a similar series of kinæsthetic sensations. This happens, *e.g.* if we first look at a sounding bell with our ears stopped, and then unstop our ears and shut our eyes. Thus we come to think of centres of discontinuity which can be approached from all sides, and which are not merely centres for colour or for sound, but are centres for both.

If we now ask ourselves why colours are held to be on the bounding surfaces of such central volumes, and not anywhere else, whilst sounds are held to be both in and all round the sounding centre, the answer is plain. Visual sensa have sensible depth ; this steadily diminishes in the successive sensa that we sense as we approach a centre, but never vanishes altogether till we are too near the centre to sense any sensum of the series at all. On the other hand, noises have no fixed boundaries ; they do not exclude each other from the same sensible place ; and they do not, I think, have sensible "depth." We have thus no ground for saying that we approach the *sound* when we approach the *sounding centre*. A part of the sound is held to be wherever we are when we hear it ; it merely is present in greater density at places nearer the sounding centre.

Let us next say a word or two about our sensation of radiant heat. We have here series of sensa of the same kind as we have with sound. They lead us again to the notion of centres of discontinuity, and in general to centres which are common to radiant heat, sound, and sight. But there is one interesting and important peculiarity in the case of heat. If we start at a distance from a centre we feel a heat sensum ; and, as we

approach, our successive heat sensa are more and more intense, in the usual way. Now, as usual, when we get to a certain point in the series we can sense tactual sensa, if we stretch out our hands in front of us. These sensa will usually be intensely and painfully hot. The interesting point is that, in this case, heat is felt *both* in the surrounding space and on the surface of the central volume. There is no sensible depth in the field of heat sensa, so that, as with sound, we do not localise the successive sensa on the central volume. On the other hand, when we do feel the central volume, the tactual sensa are themselves hot. So the heat is regarded as *both* filling the surrounding space and residing in or on the central volume. Now common-sense regards what can be felt as *the* physical object *par excellence*, and the place to which one has to move in order to sense the tactual sensa as the place of the object. Owing to the fact of visual depth, and its gradual decrease as such central volumes are approached, common-sense regards all the successive visual sensa as localised on this volume. It therefore says that the central volume *is coloured*, not that it *causes colour*. In the case of the bell it does not say that this is *endowed with sound*, but that it is the *cause* of the surrounding space being filled with sound. In the case of heat it thinks of the central volume as both *being hot* and *causing* the surrounding space to be filled with heat. The discrete side of the common-sense view of the physical world is based on the peculiarities of the visual field, and on the fact that long intervals of free movement often come between tactual sensations. The continuous side of the common-sense view of the physical world is based on the peculiarities of the fields of radiant heat and sound. Heat sensations in some way form a connecting link between the two aspects of nature, since they are felt both *on* and *between* the centres of discontinuity.

It is obvious that these two sides of the common-sense view correspond to real facts in nature. But we may

reasonably suspect that the separation between them has been made too sharp, as all separations that are made primarily in the interests of practice tend to be. As a matter of fact, the common-sense view has been based mainly on experiences of touch, sight, and movement. Pervasive media, like air and ether, have only been recognised in historical times. Thus the continuous and transmissive side of nature has had to be fitted into a prehistoric metaphysic of the external world, made up mainly to deal with our experiences of visible and tangible volumes with sharp outlines. Atomic theories are so much more comfortable to most of us than hydrodynamic theories, because they fit in so much better with the scheme that we have inherited from the practical philosophers of the Stone Age. We learn, as time goes on, that light itself travels through a medium with a velocity, that colours seen depend on events in central volumes, just as do sounds heard, and that these colours may turn up in places where no correlated tactual sensa can be felt. All this will have to be dealt with later, more especially when we come to treat of date and duration. But, in the meanwhile, we may offer the suggestion that a good deal of our difficulty with the philosophy of the external world is due to the fact that we are trying to fit new data into a scheme based on experiences which did not include them, and which ignored or minimised the sensible facts, such as images, shadows, echoes, etc., to deal with which new concepts are needed. In just the same way we insist on forcing the facts of modern society into the ethical and political framework of a simpler age, without even the excuse that this "works well in practice."

So far, we have confined ourselves to the case of a solitary observer, immersed in a homogeneous medium, such as air, and dealing with resting objects. These are, of course, very common and practically important conditions, and the corresponding experiences are there-

fore common, and have left their traces deeply on everyone. I have tried to show that such an observer will soon reach the notion of "centres of discontinuity," dotted about in various places which he can reach by movement; and that his successive visual sensa fall into series which we will localise on the surfaces of these central volumes. Further, we have seen that the senses of hearing and of feeling heat will reinforce this notion, and will lead him to recognise these centres as common to the sensa of different senses. In particular, heat and sound will combine to give him the notion of centres surrounded with "physical fields." Sight, for reasons mentioned above, does not give to unsophisticated people the notion of a physical field; and when the advance of science makes it necessary to introduce this, considerable difficulties are felt in reconciling the omnipresence and the finite velocity of the light field with the strict localisation of colours on central volumes remote from the observer. We may say, if we like, that colour belongs *physically* to the continuous side of nature, but that it has so far belonged *epistemologically* to the discrete side of nature.

We can now pass to the case of a number of observers; and thence to the more complex cases of non-homogeneous media, which considerably "stain the white radiance" of our original view about sight and the localisation of its objects. Even with the solitary observer in the homogeneous medium we have passed to a new meaning of "place" for visual sensa. The first and most primitive meaning was the place of a single visual sensum in its own visual field. We have passed beyond this to a group of visual sensa, each selected out of different sensible fields of the same observer. The members of such a group are said to be in the same place, through their correlation with each other and with the movements of the observer. The "place" referred to here is clearly not a place in any visual field, but is a place in the continuum of possible positions of

the observer's body. And the presence of a visual sensum at such a place is not an ultimate unanalysable relation, like its sensible presence at a place in its own visual field. On the contrary, we have just been analysing the meaning of the statement that a visual sensum is present at a certain place in the movement continuum, and have found that it means that the sensum in question is one of a set of sensa belonging to successive visual fields and connected with each other and with the observer's movements in the ways indicated above.

When a set of visual sensa from successive fields of a single observer have the sort of relations that we have been describing, we will say that they are *optically compresent with respect to that observer.* Each member of the set may be said to be *optically present* at the place in the continuum of possible positions of the observer's body which he reaches when the character of the set begins to change abruptly. Looking at the matter from the point of view of this place in the movement-continuum, we may say that it is *optically occupied* by sensa of such and such a kind from such and such a direction. When we have a number of such sets, which all converge on a central volume wherever the observer may start, we will say that this place is "*optically filled*" with sensa of a certain kind. We shall see later that a place may be optically occupied without being optically filled. We have seen that, as a rule, when a place in the movement-continuum is optically filled, correlated tactual sensa are present at that place. (We have not as yet considered what is meant by saying that tactual sensa are present at a place in the movement-continuum, but we will for the moment take this notion for granted. We have also not as yet adequately discussed the notion of place in the movement-continuum. To these points we shall return later.)

Now, under normal conditions, we can not only find groups of optically compresent sensa in the suc-

cessive visual fields of a single observer. We can also find something of the same kind in the fields of different observers. Let us consider what is meant by saying that the sensa s_A and s_B, belonging to visual fields f_A and f_B of the observers A and B respectively, are in the same place. We will suppose that A and B have turned their heads in such directions that s_A is in the middle of f_A and s_B in the middle of f_B. If they change places and repeat the process, A's new sensum will, as a rule, resemble B's old one in shape, and conversely. Suppose that, when they have both turned their heads so as to sense fields with these correlated sensa at their respective centres, they start to walk, following their noses. Let A do this till he senses the sensum s_A'', which is the most distinct of the series. Let him then stop, and let B now start to follow *his* nose. B's body will, in general, get nearer and nearer to A's, and by the time that B senses his most distinct sensum s_B'', they will be nearly in contact. If they now follow up their respective courses they will certainly run into each other. If they both stretch out their hands they will, in general, both sense tactual sensa correlated in shape with their visual sensa. Thus the notion of a common centre in the movement-continuum, at which a number of visual sensa are optically compresent, is extended to include series of optically compresent sensa belonging to the fields of different observers as well as to those of a single observer.

Now it will be noticed that the place which a group of optically compresent sensa are said to occupy is defined by bodily movement. I have called the continuum of possible positions of an observer's body "the movement-continuum." I think that "place," in the physical sense, refers primarily to places in this continuum. Before we can deal with the more complicated cases of visual sensa sensed by an observer who is not surrounded by a homogeneous medium, we must get clearer about the notion of place in the movement-

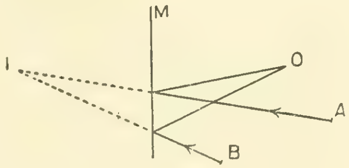
continuum. The experiences of turning one's head so much and then walking so far in a straight line are not in themselves spatial experiences. They are simply series of kinæsthetic and muscular sensations, different stages of which fall into different specious presents. They last for sensibly different times, and tire us to sensibly different degrees. How do they come to lead to the notion of a continuum of physical places, which are common property to all the observers and are co-existent? We cannot fully deal with this question till we have dealt with the dates and durations of sensa and of physical objects; but we can at least say this much: These series of *successive* kinæsthetic sensations would not lead to the notion of a continuum of *contemporary* places if it were not for their correlation with experiences of sight. All the fundamental concepts needed for dealing with Space have their origin, and their only *literal* exemplification, in the visual field. Space is thought of as a whole of contemporary parts, spread out at various distances and in various directions. A whole of this kind is sensed, if I am right, at each moment by sight, and in no other way. Turnings of the head are interpreted in terms of direction because (*a*) different sensa do have different visible directions in the same visual field; and (*b*) because with every turn of the head is correlated a change in the sensible position of some sensum within the field of view. Or, to put it more accurately, when we turn our heads a field f_1, with a sensum s_1 at a certain sensible place in it, can be replaced by a field f_2, with a similar sensum s_2 in a different place in it, *e.g.* in the middle. Again, a series of kinæsthetic sensations is interpreted as the traversing of a physical line of a certain length by the observer, because the sensible depths of the similar sensa $s_1 \ldots\ldots s_n$ in the middle of the successive fields $f_1 \ldots\ldots f_n$ continually diminish as the series lasts longer. Sight and movement are thus under reciprocal obligations. Were it not for sight, with its

extended fields of contemporary parts with different sensible depths and in different sensible directions, we should lack the very concepts needed for interpreting the movement-continuum spatially. On the other hand, were it not for the existence of groups of visual sensa, correlated with each other and with movements, in the way described, we should never have reached the notion of the optical compresence in the same place of visual sensa from different fields.

But, although the facts about visual sensa which lead to the recognition of "centres" in which groups of visual sensa are optically compresent, are necessary in order that the movement-continuum may be interpreted spatially, we must not suppose that all places in the movement-continuum are optically full or even optically occupied at all. The vast majority of them are not. Moreover, some which are optically occupied from several directions are yet not centres at which correlated tactual sensa are present. Let me illustrate the first point. If I direct my movements by a certain series of optically compresent sensa in the way described, but stop before I reach the end of the series, I have reached a place in the movement-continuum. But I have not arrived at the place in which the sensa of this series are optically compresent, and when I stretch out my hands I may feel nothing at all. And the place in the movement-continuum at which I have stopped may quite well not be occupied by *any* visual sensa of any series. What do we say under such circumstances? We say that we have indeed reached a physical place, for we have walked so far, and in such and such a direction. But we add that this place is neither optically nor tactually occupied. If *no* places had been optically or tactually occupied, we should almost certainly not have interpreted the movement-continuum spatially, or have arrived at anything like our present conception of the external world. As it is, a large number, though a minority, of places in the

movement-continuum are optically occupied; many are optically filled; and most of these are also centres for sound and heat, and are also tactually occupied. This fact gives us the contrast between the filled and the empty parts of the movement-continuum, and helps us to conceive it as a Space dotted about with physical objects in definite places and with definite boundaries.

We are now in a position to deal with the less usual forms of optical presence. These arise when, as the physicist would say, we are surrounded by a non-homogeneous medium. Our present task, however, is to describe as accurately as possible the actual facts about our visual sensa, and not to offer causal explanations of them in terms of their correlations with physical events. To begin with a very simple case, let us suppose that I am looking at the image of a luminous point in a plane mirror. I can, as before, turn my head in such a way that I sense a visual field f_1 with a sensum s_1 in the middle of it, similar to the sensum s_0 that originally attracted my attention. Having done this, I can, as before, follow my nose. Up to a point my experiences will be exactly like those which we have already described. There will be the same kind of series of sensa $s_1 \ldots\ldots s_n$, qualitatively much alike, each in the middle of its field, of steadily decreasing visual depth, and so on. But at a certain stage in the series I shall suddenly sense certain tactual sensa, quite uncorrelated with the visual sensa of the series (*i.e.* I shall "bump into the mirror"). This is illustrated by the figure below:



If I, or anyone else, were to start from B instead of from A, the same sort of experiences would be

enjoyed. This, however, is by no means all. A and B might both have experiences of this kind if they were both looking directly at some source of light through a thin sheet of transparent glass. The difference is the following: In the former case, if A or B break through or get round the mirror and try to continue their course, there will be nothing in their visual fields corresponding to the visual sensa that led them up to the mirror. (That is to say, their visual experiences, as they move along the dotted part of the line AI or BI, are quite different from those which they had when they traversed the undotted parts of these lines.) If there were merely a thin sheet of transparent glass at M, and A and B were viewing through it a source of light at I, the series of visual sensa would go on steadily after they had broken through or got round the obstacle.

The next point to notice is that the courses of A, B, C, etc., who start from the same side of the mirror, really do converge on a common place in the movement-continuum. If they pursued them through the mirror or the glass they really would meet at I. The difference in the two cases would be this: If they were looking at something directly through a thin piece of glass, the series of visual sensa of each of them would end at about the time when their bodies came in contact with each other, and correlated tactual sensa could be sensed by each if he stretched his hand forward. If they are looking at a mirror-image the series of visual sensa which leads them up to the mirror not only ceases abruptly as soon as they get through or past it; they also find that, when they meet, they either sense no tactual sensa at all, or, if they sense any, these are quite uncorrelated with the visual sensa that originally guided them on their respective ways. If they want to sense correlated tactual sensa, they will have to go to quite a different place in the movement-continuum, and one that is not on their course of movement at all, viz., the place O

in the figure. Now this place O, which is on A's and B's side of the mirror, is also a place in which visual sensa, much like those that guided A and B up to the mirror, are optically compresent. But, as we have remarked, it is in quite a different direction from those followed by A and B; and people who walked up to *it* would sense tactual sensa correlated with the visual sensa that led them to it, and therefore also correlated with the visual sensa that led A and B *away* from it towards I.

There is one further point to notice about I as compared with O. Not only are there no tactual sensa at I correlated with the visual sensa that guide observers from the other side of the mirror on their paths towards I; there is also a purely optical peculiarity about I. The place O is optically *filled* with visual sensa of the kind in question. That is, any observer, no matter in what direction he may approach O, will sooner or later begin to sense a series of visual sensa of this kind, which are optically compresent at O. This is far from being true of I. I is not a centre which is occupied by visual sensa of the kind in question for *all* observers, or even for the latter parts of the course of *any* observer. People at the back of the mirror, who look directly at the place I, either see nothing there or else they sense sensa which have no resemblance to those which A and B sense on the earlier part of their courses. Again, A and B, during the latter part of their courses, sense no such sensa as they did when they were on the reflecting side of the mirror. We must say, then, that I is *occupied* by the sort of sensa that constitute the mirror-image, from certain places, but by no means from all; whilst it may be *filled* with visual sensa of quite a different kind. On the other hand, O is not merely occupied, but is filled, with such visual sensa as constitute the mirror-image. (For the moment I neglect the inversion of the image, which of course makes a characteristic difference between the

sensa that fill O and the otherwise similar sensa that optically occupy I from places on the reflecting side of the mirror.)

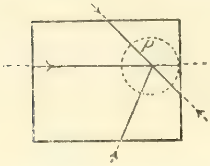
We may sum up the peculiarities of mirror-images with respect to place, as follows: (i) The usual correlation between visual and tactual sensa breaks down. Usually, when visual sensa are optically compresent at a certain place, correlated tactual sensa can be sensed by an observer who walks up to that place. If, however, you want to sense tactual sensa correlated with the visual sensa that constitute a mirror-image, you must go to quite a different place from that at which these visual sensa are optically compresent. This is, of course, puzzling, because unusual; but there is no theoretical difficulty in the fact that two sorts of sensa, which are generally compresent, should sometimes not be so. People whom we meet are generally compresent with their trousers, but this rule is liable to break down in swimming-baths. (ii) The optical places of mirror-images are never optically *filled* with the sensa that constitute the image, but are only occupied by such sensa from certain directions and from the remoter places on these directions. On the other hand, they may be at the same time optically filled with visual sensa that are not in the least like the mirror-image, but *are* correlated with tactual sensa which can be sensed by people who walk to these places.

We can now ask: What is it precisely that the laws of geometrical optics tell us about mirror-images? The answer is simple. They tell us where sources would have to be placed, and what tangible shapes they would need to have, in order that an observer who stands in a given position shall continue to sense the same visual sensum when the heterogeneous medium, with which he is in fact surrounded, is replaced by air. If we like to use the convenient language of the general Theory of Relativity, we can say that the introduction of suitable sources in suitable places in a homo-

geneous medium will always "transform away" (*i.e.* be equivalent to) the effects of any heterogeneous medium for any *one* visual sensum of any *one* observer in any *one* position. In favourable cases the transformation may apply to many sensa of many observers in many positions. But no arrangement of sources in a homogeneous medium will be equivalent to the effects of a heterogeneous medium for *all* observers in *all* positions. For instance, if we remove the mirror M and put a luminous point of the right colour at I, A's and B's visual sensa will be unchanged; but very different sensa will now be introduced into the fields of observers at the back of the mirror. The laws of geometrical optics are then simply the rules according to which we can calculate the tactual shapes and the positions of such hypothetical sources as would transform away the effects of a heterogeneous medium for a given sensum of a given observer in a given place in the movement-continuum.

(*c*) *The Relation of Optical Occupation.*—I think that we are now in a position to go a step further in our analysis of the optical places of visual sensa. We notice that three types of case can arise, ranging from the completely normal, through the mildly abnormal, to the wildly abnormal. (i) There is the case of seeing things by direct vision in a homogeneous medium. Here all observers in all directions (provided they be not too far off) can sense very similar sensa, and can bring them into the middles of their respective fields of view; and the paths of all these observers converge to a common place in the movement-continuum, at which all the sensa of all these series are optically compresent. The proviso that the observers are not to be too far off is added in order to allow for the possible interposition of opaque obstacles between the place where the observer is and the centre of optical compresence. If a luminous point be inside a room, it is true that the place where it is said to be is optically occupied by sensa of similar

quality from all *directions*; it is not true, however, that it is occupied by such sensa *from all places* on any one of these directions. It is not so occupied from places that are outside the room. What we can say is that there is some finite distance r, such that the place in question is optically occupied by such sensa from all places within a sphere of radius r drawn with this place as centre. The figure below illustrates this restriction.

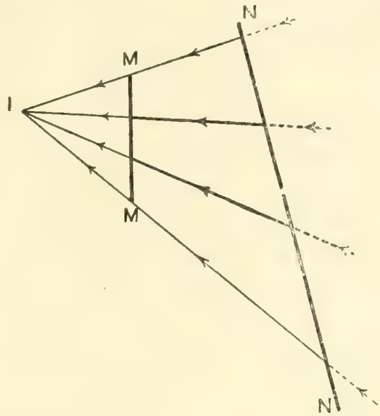


The dotted parts of the lines are the positions from which P is not optically occupied by sensa of the sort with which it is optically filled.

(ii) In the case of seeing a mirror-image there is a certain place behind the mirror which (a) is occupied by similar visual sensa from *many*, but not from *all*, directions which converge on the point. (b) It is only occupied by visual sensa of this kind from certain places on any one of these directions, and no series of such places extends up to the place where the image is said to be. On the contrary, these series always end abruptly at a finite distance from the place. (c) The place of the mirror-image may, though it need not, be also a place of complete optical compresence from all directions. But, if so, the sensa with which it is optically filled will be quite unlike those which optically occupy it from places on the reflecting side of the mirror. In the figure below, M is a mirror, N an opaque obstacle, and I the place of a mirror-image. The full thick part of a line represents the places on it from which I is optically occupied by the sensa which constitute the mirror-image. The full thin part represents the places from which it is optically occupied by sensa of the sort with which it is optically filled. The dotted parts represent places from which it is occupied by neither kind of sensa.

(iii) Lastly, with distorted mirrors or other kinds of more heterogeneous media, any observer may find that

he has continually to turn his head at each step, if he wants to sense a series of visual fields with at all similar sensa at their centres. In such cases the observers will



also generally find that their sensa are affected with sensible movement as they turn their heads.

We thus have a series of cases, ranging from the complete tameness of (i) to the extreme wildness of (iii). Now it seems to me that the psychological and the logical order are here opposite to each other. Psychologically our concept of Space, and of the places of things in it, is built on (i), *i.e.*, on the commonest and most practically important cases. If these had been less common and less practically important, it is doubtful whether we should have reached anything like our present view of the external world. But, logically considered, it is the wild cases, of type (iii), that are of fundamental importance. It seems pretty clear that the normal cases can only arise when certain special simplifying conditions are fulfilled, viz., those which we sum up by saying that the medium is homogeneous. These special conditions mask the real complexity of the relations involved; whereas the wilder cases exhibit these relations in their most general form. There is some hope that, if we treat the wild cases as funda-

mental, we may be able to deal with the normal ones as specially simplified instances of a more general relation; as, *e.g.*, a circle may be regarded as a specially simplified case of an ellipse. But there is very little hope that, if we take the relations involved in the normal cases as fundamental, we shall be able to interpret the abnormal cases in terms of them. And, as Critical Philosophers, it is our business to try to deal with *all* the facts, and not to hush up the existence of abnormal sensa, as though they were the peccadillos of a Cabinet Minister.

We can now say something about the logical characteristics of the relation of *optical occupation*. (1) It is a relation between a visual sensum on the one hand and a place in the movement-continuum on the other. (2) It is a many-one relation. This means that a given sensum s can only occupy optically one place in the movement-continuum, but one place in the movement-continuum can be optically occupied at the same time by many sensa. (3) I think we must also hold that the relation of optical occupation is irreducibly triadic. This means that any complete statement, which asserts this relation to hold, involves *three* terms, viz., the sensum, the place that it optically occupies, and a third term. My reason for saying this is the following: The statement that the place p is optically occupied by the sensum s seems to be incomplete; the full statement would seem to be that p is optically occupied by s from q, where q is the place in the movement-continuum occupied by the observer's body. We see this more clearly if we state exactly what we mean when we say that s optically occupies p. s will be a sensum which is sensibly present in a certain observer's visual field at the time. This observer will, in fact, be in a certain place q. To define the direction of p, the place optically occupied by s, we have to suppose that the observer turns till he senses a visual field with a sensum s', similar to s, in its centre. The

direction of p is then the direction in which he would start to walk if he followed his nose. The distance of p is determined by the sensible depth of s' in the observer's visual field. It is the distance that he *would* have to walk to reach a source *if*, in fact, the medium were homogeneous and s' were due to the transmission of light directly from this source to his eye. It seems therefore that the full meaning of the statement that s is optically present at p cannot be understood without a reference to the place q occupied by the observer in whose visual field s is sensibly present. If so, the relation of optical occupation is triadic, and the minimum complete statement is that s occupies p from q.

Of course, in a great many cases, if the observer were to walk to a place p, thus determined, he would not find any centre of discontinuity there which could be taken as the source of his original sensum s. And, in many cases, he would not find that a series of sensa like s were sensibly present in the middle of his successive visual fields as he moved in the line from q to p. This, however, does not prove that our definition of optical occupation is wrong. It merely shows that the fact that a sensum s occupies p *optically* from q is no guarantee that p is *physically* occupied by anything closely connected with s. This we already knew from our experiences with mirrors and other types of non-homogeneous medium.

We must not be frightened of triadic relations, for there are plenty of other examples of them in daily life. The relation of *giving* is an example, since it essentially involves a giver, a gift, and a recipient. The minimum intelligible statement which asserts the relation of *giving* is of the form "x gives y to z." It is true that we sometimes use apparently simpler phrases, like "Smith gives to the Additional Curates' Fund"; but these are clearly elliptical, and, when fully stated, appear in the form "Smith gives *something* to the Additional Curates' Fund." Of course, whenever x, y, and z stand in a

triadic relation, this *involves* certain dyadic relations between them by pairs; but the assertion of the triadic relation is not analysable into the conjoint assertion of these dyadic relations. The latter are derived from the former, and the former is not built up out of the latter. Contrast the relation of "uncle" with that of "giving." Both involve three terms. For to say that x is uncle of z means that x is brother of some third person y, who is a parent of z. This does not make the avuncular relation triadic; for it is completely analysable into the conjoint assertion of these two dyadic relations, and they are not merely derived from it.

Now we are very liable to ignore the fact that a relation is polyadic and to treat it as dyadic. This happens if two of the terms mainly interest us and the rest are uninteresting or generally constant. When this condition ceases to be fulfilled we are liable to find apparent contradictions, which can only be avoided by recognising the polyadicity of the relation. When we say that A is to the right of B, we often ignore the fact that we are really asserting a triadic relation between A, B, and our own hands. Eventually we meet someone as sane as ourselves, who insists that A is to the left of B. This is a contradiction, until we take into account the neglected third term, which is different in the two cases, and see that both parties may be right when their full meanings are made explicit.

If we accept the view that the relation of optical occupation between visual sensa and places in the movement-continuum is triadic, there is no difficulty in the fact that a place may be at once optically filled with sensa of a certain kind and optically occupied from many places with sensa of quite a different kind, which have no connexion with the physical filling of this place. P is optically filled with sensa of the kind k if there is a closed surface in the movement-continuum such that it contains P, and such that P is optically occupied by sensa of the kind k from all places between the outside

of P and the inside of this surface. This is quite compatible with the fact that there are other places in the movement-continuum from which P is not occupied by sensa at all. It is also quite compatible with P being optically occupied from many other places with sensa of a different kind k'. This is what happens in the case of mirror-images. With a plane mirror the situation is as follows: There is a set of places from each of which a sensum of the kind k' is optically present at P. These places are on lines of approach which converge on P. But (*a*) all the lines on which such places are situated are confined within a certain solid angle with P as vertex; and (*b*) even for lines within this region the series of places from which sensa of the kind k' are optically present at P does not reach P, but stops short at a finite distance from it.

The question might now perhaps be raised: "Is it enough to suppose that the relation between a visual sensum and a place which it occupies in the movement-continuum is triadic?" Ought we not, in the case of the mirror-image, for instance, to bring in the positions of the source and the mirror as well as that of the observer, and thus make the relation at least pentadic? This is a plausible question, but I think that it rests on a confusion. Undoubtedly, if we want to *predict* in what place a sensum of a certain kind will be optically present from the place of a certain observer we need to know the positions of the source and the mirror. But these are not involved in the *meaning* of the statement that such and such a sensum is optically present in such and such a place. We saw that a reference to the place of the observer *is* an essential part of the meaning of this statement. But the parts played by the source and the mirror are merely *causal* and not *constitutive*. This is clear from the fact that we have been able to give a satisfactory definition of optical occupation without mentioning the positions of the source or the mirror. The way in which these do become relevant is the

following: The positions of the source and of the mirror do determine causally, according to the physical laws of light, the sensible place of the sensum s in o's visual field. And the place p in the movement-continuum, which is optically occupied by s from where the observer is, depends (by definition) on the sensible place of s in o's visual field. But it is one thing to say that the positions of the source and the mirror are factors which *causally determine* the nature of the sensum which optically occupies a particular place p from another place q, and quite another thing to say that the positions of source and mirror have to be stated before the proposition that s optically occupies p from q can be *understood*. If the latter were true, the relation between a sensum and its optical place would be at least pentadic, for the minimum intelligible statement about optical occupation would be of the form "s optically occupies p from q with respect to the medium m and the source σ." But this does not seem to be true, and therefore I see no reason at present to hold that the relation of optical occupation is more than triadic.

(*d*) *Physical Place.*—Having dealt with the puzzling, but most illuminating, case of abnormal optical occupation, we can now treat the places of physical objects. Before the notion of physical place can be profitably discussed, we must form a clearer idea of what we mean by a physical object. For a physical place is the sort of place that can be occupied by a physical object. So far we have simply contrasted physical objects with the sensa which are their appearances. But it may well be that "physical object," in this sense, is a somewhat loose term, and covers several different kinds of entity. We must even be prepared for the possibility that what common-sense calls a physical object may be really a number of correlated objects of fundamentally different kinds.

That this is so will be plain, I think, if we compare the following four entities: a particular visual appear-

ance of a certain penny; an image of the penny in a plane mirror; what common-sense understands by the penny; and the atoms, electrons, etc., which science asserts to be the ultimate physical constituents of the penny. The first, no one would think of calling a physical object. The second would not indeed be called a physical object; but it is much more than a mere sensum. It can be "seen" by a number of different observers from different places in exactly the same sense in which the penny itself can be seen. And it has a certain persistence and independence. It is, in fact, a group of closely correlated visual sensa, and a certain place in the movement-continuum is optically occupied by members of this group from a great many places, although it is not filled by them. We refuse to call it a physical object, because of the lack of complete optical filling, and because of the absence of correlated tactual sensa when we come to the place which is optically occupied by sensa of such a group. I will call such a thing as a mirror-image a *Partial Optical Object:*—optical, because it consists wholly of visual sensa; partial, because it does not optically fill the place which it optically occupies.

Now what common-sense understands by a physical object, such as a penny, is something more than this in two ways at least. (1) It involves a *Complete Optical Object*, for the place where the penny is said to be is optically *filled* with correlated brown elliptical and round sensa. (2) It involves something more, which is not optical at all. The place in the movement-continuum which is marked out for us by being filled with the complete optical object very often resists our efforts to move into it. It is often a centre for sound and radiant-heat sensa. And, as a rule, we sense tactual sensa of characteristic shape and of some temperature or other when we come to this place. It is very exceptional for condition (1) to be fulfilled without condition (2); though I suppose we may say that condition (2) is evanescent

in the case of clouds and wisps of coloured vapour. Let us call the penny, as common-sense understands it, a *Perceptual Object*. Now the important thing to notice is that a perceptual object is really not one single homogeneous object, present in a place in the movement-continuum in one single sense of "presence." It is a number of interconnected objects of different types, and the different kinds of object included in it are present in different senses in the place where the perceptual object is said to be. I will call the various correlated objects which together constitute a perceptual object *constituents* of the perceptual object. It would be misleading to call them *parts* of it, because this would suggest that they literally fit together to fill up the place in which the perceptual object is said to be. This could not be true, because they are of radically different kinds, and are in this place in radically different senses. Take, for example, the perceptual object which is what common-sense means by a penny. One constituent of this is a complete optical object. This consists of visual sensa. Each of these is literally present only at a place in its own visual field. The optical object is only present at the place in the movement-continuum in the sense that this place is optically filled by the visual sensa which together make up the complete optical object. Another constituent of the perceptual penny is a group of tactual sensa. Each of these is literally present only in its own tactual sense-field. The whole group is present at the place where the penny is said to be, in some Pickwickian sense which we have not yet defined, but which, from the nature of the case, cannot be identified either with sensible presence or with optical presence. It is because the perceptual object is not one homogeneous thing, but a complex of correlated constituent objects of various types, that science finds it necessary to pass beyond the perceptual objects of common-sense. This does not mean, as we are liable to think, that the latter are "unreal." It only means

that they are unsuitable units for scientific purposes, though admirably convenient units for the purposes of everyday life. This leads us to the last meaning of "physical object," viz., what Whitehead calls *Scientific Objects*. (Though I use this convenient expression of Whitehead's, and mean it to *apply to* much the same things as he applies it to, it does not necessarily follow that he would agree with the account that I am going to give of the *concept* of such objects.)

Science tells us that a penny "consists of" large numbers of colourless particles, moving about with great velocities in characteristic ways. This is understood both by science and common-sense to mean that the colourless particles are parts of the perceptible brown penny in the same literal sense in which a visual appearance of the King's head is a part of the visual appearance of the penny. It would be difficult to accept this interpretation, even on a naïvely realistic view of pennies and our perception of them. It is not easy to believe that the brown continuous surface of the penny, which, on that view, we sense, can literally be composed of colourless particles. Anyhow, this simple-minded interpretation of the scientific statement becomes impossible when we remember that the perceptual penny is not one homogeneous object, but is a complex of connected constituent objects of different types, which all occupy a place in the movement-continuum in different Pickwickian senses. It is clear that nothing could be a part of *all* the constituents of a perceptual object in any one sense of the word "part," whether literal or Pickwickian. If it be literally part of *one* of the constituents, it can only be a part of the others in as many different Pickwickian senses as there are different types of constituent. Moreover, some at least of the constituents are such that nothing could *literally* be a part of them. One constituent, *e.g.*, of a perceptual object is a complete optical object. Nothing could claim to be a literal part of this except one of the visual

appearances of the perceptual object. And even these are not literally *parts* of the complete optical object. A visual appearance of a penny is a "part" of the complete optical object only in the sense that the latter is a group of optically compresent sensa of which this appearance is one *member*. But the various members do not literally fit together to make up a surface, and therefore they are not *literally* parts of the complete optical object.

We can now return to the statement that perceptual objects, like pennies, are "composed of" scientific objects, like electrons. From what we have just said, this cannot mean more than that the scientific objects are literally parts of *one* of the constituents of a perceptual object. It is further quite clear that they are not literally parts, or even members, of the *optical* constituent of the perceptual object. This, I take it, is why there is no objection to the view that a brown penny is composed of colourless electrons. The brownness belongs to the optical constituent; and the electrons are not literally parts of this, but at most of some other constituent of the perceptual object.

Now I think that by a scientific object we mean something that *literally* occupies a place in the movement-continuum. And by this I mean that it occupies it in the same indefinable way in which a sensum occupies its sensible place in its own field. If this be right, the relation between the place of the perceptual object and its component scientific objects may be stated as follows: The perceptual object marks out a certain region in the movement-continuum by the presence in this region of its various constituents. These constituents are all present in this place in different ways, and these ways are all definable and Pickwickian. We have attempted to define the way in which the optical constituent is present, because this is the most difficult and important case. Science conceives that the regions in the movement-continuum, thus marked out, are literally occupied by certain objects which have an important

causal bearing on the nature of the sensa which occupy such regions in their various Pickwickian ways. These supposed objects, defined as the *literal* occupants of places in the movement-continuum, are what we mean by scientific objects. And a perceptual object is composed of certain scientific objects, in the sense that the latter *literally* occupy that region of the movement-continuum which the constituents of the former occupy *in Pickwickian senses.*

(*e*) *Summary of Conclusions about Place.*—There is one and only one literal sense of "being in a place." This is not definable, but it is exemplified in our sense-experience most clearly in the presence of a visual sensum at a certain sensible place in its visual field. The concept of being in a place is based on our sensible acquaintance with such instances as this. It can then be applied in thought to types of object and of continuum which we cannot sense as simultaneous wholes. Again, there is one and only one kind of place which we deal with when once we leave individual sensa and their fields and pass to physical objects in the widest sense of the term. This is a place in the continuum of possible positions of our bodies as we move. This continuum is not sensed as a simultaneous whole; but our successive experiences of motion are synthesised under the concept of a spatial whole, through analogy with visual fields which we can sense simultaneously. Now, although there is only one literal sense of being in a place; and although by "place" we always mean "place in the movement-continuum, spatially conceived," so soon as we leave the individual sense-field; still there are many derivative, definable, and Pickwickian senses of "being in a place." Whenever we talk of any sensum occupying a place in the movement-continuum, we are using terms in a Pickwickian manner, and are bound to define them. And for different kinds of sensa different Pickwickian kinds of occupation will have to be defined.

Now there are certain correlations between the sensa of successive fields sensed by the same observer, between contemporary sensa of different observers, and between sensa of different kinds, which constantly occur in real life, and make these definitions possible and useful. But we are liable to overlook cases where these correlations break down in whole or in part, and thus to produce an illusory simplification. This mistake is avoided by considering such facts as mirror-images. We found that the perceptual objects of everyday life are not homogeneous, but are really composed of a number of correlated constituent objects, all occupying, in various Pickwickian senses, the same region of the movement-continuum. A mirror-image bears a close resemblance to the complete optical object which is one of the constituents of an ordinary perceptual object. It differs from a perceptual object in three ways: (1) It is not a complete optical object, but only a partial one. (2) The place which it optically occupies is not also occupied by correlated tactual and other types of object. (3) There is good reason to think that the place of a perceptual object is literally occupied by certain scientific objects, which are intimately connected causally with the sensa which occupy this place in Pickwickian ways. In the case of a mirror-image, the place which is optically occupied by the sensa which make up the image may or may not also be literally occupied by scientific objects. But, on either alternative, the nature of the sensa is *not* causally determined by the scientific objects which occupy *this* place, and *is* causally determined by the scientific objects which occupy certain *other* places, viz., the places where the source and the mirror are perceptually present. Finally, just as a place in the movement-continuum may be optically occupied without containing any relevant scientific objects, so there may be many places in the movement-continuum which contain important scientific objects without being either optically or tactually occupied. If there had been no perceptual objects, or if the relevant

scientific objects had not as a rule occupied the region marked out for us by the perceptual objects to which they are most relevant, we should hardly have reached the notion of scientific objects at all. But, once having reached this notion from reflecting on perceptual objects, there is no reason why we should not apply it to regions which are not occupied by perceptual objects at all. Nevertheless, this is a late development of human thought, which has happened well within historical times, whereas the recognition of perceptual objects is, of course, prehistoric and almost certainly pre-human.

The Concept of Shape.—It remains to consider what is meant by "shape," and what is the exact cash value of common statements about shape, such as "This penny is round." The notion of shape is one of the many points where the traditional separation between Space and Time wears very thin. This is readily seen if we ask: "What is the shape of a cloud of coloured vapour?" As the outlines of a cloud are continually shifting, there is nothing that can strictly be called *the* shape of it. We can, however, divide up the history of the cloud into shorter and shorter successive sections, and talk of the shape of each of these. Shape only becomes a perfectly definite concept when it refers to a *momentary* extended object; it can therefore only be defined strictly by the use of Extensive Abstraction. It is true, however, that there are many objects, such as pennies, for which the shapes of successive momentary sections are practically identical over a long slice of history. In such cases we can talk of *the* shape of the object. We can say that a penny has a definite shape, and that this is circular. We have now to consider the precise meaning of such statements.

(*a*) *Sensible Shape.*—Just as there is one and only one non-Pickwickian sense of being in a place, so there is only one literal sense of having a shape. We cannot define "shape" in its literal sense, any more than we

can define "being in a place" in its literal sense. But we can and do become acquainted with concrete instances of shape in our sense-fields. The literal meaning of shape is best illustrated by a visual sensum which persists unchanged throughout the whole of the short duration of a single sense-field. It will be remembered that, in the present chapter, we are making the simplifying assumption that sense-fields and the sensa which they contain are literally momentary. This assumption will be corrected in the next chapter. But in the meanwhile we may say that *Sensible Shape* is the sort of shape possessed by visual and other sensa, and that this is the fundamental meaning of shape.

(*b*) *Optical Shape.*—We talk of a number of different observers "seeing the same object from different places." We have already discussed the cash value of this statement with sufficient accuracy for the purpose of defining optical occupation. For the present purpose we must go a little further and draw a distinction which we have hitherto ignored for the sake of simplicity. When several people are said to "see the same object," this sometimes means that they all "see the same part of the object," and it sometimes means that they "see different parts of the same object." Moreover, when they are seeing different parts of the same object, it would be held that sometimes the parts which they see are entirely separate, and that sometimes they partially overlap each other. The following examples will illustrate these distinctions: (1) If a penny be laid on the table and a number of people stand round and look at it, we should say that they all "see the whole of the upper surface of the penny." (2) If I am in my rooms with the door shut, and I look at the door from inside the room whilst you look at it from outside in the passage, we should be said to be "seeing wholly separate parts of the same object." (3) If a cricket-ball be put on the table and a number

of people stand round and look at it, we should say that they all "see partially different parts of it, but that the parts seen by adjacent observers partially overlap." It is quite evident that these three different types of statement express three genuinely different situations, all of which often arise in real life. On the naïve view, that we literally sense parts of the surfaces of physical objects when we look at them, the meanings of such statements are tolerably obvious. But we have long ago deserted that view; and indeed one of the reasons which made us do so was the differences in sensible shape of the sensa of various observers who were all "seeing the whole of the upper surface of a penny." It is therefore necessary for us to define Pickwickian senses in which such statements are true.

A and B may be said to see the same part of a perceptual object when the visual sensa s_A and s_B, which are the appearances of this object to A and B respectively, are optically present in precisely the same region of the movement-continuum. It might be said: "How is this possible, when s_A may be circular and s_B elliptical; or, again, both may be circular, but s_A much bigger than s_B?" This objection rests on a confusion between optical and literal occupation. There is nothing in the definition of optical occupation to prevent precisely the same region of the movement-continuum being optically occupied from different places with sensa of various sensible shapes and sizes. What would be impossible is that either (*a*) the same place in a sense-field should be sensibly occupied by two sensa of different shape or size; or (*b*) that the same region of the movement-continuum should be physically occupied by scientific objects of different shape or size. It is now easy to deal with the other two cases. We see wholly different parts of a perceptual object if the visual sensa, which are the appearances of this object to us, are optically present in wholly separate regions of the movement-continuum. Lastly, A and B see partially overlapping parts of a

perceptual object if (*a*) the sensa s_A and s_B are optically present in different regions of the movement-continuum; (*b*) these regions partly overlap; and (*c*) the overlapping part is optically occupied by a part of s_A and by a part of s . What we must clearly understand is that literally it is nonsense to suggest that the various sensa which constitute a complete optical object themselves overlap and together make up a single surface.

It is hardly worth while to take great trouble to define *the* optical shape of a perceptual object. This would involve defining some Pickwickian sense in which we could talk of *the* shape of the complete optical object which is a constituent of the given perceptual object. Now common-sense would admit that no one can literally see the whole of any perceptual object from any one position. And it would admit that the visual shape and size of any part depend on the position of the observer. In fact we only use visual shape and size as indications (trustworthy enough under normal conditions, if suitably corrected) of the shape of the perceptual object. And by the shape of the perceptual object common-sense understands its felt shape. It is possible, and perhaps useful, to define the optical shape and size of a part of a perceptual object from a given direction. This might be done as follows: If we look at the place where a perceptual object is, bring the visual appearance of the object into the middle of our visual field, and then follow our noses, we do sense a series of visual fields, each containing an appearance of the object. These sensa, as we have already seen, do increase to a maximum of size and brightness as we approach the place which they optically occupy. We might, perhaps, take the size and shape of the largest and clearest sensum of such a series as what is meant by the optical size and shape from a given direction of a certain part of the perceptual object. But I do not think that it would be possible to generalise this definition, so as to give a meaning to *the* size and shape of a complete optical object.

(*c*) *Physical Shape.*—We have said that common-sense identifies the "real" shape of a perceptual object with its felt shape. This statement requires a good deal of analysis. The first thing to notice is that we are much more inclined to believe that we feel literal parts of the surfaces of physical objects than that we see them. Mirror-images, and the variations of visual shape and size with the position of the observer, make it fairly evident, even to common-sense, that visual sensa are not literally parts of the surfaces of perceptual objects, though, of course, common-sense does not understand what radical changes a consistent application of this conclusion involves. But we are convinced that what we touch is literally a part of the surface of a physical object. I believe that, with suitable explanations and qualifications, some such view can be held; but we must gradually work up to it, and make the necessary distinctions as we go along.

(1) There are tactual fields, just as there are visual fields. And within them there are sometimes outstanding tactual sensa, with recognisable sensible shape and position within the field. Tactual sensa stand out from the rest of the tactual field, if they be markedly different in temperature or in "feel" from the rest. These remarks would be illustrated by laying one's hand on a table with a small bit of ice lying on it or with a nail sticking up from it. In each case we should sense a tactual field with a certain outstanding tactual sensum at a certain sensible place within it. In the first case the sensum would stand out by its coldness from the background, and it would have a sensible shape. In the second a sensum would stand out from the background by its peculiar "prickly feel." But, in the ordinary man, the tactual field is much less clearly differentiated than the visual field, and sensible tactual position and shape are far vaguer than the sensible shapes and positions of visual sensa. Very possibly this is not true of blind men. The tactual field, such

as we have just been describing, is connected with what psychologists call "passive touch"; and it is generally admitted that passive touch by itself gives very vague information about shape and size.

(2) Just as visual sensa are literally present only in their own fields, so tactual sensa are literally present only in tactual fields. When we say that there is a cold round tactual sensum at a certain place in the movement-continuum, we are necessarily speaking in a Pickwickian sense, as much as when we say that there is an elliptical brown visual sensum there. This Pickwickian sense is fairly obvious. A certain tactual sensum may be said to occupy that place in the movement-continuum to which I have to move my hand before I can sense this sensum. The total region in which a certain perceptual object is present may, in this sense, be occupied in different parts by a great number of different tactual sensa from contemporary fields of different observers and from successive fields of a single observer. The whole of such a group of tactual sensa would be the *Tangible Constituent*, which, along with the complete optical object and perhaps other constituents, makes up the perceptual object.

(3) It would generally be admitted that it is by "active touch," *i.e.*, by passing our fingers over surfaces that we learn about the "real shapes" of objects like pennies. Now active touch is partly a movement-experience and partly a tactual experience. The purely tactual side of it is illustrated in isolation in passive touch, and we have seen how very little it has to tell the normal man about shape and size. But active touch is movement of very much the same kind as we experience when we walk about, accompanied by sensations of temperature, pressure, "sharpness," "bluntness," etc. We find that there are certain regions of the movement-continuum into which we cannot enter or push our hands. Our previously free course is stopped. This stoppage is accompanied and emphasised by

tactual sensations of various kinds. It is always accompanied by pressure-sensations, which grow in intensity the more we try to penetrate the region in question. When we actively feel a body we are trying to penetrate a certain region of the movement-continuum from various directions, and are failing to do so. And our failure is marked by characteristic tactual sensations. The points on its surface are the points at which attempted courses of further movement are stopped. Thus, it seems to me that what we feel when we are said to be actively exploring a certain perceptual object is a closed surface in the movement-continuum. The felt boundaries are the boundaries of a volume which is *in* the movement-continuum in the same literal sense in which a tactual sensum is in its tactual field or a visual sensum in its visual field. The optical constituent and the tangible constituent of the perceptual object are on the surface of this felt region in their respective Pickwickian ways, whilst relevant scientific objects are within this region in a perfectly literal sense.

There is one important point to remember here. The experience of being stopped when we try to penetrate a certain region of the movement-continuum from various directions is not one simultaneous experience, but is a series of *successive* attempts and failures, accompanied by characteristic tactual sensations. On the other hand, the region which we are said to feel is conceived as a network of *contemporary* points. If we had not got the concepts of shape and volume from our visual, and in a much smaller degree, our tactual fields, we should never have been able to interpret these successive stoppages as a network of contemporary points in a kind of space. This is simply a further illustration of the general fact, already noted, that apart from the characteristic peculiarities of visual fields and their correlations with our bodily movements we should never have interpreted the movement-continuum spatially at all.

(*d*) *Summary of Conclusions about Shape*.—Shape has a perfectly definite meaning only as applied to extensive wholes of co-existent parts. It is therefore impossible to deal with it adequately apart from time. Strictly speaking, only momentary extended *events* have shape, and we can only talk of the shape of a persistent *object* on the assumption that successive momentary sections of its history are extended events with the same shape. Leaving these temporal complications aside till the next chapter, we may say that we reach the concept of shape by acquaintance with particular instances of it in the form of visual and (to a much less degree) tactual sensa. Having reached the concept in this way, we can, as usual, proceed to apply it to other cases which we cannot sense.

The notion of the shape of a perceptual object has the same kind of confusion as the perceptual object itself. For the latter is a *compositum* of constituent objects of various types. Each of these constituent objects will have a shape only in a Pickwickian sense, if at all. And the Pickwickian sense will be different for each different type of constituent object. It proved to be unprofitable, and perhaps impossible, to define a meaning for the shape of the optical constituent or the tangible constituent. In fact, what is meant by the shape of a perceptual object seems not to be the shape of any of its constituent objects. It is rather the shape of a certain region of discontinuity within the movement-continuum. This is the region on whose surface the optical and tangible components of the perceptual object are present in the Pickwickian senses of "presence" appropriate to each. And within this volume are supposed to reside those scientific objects which are mainly relevant in determining the optical and tangible filling of the region.

The boundaries of such regions of the movement-continuum are learnt by active exploration. Attempts at further movement are here stopped, and the stoppage

is emphasised by the accompanying tactual sensations. The interpretation of these successive stoppages as a network of contemporary points within the movement-continuum involves the application of concepts derived mainly from the visual field, and the same is true of the spatial interpretation of the movement-continuum itself. The shapes of visual sensa are taken as indications of the shape of this region in the movement-continuum, but are admitted by common-sense to need correction, a correction which we apply automatically and properly in familiar cases.

This is as far as we can profitably go without considering the temporal characteristics of sensa, physical objects, and physical events. With these we shall deal in the next chapter.

The following additional works may be consulted with advantage:

G. F. STOUT, *Manual of Psychology*, Book III. Part II., Caps. III. and IV.

W. JAMES, *Principles of Psychology*, Chapter on Space.

BERKELEY, *Theory of Vision.*

CHAPTER X

"She is settling fast," said the First Lieutenant as he returned from shaving.

"*Fast*, Mr Spoker?" asked the Captain. "The expression is a strange one, for Time (if you will think of it) is only relative."

(R. L. STEVENSON, *The Sinking Ship*.)

The Dates and Durations of Sensa and of Physical Objects and Events

WE have now to raise the same kind of questions about date and duration as we have just been raising about place and shape. As in the last chapter we were learning something fresh, not only about Matter, but also about Space, so here we are going to dig beneath the traditional concepts of Time and Change which were treated in Chapter II. We shall also be correcting certain simplifying assumptions which were made in the last chapter, such, *e.g.*, as the assumption that our successive sensible fields are literally momentary.

Comparison of Spatial and Temporal Characteristics of Sensa.—Let us begin with the temporal characteristics which belong to sensa in the same direct and literal way in which sensible place in their own fields belongs to them. There are three ways in which temporal characteristics are more pervasive than spatial ones. (i) Only objects have places and shapes in a literal or even a Pickwickian sense. Mental acts, like believing, wishing, etc., neither have sensible places, such as sensa have in their own fields, nor are they commonly held to be in physical Space, even in a Pickwickian sense. This is denied by Alexander, but I am quite

unconvinced by his arguments. It is no doubt possible to give a Pickwickian meaning to the statement that our mental acts are in our heads, but we make so little scientific use of such statements that it is hardly worth troubling to do so. On the other hand, it seems to me that mental acts have dates in the same literal sense as sensa and other objects, which are not acts. When I say that I began to think of my dinner at the moment when I heard a noise, I am asserting that a certain act of thought and a certain sensation of sound were contemporary; and this is an expression of an immediate experience, and has nothing Pickwickian about it. (ii) The spatial characteristics of the sensa of one sense do not literally extend to those of another sense, even in the case of a single observer. My visual sensa have places in my visual field, and my tactual sensa have places in my tactual field; there is no place in which both are literally present. We do, indeed, come to say that certain visual sensa are compresent with certain tactual ones; but, as we have seen, this only means that both are present, in different Pickwickian senses, in a region of the movement-continuum. This is not the kind of fact that can be directly sensed. On the other hand, it does seem to me that temporal relations do literally connect sensa belonging to different senses of the same observer. I can often judge quite immediately that a certain noise that I sense is contemporary with a certain flash that I sense, and is later than a certain twinge of toothache which I remember. Here I seem to be using the names of these temporal relations quite literally, and in no Pickwickian sense. On the other hand, temporal relations do not literally stretch across from one observer to another. You and I may judge that two visual sensa, one of which was sensed by you and the other by me, were contemporary; and you may judge that your visual sensum was contemporary with a twinge of toothache that you felt. But my flash and yours are not contemporary, in the same literal sense

in which your flash and your toothache are contemporary. Temporal relations between the sensa or the mental acts of two different observers have to be defined in terms of a good many other facts beside the two which they are said to relate, just as we found with spatial relations between the sensa of different observers. (iii) Spatial relations do not literally extend from the sensa of one field of a certain observer to the sensa of a later field of the same sense of the same observer. It is only in a Pickwickian sense that we can say that a certain visual sensum of mine is compresent with another visual sensum of mine, which belongs to a later field. On the other hand, direct memory seems often to bridge the gap between two of our sensa of different dates, and to enable us to judge directly that one is literally later than the other.

Sensible Duration: (*a*) *Sensa and Sense-objects.*—We assumed temporarily, and for the sake of simplicity, in the last chapter that our successive sensible fields are literally momentary, and that a sensum in one field is *ipso facto* different from any sensum in another field. We must now get behind these simplifying assumptions. The second of them is partly a matter of definition. It is obvious that what is now past cannot be precisely and numerically the same as what is now present, even though the sensible qualities and shapes of both should be exactly the same, and though they should occupy precisely similar sensible places in their respective sensible fields. I am therefore justified in using the term "sensum" in such a way that they shall be called different sensa. This is, of course, without prejudice to the fact that the resemblances and the continuity between the members of a series of different *sensa* in successive fields may be such that it is possible and useful to speak of a single persistent *sense-object*, of whose history the sensa of the series are different and successive slices. When there is a series of sensa

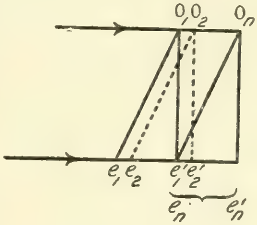
$s_1 \ldots\ldots s_n$ in a set of successive fields of an observer O, and when there is enough qualitative likeness between adjacent sensa of the series, we can say that a sense-object S exists and persists, and that these sensa are successive parts of its history. If all the sensa of the series be indistinguishable in their qualities, we can say that the sense-object S has persisted unchanged throughout a certain duration. If the successive sensa have different places in their respective fields, and if certain further conditions be fulfilled, we can say that the sense-object S has moved. The sort of continuity that is required of the sensa $s_1 \ldots\ldots s_n$ in order that they shall all count as parts of the history of a single sense-object S, is that the nearer together two sensa are in the series the more alike are their sensible places in their respective fields. If this condition be fulfilled, we say that there is a single sense-object, and if the successive sensible places are different, we say that it has moved. We can, of course, remember the place of a sensum s_r in its field f_r, and compare it with that of s_{r+1} in its field f_{r+1}. This is not generally an act of deliberate memory and comparison, but we automatically notice if s_{r+1}'s position in f_{r+1} is greatly different from s_r's position in f_r. If the fields which come after a certain field f_n do not contain sensa with the right sort of resemblance and continuity with the previous s's, we say that the sense-object S has ceased to exist. As we have already explained, nothing that has ever existed really ceases to exist. The parts of its history that have become, merely recede into the more and more distant past; and nothing that henceforward becomes, is of such a nature that it adds on to these past events to make a continuation of *that particular* sense-object. It were therefore less misleading to say that the sense-object in question ceases to *per*sist. The past, like the unhappy Theseus, "*Sedet, æternumque sedebit.*"

(*b*) *Duration of Sense-fields and of Sensa.*—On the assumption that sensible fields are literally momentary,

it follows that sensa are also literally momentary. But this assumption must now be dropped, and we must come closer to the actual facts of sensible experience. A sensible event has a finite duration, which may roughly be defined as the time during which it is sensed, as distinct from being remembered. The two kinds of act are markedly different when a long gap of time separates the act of remembering from the object remembered. As the time-lapse between act and object decreases, the distinction between sensing and remembering grows fainter, and no absolutely sharp line can be drawn where one ends and the other begins. Still, it is certain that what can be sensed at any moment stretches a little way back behind that moment. This is the phenomenon to which we have already referred as the Specious Present. I do not find the accounts of the Specious Present given by psychologists very clear, and I shall therefore try to illustrate the matter in my own way, which will lead us to definitions of momentary fields and momentary acts of sensing. It is obvious that, if we are to hold that all object-events are really of finite duration, and that momentary *objects* are to be defined by Extensive Abstraction, we ought to take up the same attitude towards *acts*. I shall begin by assuming literally momentary acts of sensing, and shall then correct this abstraction.

Let us represent the history of O's acts by a directed line OO. Let us represent the history of his sensible fields by a parallel line *ee*. Let O_1, on the upper line, represent a momentary act of sensing done by O at a moment t'_1. I take it to be a fact that this act grasps an event of finite duration which stretches back from the moment t'_1 to a moment t_1, which is earlier by an amount τ. This duration τ is the length of O's Specious Present. I call this event $e_1e'_1$, and I represent the act of sensing which grasps it as a whole by the right-angled triangle $e_1O_1e'_1$, with $e_1e'_1$ as base and O_1 as vertex.

Let us now suppose that, at a slightly later date (separated by less than the length of the Specious Present), O performs another act of sensing. We will represent this by the dotted triangle $e_2O_2e'_2$, which is similar to $e_1O_1e'_1$. This grasps an event of duration τ, stretching back from the moment when the act happens. The event is represented by $e_2e'_2$. Now it is evident that there is a part $e_2e'_1$, which is common to the two events $e_1e'_1$ and $e_2e'_2$. This part is sensed by both the acts O_1 and O_2. On the other hand, there is a part e_1e_2 of the first event which is not sensed by the second act, and a part $e'_1e'_2$ of the second event which is not sensed by the first act. It will be noticed that the duration of $e_2e'_1$, the event which is sensed by both O_1 and O_2, is such that, when added to the time that elapses between the two acts, it makes up the duration of O's Specious Present. If we finally take an act O_n, separated from O_1 by the length of the Specious Present, the event $e_ne'_n$ which it grasps has nothing in common with $e_1e'_1$, except the single point which is labelled both e'_1 and e_n. Thus, if two acts of sensing by O be separated by the length of O's Specious Present, the only "event" that is sensed in both of them is a "momentary event." In general, we notice that the shorter the time-lapse between two of O's acts of sensing, the longer is the event which is sensed in both of them; and that, as the lapse tends to nothing, the duration of the event tends to τ.



(*c*) *Momentary Fields and Momentary Acts of Sensing.*—We are now able to remove the supposition of literally momentary acts, and to define by Extensive Abstraction both momentary acts and momentary fields. If the reader will look back at the diagram he will see that the event $e_2e'_1$, which is common to the two acts of sensing O_1 and O_2, is *a fortiori* common to O_1 and any

act that happens between O_1 and O_2. For it will be a proper part of the longer event which is common to this pair of more closely adjacent acts. If we imagine a continuous series of momentary acts between O_1 and O_2 we can regard them as momentary sections of an act or process of finite duration, and can say that the finite event $e_2e'_1$ is present throughout the whole of this process of sensing. The parts e_1e_2 and $e'_1e'_2$ form a kind of penumbra; the latter was not present at the beginning, and the former is not present at the end, of this finite process of sensing; but the part $e_2e'_1$ is present all through. A momentary sensible field may thus be roughly defined as the limit which the event that is present throughout the whole of a process of sensing approaches, as the duration of the process of sensing approaches to the length of the observer's Specious Present. The reference to limits can then be got rid of in the usual way by Extensive Abstraction. The momentary field e'_1 might finally be defined as follows: It is a class of events such that each member of it is present throughout the whole of *some* process of sensing which begins at t'_1 and does not last longer than O's Specious Present.

In the same kind of way we can define a momentary act of sensing. The longer an event the shorter is the process of sensing throughout the whole of which it is present. As the length of the sensed event approaches that of the Specious Present, the duration of the process of sensing throughout the whole of which the event is present approaches to nothing. We could, therefore, roughly define a momentary act of sensing as the limit which a process of sensing approaches as the duration of the event which is present throughout the whole of this process approaches to that of the observer's Specious Present. The reference to limits can then be got rid of in the usual way. The momentary act O_1 might ultimately be defined as follows: It is a class of acts such that throughout each member of it there is present *some*

event which ends at t'_1 and does not last longer than the duration of O's Specious Present.

In real life we may assume that our acts of sensing are not momentary, but are processes that last for a finite time. What we choose to count as one process of sensing, of course, depends on many factors, of which the most important is probably unity of interest. If our account of the Specious Present be right, the fundamental fact is that a process of sensing which lasts for a finite time (provided it be shorter than the duration of the Specious Present) will actually *sense* a certain event of finite duration throughout the whole time that the process lasts. Since, however, we have succeeded in defining momentary acts and momentary sensible fields in terms of processes of sensing and sensible fields of finite duration, we are henceforth at liberty to use the momentary conceptions whenever we find it convenient to do so.

(*d*) *Sensible Change.*—We are now in a position to deal with sensible change and movement. We have already defined what is meant by the statement that a sense-object *has* changed or moved. We saw that it depended on a comparison between the sensible positions and other qualities of sensa in successive fields. But it is a notorious fact that we do not merely notice that something *has* moved or otherwise changed; we also often see something *moving* or *changing*. This happens if we look at the second-hand of a watch or look at a flickering flame. These are experiences of a quite unique kind; we could no more describe what we sense in them to a man who had never had such experiences than we could describe a red colour to a man born blind. It is also clear that to see a second-hand *moving* is a quite different thing from "seeing" that an hour-hand *has* moved. In the one case we are concerned with something that happens within a single sensible field; in the other we are concerned with a comparison between the contents of two different sensible fields. Now we have just seen

that, in the total event which is sensed by a process that lasts for less time than the duration of the Specious Present, there is a finite part which is sensibly present throughout the whole process of sensing. Even if a certain process of sensing goes on for longer than a Specious Present, there will be parts of it that are shorter than the duration of a Specious Present, and *some* event of finite duration will be sensed throughout any one of these shorter parts of the total process. Let us consider any such finite event, which is sensed throughout the whole of a finite process of sensing. It will constitute a sensible field, and it lasts for a finite time. It can therefore be divided into successive fields of shorter duration, which together make it up. If anything in one of its earlier sections be qualitatively different from anything in any of its later sections there will be change *within* the original finite field. But the whole of this field is sensed throughout a finite process of sensing. Thus the qualitative differences between its earlier and its later sections will be sensed *together*; *i.e.* the observer will actually sense the changing and will not merely notice that something *has* changed. We can now easily see why a change must surpass a certain minimum speed if it is to be sensed as such. If a change takes place slowly, this means that closely adjacent events are qualitatively very little different from each other. It may therefore happen that two events are not qualitatively distinguishable by us unless they are separated by more than the duration of a Specious Present. If this be so, these two qualitatively distinguishable sections of a single long event are too far separated to be sensed together even by a momentary act. *A fortiori* they could not be sensed throughout the whole of any process of sensing which lasts for a finite time, as all real acts of sensing do. Thus we may be able in such a case to judge by memory and comparison that something *has changed*, but we shall not be able to sense its *changing*.

The fact that, in favourable cases, changes can actually be sensed, is of great importance in developing the concept of change in general. A sufficiently short act of sensing senses a field of finite duration. This field is divisible into earlier and later parts, which together make it up. Now, since I sense this finite field as a whole, I actually sense the way in which its earlier half joins up with its later half to make up the whole. By analogy with this, I am able to conceive how two successive adjacent fields, which no act, however short, can sense together, are joined up with each other in nature to form a single long event. I thus interpret those qualitative differences, which I can notice only between successively sensed fields, in terms of the changes which I can actually sense within a field that is short enough to be sensed as a whole by an act of finite duration. If there were no sensible change, it would still be true that a sufficiently short act of sensing senses a field of finite duration ; but it would be extremely difficult for us to recognise that this was divisible into successive shorter sections which join up with each other to make the finite field. For there would be no recognisable qualitative difference between the earlier and the later sections. In this case, it would be extremely difficult for us to conceive the way in which a finite field, which is now sensed, joins on to an earlier finite field, which is now only remembered. It would be proportionately difficult for us to interpret any qualitative differences that we might find between two such fields in terms of slow continuous change.

(*e*) *Conclusions about Sensible Duration.*—We have now, I think, got all the facts that are needed to deal with the concept of the duration of sensa. A sensible field is the total event that is sensed throughout the whole of any process of sensing. No process which lasts for longer than the duration of a Specious Present senses a single sensible field, and no sensible field can last longer than the duration of a Specious Present. But,

on the other hand, every process of sensing that lasts for a shorter time than a Specious Present senses throughout the whole of it a sensible field of finite duration. Since we can always divide up a process of sensing into successive bits, each of which is shorter than a Specious Present, we can always divide up the total event that an observer has sensed in the course of a long process of sensing into successive sensible fields, each of a finite duration less than that of the Specious Present. There is thus a maximum possible duration for a sensible field, but any sensible field is divisible into shorter fields which join together at their ends to make up the whole. This divisibility is made obvious to us by the fact of sensible change, and the mode of junction of successive adjacent fields is conceived to be analogous to that which is actually sensed in the case of the earlier and the later half of a single sensible field.

Now we have already seen that even a momentary sensible field (especially, for example, a visual one) is spatially extended. We have now seen that any real sensible field has a certain duration, which cannot exceed that of the observer's Specious Present. It is thus also temporally "extended." It may thus be regarded as a four-dimensional spatio-temporal whole. I define a sensum as a part of a sensible field. Now, if we consider an ordinary three-dimensional volume, like a cube, and neglect the question of duration altogether, we see that anything that is literally a part of it must be a three-dimensional volume too. For it is only such things that could literally fit together to make up the cube. Plane sections of the cube are not parts of it in this literal sense, though it is perfectly easy to define by Extensive Abstraction Pickwickian senses in which planes, lines, and points can be truly and usefully said to be "parts" of volumes. In the same way, it is clear that the only sort of thing that can literally be a part of a spatio-temporal whole, like a sensible field, must be something that is extended in

time as well as in space. Any actual sensum is therefore extended both spatially and temporally. Granted that no sensum is to be held to last longer than the sensible field of which it is a part, we have still to ask what is meant by the statement that one sensum persists through the whole of a certain sensible field and that another sensum does not. The following cases can arise: (i) A certain place in a sensible field may be occupied by a sense-quality (*e.g.*, a colour of a certain definite shade, brightness, and saturation) throughout the whole duration of the sensible field. We should then say that a sensum of this colour has persisted and rested in one sensible place throughout the whole duration of the field. Of such a sensum we can only say that it cannot last longer than the sensible field of which it is a part (and therefore not longer than the duration of a Specious Present), though, of course, it may be continued by qualitatively indistinguishable sensa, occupying similar sensible places in successive sensible fields. (ii) A certain place might be sensibly occupied by a continuously changing sense-quality throughout the whole duration of the sensible field. This means roughly that, if we divide up the history of this place throughout the duration of the field into successive thinner sections, any two sections will be occupied by a different sense-quality, but the thinner we make the sections the more nearly alike will be the sense-qualities that occupy this place throughout adjacent sections. In this case we should actually "sense the change of quality." The sensible identity of place, and the continuity of the sense-quality, would generally be regarded as sufficient to justify us in saying that a single sensum has persisted throughout the sensible field and has rested in one sensible place, but that it sensibly and continuously changes in quality. (iii) It might be possible to divide the history of a certain sensible place in a sensible field into three successive sections, of which the first is occupied by a quality q_1,

the second by a markedly different quality q_2, and the third by a markedly different quality q_3. We should then say that there were three successive sensa, each of which persisted for so long, and then was succeeded by another. If the middle one of these sections should be excessively short, we could say that we had sensed a "*sense-flash* of quality q_2 at this sensible place." (iv) It might happen that, as we divide up the sensible field into successive thinner sections, we find that in each section there is a sensible place occupied by the same sense-quality. Moreover, the shapes of these sensible places might be indistinguishable. But the sensible places occupied by this quality in successive sections of the sensible field might differ. And it might be found that the thinner we made the sections the more nearly alike were the sensible places occupied by this quality in adjacent sections. On the grounds of this continuity of place and identity of shape and sensible quality, we should be justified in saying that we were dealing with a single sensum, which persists throughout the whole of the sensible field. But we should actually sense its movement; and should therefore say that a moving sensum of such and such shape and sensible quality persisted throughout the whole of this sensible field. In real life it is unlikely that the shapes of the successive places would be exactly alike, or that precisely the same sense-quality would occupy each of them. But, provided that the change of shape and of sense-quality was continuous in the sense defined above, we should still say that we were dealing with a single sensum; but should add that it changes sensibly in shape and quality as it sensibly moves. Of course a moving or qualitatively changing sensum need not persist throughout the whole of a sensible field, any more than a resting or qualitatively fixed one need do so. The change may begin after the beginning and end before the end of the sensible field in question.

I think that we have now said all that is necessary

about the duration of sensa. As in all questions of duration, the answer depends in part on mere matters of definition. When we ask how long so and so lasts, we have first to lay down our criterion of identity for so and so. If anything lasts at all, the successive parts of its history are necessarily *numerically* different, or they could not be successive. Our criterion of identity *must*, therefore, depend on identity of quality, in a wide sense of that word, which includes shape and place. Thus the question is: "How much qualitative difference can we allow between successive slices of a long event before it ceases to be appropriate to call the whole event the history of *so and so?*" Obviously, this is a question which admits of various answers; but no one holds that complete qualitative identity of successive events is necessary if they are all to be regarded as parts of the history of one persistent object. I have defined the word *sensum* in such a way that nothing which cannot be sensed throughout the whole of *some* process of sensing is to be called one sensum, no matter how great the qualitative resemblance and the continuity between successive slices of this long event may be. Such a long event may count as the history of a single *sense-object;* because the kind of identity needed for the persistence of a sense-object, as defined by me, is different from that required for the persistence of a sensum. Within these limits, however, I have not considered that complete identity of place, shape, or sense-quality is essential to the identity of a sensum. I therefore recognise the existence of sensibly moving and sensibly changing sensa. Since the experiences of sensible change and movement are peculiar and important, and since they occur within fields that are sensed as wholes by processes of sensing of finite duration, this seems to be the most reasonable course to take. Anyone who disapproves of it has merely to make appropriate modifications in his definition of the word *sensum;* he will still have to recognise and deal as

best he can with all the facts which we have been passing under review.

Dating of Sensa.—We can now turn to the subject of date. The notion of date only becomes perfectly definite when we deal with momentary events; and no actual events are momentary. It therefore has to be defined by Extensive Abstraction. We will first consider the dating of sensa which are sensed by a single observer, and we will then pass to the concept of temporal relations between sensa of different observers. When a meaning can be assigned to the statement that a sensum s_1, which is sensed by O_1, is contemporary with s_2, which is sensed by O_2, and later than s_3, which was sensed by O_3, it will be possible to see what is meant by the notion of a date which is neutral as between various observers. But I must just say a word about the dates of *acts of sensing*.

(*a*) *Temporal Relation between Act of Sensing and Sensum.*—If the reader will refer back to the diagram, by which we illustrated the facts of the Specious Present, he will see that we there tacitly assumed that a momentary act of sensing would be contemporary with the end-point of the finite event which it senses. This is implied by making lines, like $O_1e'_1$ in the diagram, normal to the line of objects sensed. I suppose that it is possible that an act of sensing might be later by a finite amount than the whole of the event that it senses. It could not, of course, on our view of the future, be *earlier* than any part of what it senses. For, when the act is present, there is nothing later than it; and to sense what has not yet become, would be literally to sense *nothing*. Our assumption seems to be the most reasonable one to make. On the one hand, there is, so far as I know, nothing conclusive against it. On the other hand, the distinguishing mark of an act of *memory* is that it is separated by a finite time-lapse from the latest part of the event which it remembers. Hence,

any other assumption than that which we made, would render it difficult to distinguish, even in theory, between an act of sensing and an act of remembering. The *practical* difficulty which there sometimes is in drawing this distinction can easily be accounted for on our view. We can well suppose that, as the gap between an act of remembering and the end of the event remembered gets shorter and shorter, it will be more and more difficult to distinguish the act of remembering from an act of sensing, in which, if we are right, the gap vanishes altogether. I shall therefore take it that the assumption tacitly made in the diagram is justified. In general, then, we may say that the beginning of a process of sensing, throughout the whole of which an event of finite duration is sensed, is contemporary with the end of the event in question. Thus, in the diagram, O_1, the beginning of the act O_1O_2, is contemporary with e'_1, the end of the event $e_2e'_1$, which is sensed throughout the whole of this process. This will suffice as to the connexion between the dates of an act of sensing and of an event sensed by it ; a question to which nothing comparable arises when we deal with Space, since mental acts do not have places, as they have dates.

(*b*) *Temporal Relations within a Sense-field.*—Having cleared this point out of the way, let us consider the dating of sensa that are sensed during the life-history of a single observer. This inquiry falls into two parts. We have first to consider the dating of sensa that fall within a single sensible field of the observer, and then to consider the extension of this to sensa that do not fall into the same sensible field but into successive ones. I must first clear up a slight ambiguity in the term *sensible field*. In the last chapter we counted the fields of two different senses, *e.g.*, an auditory and a visual field of the same observer, as different sensible fields which do not form parts of a single larger whole. This is true as regards spatial characteristics, which we

were then considering; since sensible spatial relations do not connect the sensa of one sense with those of another. But, as regards temporal characteristics, the distinction between the sensible fields of different senses ceases to be of importance. A noise that I sense auditorily may be sensibly and literally contemporary with a flash of colour that I sense visually. We can therefore say that the *special* sensible fields of the various senses form parts of a single *general* sensible field, so far as temporal characteristics are concerned. When I speak of a sensible field in the sequel, I shall mean a general sensible field, unless the context makes it plain that I am referring to some special one, such as that of sight or that of hearing.

Let us then take a certain sensible field of a certain observer. As we have explained, this is of finite duration and its parts of finite duration are sensa. Some of these endure throughout the whole of it, others do not. Of two sensa, neither of which endures throughout the whole of this field, one may be completely separated from the other, *i.e.*, one may cease and some third sensum may intervene before the other begins. On the other hand, the end of one may exactly coincide with the beginning of the other. Or, finally, the two may partially or totally overlap. These various temporal relations between sensa of finite duration that fall into the same sensible field can be and are directly sensed, just as the spatial relations between two coloured patches in the same visual field can be. Two sensa would be said to be *sensibly simultaneous* if each completely overlaps the other. If one sensum only partially overlaps another, there is a shorter part of one which completely overlaps and is completely overlapped by a certain shorter part of the other. Thus these two parts will be sensibly simultaneous, though the wholes are not. It will be seen that sensa which are sensibly simultaneous both persist through the same slice of the sensible field. As this slice is made thinner and thinner, the sensa that

persist through it are made shorter and shorter. Proceeding to the limit, we get the notion of exact simultaneity between momentary events. The reference to limits can then be removed by Extensive Abstraction. The details of the process will be found in Whitehead.

(*b*) *Temporal Relations within a Sense-history.*—We can see roughly how, in this way, the sensa that fall within a single sensible field can be arranged in a temporal order and dated. We have now merely to extend this to successive fields of the same observer. Any sensum in a later field is later than any sensum in an earlier field. A field is later than another if it was sensed when the other could only be remembered. (This is not the *meaning* of being later, as we have seen, but it is a criterion of it that we can and do use in practice.) Now we have seen that earlier and later sections of any one sensible field can be distinguished and dated. Successive fields of the same observer are conceived as joining on to each other in the same way in which successive sections of the same field are actually sensed to join up with each other and to constitute that field. Thus we conceive of the total event, that is gradually and piecemeal sensed by an observer in the course of his life, as being completely analogous in its temporal characteristics to those short sections of it which can be sensed as wholes throughout the whole of a single process of sensing.

The particular duration of an observer's Specious Present may fairly be regarded as a peculiarity of himself or of his species. It is known that this duration is much the same for all men under normal conditions. It is known that it is short as compared with the duration of most events that are practically interesting to us, but long as compared with that of many events—such as a single vibration of an electron—which are of great scientific importance. (These statements can, of course, only receive a perfectly definite meaning at a later stage, when the temporal characteristics of physical objects and

events have been discussed.) In the meanwhile it is a fact that we can easily conceive of Specious Presents which are longer than our own. In particular, we can imagine ourselves replaced by an observer who differs in no respect from us except that his Specious Present covers the whole of his history. Such a man would still distinguish the present from the past and the future, and the less from the more remote past. But, whilst the distinction between present or past and future would be as important for him as for us, since it is the distinction between something and nothing, the difference between present and past would be much less important for him than for us. With us the sinking of an event into the past is accompanied by a change in our mode of cognising it. We have to cognise it by memory or inference, if at all ; and the further it sinks into the past the vaguer is our knowledge of it likely to become. But the hypothetical observer would sense the whole of his past history at every moment, and therefore would have the same full knowledge of its earliest parts as of those that have only just become. This conception of an observer with an indefinitely long Specious Present is useful, because we *conceive* the whole content of our history to be such as this observer would *sense* it to be.

(*c*) *Neutral Temporal Relations*.—We have now to deal with the temporal relations between sensa of different observers. Let us call the whole series of sensible fields which an observer O senses in the course of his life, O's *sense-history*. We have seen that, within any sense-history, momentary sections can be defined and dated by Extensive Abstraction. We have now to take into account the existence of a number of observers, each with his own sense-history. Our task is to treat the temporal relations between a certain event in one sense-history and a certain event in another. Let us start with the fundamental relation of simultaneity. This is illustrated in its most literal sense by sensa in the same field ; the question is, how far it can

be extended to a pair of sensa, one from the field of one observer and the other from the field of another observer.

We will begin by pointing out a complication which did not arise over spatial relations. When we discussed in the last chapter the meaning of the statement that visual sensa from several different fields are "in the same place," it was clear that we were giving a *definition* and not a mere *test*. This is perfectly evident from the following consideration: Two different visual appearances of a penny are at once *sensibly* present in different places and *optically* present in the same place. This would be a sheer contradiction if optical and sensible presence had the same meaning. Thus, when we say that, under such and such conditions, two visual sensa are optically compresent, the conditions are part of the definition of what is meant by "optical compresence." It is impossible to hold that optical presence really means the same thing as sensible presence, and that the conditions mentioned are simply *tests*, by which we can establish that this relation holds in cases where the evidence of direct sense-awareness fails us.

Now, when we deal with temporal relations, and try to state the conditions under which two sensa in different sense-histories are said to be contemporary, it is by no means obvious whether we are *defining a new sense of simultaneity*, or merely giving a *test* by which the fact of simultaneity, *in the old sense of the word*, can be established in cases where it cannot be directly sensed. I think that failure to distinguish clearly these two possibilities has caused much confusion in the writers and readers of books on the Theory of Relativity. It is very much more plausible to hold that "simultaneity" always means the same in all its applications, than to hold that "compresence" means the same always and everywhere. For it is admitted that sensa belonging to different senses of the same observer can be con-

temporary with each other, in precisely the same way in which two visual or two tactual sensa of the same observer can be contemporary. It is therefore not glaringly absurd to suggest that sensa belonging to different sense-histories may be contemporary in the same way in which sensa in the same sense-history can be so. In that case the conditions under which two sensa belonging to different sense-histories are said to be simultaneous do not define a new meaning of "simultaneity," but merely give a test for simultaneity, in the old meaning of the word, which we use in those unfavourable cases where the relation cannot be directly sensed.

The only way of deciding between the two alternatives would be the following: The relation of sensible simultaneity has certain logical characteristics. For instance, it is transitive, *i.e.*, if A has it to B, and B has it to C, then A necessarily has it to C. If we found that "simultaneity," as tested by the conditions commonly laid down, did not have all these logical characteristics, we could conclude that we were dealing with a new meaning of "simultaneity." This would not, of course, preclude the possibility that sensa from different sense-histories have also in fact the relation of simultaneity, in the original sense. But it would show that the conditions laid down were not a test for that relation. And it might turn out that no conditions that we could think of would be a test for that relation between sensa belonging to different histories. In that case, it would be a mere personal idiosyncrasy to hold that simultaneity, in the original sense, ever holds between sensa in different histories; and it would be better to regard the conditions laid down as defining a new sense of "simultaneity." For the present we must confine ourselves to the question of fact: "Under what conditions do people hold that sensa from different sense-histories are contemporary?" We may later on raise the question whether these conditions are simply a test for simultaneity, in the original

sense of the word, or whether they define a new meaning of "simultaneity." I will use the vague word *determine*, to cover both "being a test for" and "being a condition of" so and so.

Under what conditions do two observers in fact judge that they sense two contemporary sensa? Often two men assert that they both "see *the same* flash" or "hear *the same* noise." If this means literally that the two men sense precisely and numerically the same visual or auditory sensum, and if their statement be true when so interpreted, it is easy to lay down the conditions under which sensa from their respective sense-histories would be said to be simultaneous. If A's twinge of toothache be sensibly contemporary with this common sensum, and B's twinge of stomach-ache be also sensibly contemporary with it, we might say that A's toothache and B's stomach-ache are *neutrally* contemporary with each other.

Now there is no doubt at all that it is under conditions of this kind that sensa belonging to different sense-histories are said to be "simultaneous." But it will take us some time to find the exact meaning of these conditions, and to make sure what are the properties of "simultaneity" thus established. Evidently the first question that arises is: What is meant by the common statement that two observers "hear the same noise" or "see the same flash"? Do they mean that they sense a single sensum which is common to the sense-histories of both of them? And, whether they mean it or not, is it ever true? As ordinary people do not explicitly draw a distinction between sensa and physical objects, it is difficult to say whether they mean that they sense a common visual sensum when they assert that they see the same flash. But, as it is quite certain that by words like "seeing" and "hearing," people commonly mean to refer to acts of perceiving and not to acts of sensing, it is probable that by "the same flash" or "the same noise" they intend to refer to a common *physical event*

and not necessarily to a common *sensum*. In that case no such simple interpretation of the statement that A's toothache and B's stomach-ache are contemporary, as was offered above, can be accepted. For we should need to know how to determine whether two sensa are contemporary with the same *physical event* before we could determine whether they are contemporary with each other. Now, at present, all that we know is what is meant by one sensum of an observer being simultaneous with another *sensum* of *that* observer. Hence to determine neutral simultaneity between two sensa in terms of the simultaneity of each with a common physical event tells us nothing, since it involves simultaneity in a sense which has not yet been determined.

Let us then ask ourselves what is the exact cash value of the statement that A and B hear the same noise. I would like to point out at the beginning that nothing that has been said so far about sensa and sensible fields precludes the *possibility* that one and the same sensum should be in several sensible fields of different observers. A sensum is defined as a part of some sensible field; this clearly leaves open the possibility that two or more sensible fields, sensed by different observers, might have a part in common. If so, there are sensa common to several fields of several different observers. Whether this is an actual fact remains to be seen.

It is fairly easy to show, subject to certain subtle qualifications, that when a number of observers say that they hear the same noise and that they see the same flash, this cannot mean *both* that they all sense the same auditory sensum *and* that they all sense the same visual sensum. For, as we shall see in a moment, it is very difficult to reconcile this view with all the facts. Let us suppose that I fire a pistol, and that there is a number of other observers dotted about at different places. All the observers, including myself, will sense a short auditory sensum and a short visual sensum. These will be sensibly contemporary for me; for an observer

at some distance from me they will only partially overlap, the visual sensum beginning before the auditory one does so. For an observer still further off, the visual sensum will totally precede the auditory one, though both may be in the same sensible field. Finally, for a very distant observer the visual sensum may fall into a different (and earlier) field from that into which the auditory sensum falls. Nevertheless, all the observers, on comparing notes, will say that they heard the same noise and saw the same flash. Now, if this literally means that there is one single visual sensum which they all sense, and one single auditory sensum which they all sense, we shall have to hold that the same pair of sensa can be both sensibly simultaneous, partially overlapping, and completely separated in time. Now these relations seem to be incompatible with each other, and therefore we seem forced to conclude that, when several observers say that they see the same flash and hear the same noise, this cannot mean *both* that they all sense one and the same visual sensum, and that they all sense one and the same auditory sensum. Theoretically, it would be possible to interpret *one* of these statements (*e.g.*, that they all saw the same flash) in this literal way, provided we did not interpret the other (viz., that they all heard the same noise) literally. But, even apart from the additional facts which have led physicists to ascribe a finite velocity to light as well as to sound, such a course would hardly be reasonable. If *at least one* of the statements, that we all hear the same noise and that we all see the same flash, *must* be interpreted in some Pickwickian manner, it is hardly reasonable to suppose that the other can be interpreted literally.

Is there any way out of the conclusion that to hear the same sound and to see the same flash cannot mean that a number of observers literally sense a single visual and a single auditory sensum? So far as I can see, there are at least two alternative ways in which this conclusion could be avoided. One would be to hold that sensa can

be sensed at various times after they have ceased to persist, and that the further a man is from a source of sound, the greater is the gap between his act of sensing and the end of the auditory sensum which it senses. I do not think that this is a satisfactory alternative, for reasons which I have given earlier in this chapter, when I tried to justify the view that the beginning of a process of sensing, throughout which a finite event is sensed, is contemporary with the end of that event.

The second alternative is a much more important one. It is to adopt the usual expedient, which has already been mentioned as useful when two entities seem to have incompatible relations to each other. This expedient is to assume that what has been taken to be a dyadic relation between these two entities is really irreducibly polyadic, and involves some other term or terms beside the two entities in question. It is undoubtedly true that the same pair of sensa cannot be simultaneous, and partially overlapping, and wholly separated, *with respect to the sense-history of a single observer*. But suppose that this pair of sensa belongs to the sense-histories of several observers, and that the temporal relations in question are really irreducibly triadic. Suppose that the minimum intelligible statement that can be made about the temporal relations of two sensa in a sense-history is of the form "s_1 is contemporary with s_2 (or partially overlaps it, or wholly succeeds it, as the case may be) with respect to the sense-history h." In that case there need be no inconsistency in the same pair of sensa being contemporary with respect to one sense-history, partially overlapping with respect to another, and completely separated with respect to a third sense-history. We see then that our argument from the facts of sound does not conclusively prove that, when a number of observers say that they all hear the same sound and see the same flash, they cannot all be sensing precisely the same auditory sensum and precisely the same visual sensum. It does,

however, tie us down to one of two alternatives. Either this conclusion must be accepted, or we must give up the common-sense notion that the temporal relations between the sensa in the same sense-history are dyadic, and must substitute for it the view that they are at least triadic, and that the third term which is always involved is some sense-history in which both the sensa are contained.

Is there any way of deciding between these two alternatives? I think that we can at least show that the second alternative could not stand by itself, but would need to form part of a general Multiple Relation theory of sensible appearances. The various observers in my example do not really all sense auditory sensa which are exactly alike in quality. Both the auditory and the visual sensa which are sensed by very distant observers are much fainter than those which are sensed by me and by observers near me. Now, on the sensum theory, sensa have all the qualities that they appear to have. What really differs in quality cannot be numerically identical; hence a faint sensum cannot be the same sensum as a loud one, however much alike they may be in other respects. This argument would not be conclusive on a Multiple Relation theory of sensible appearance; because, on such a theory, sensa need not have the qualities that they seem to have. But I am deliberately ignoring Multiple Relation theories of sensible appearance in this book, in order to test Sensum theories, as Cardinal Newman tested the Thirty-nine Articles to see how much Catholic Truth they could be made to contain. I am as indifferent as he was to the possibility of the subject of my experiment blowing up at the end of the process; for negative results are often as valuable as positive ones. Accordingly, I think I may conclude that, on the Sensum theory of sensible appearance, it cannot be true that when a number of observers say that they see the same flash or hear the same noise they literally sense a single visual or auditory sensum common to all of them.

On either alternative the determination of neutral simultaneity between A's toothache and B's stomach-ache is going to be a much harder job than it would be if the facts about sound (and as we shall see later, about light) were different. If what we call the same noise be really a *group* of auditory sensa, the simultaneity of A's toothache and of B's stomach-ache with this noise only means that the former is sensibly contemporary with a certain auditory sensum sensed by A, and that the latter is sensibly contemporary with a different auditory sensum sensed by B. It is true that these two auditory sensa are both members of a group of sensa which are so connected with each other that the whole is called one noise. But it is by no means obvious that this rather indirect relation between A's toothache and B's stomach-ache will have the kind of properties that we demand of simultaneity. The same difficulty arises if we suppose that there is literally only one auditory sensum, which is sensed by both A and B, and that the relation of sensible simultaneity is triadic. The fact that A's toothache is contemporary with a certain auditory sensum with respect to A's sense-history, and that B's stomach-ache is contemporary with the same auditory sensum with respect to B's sense-history, does indeed constitute a relation between the toothache and the stomach-ache. But there seems no particular reason to expect that this relation will have the kind of properties that we demand of simultaneity.

Let us begin by imagining a set of observers who tried to determine neutral simultaneity entirely by sound. We need not suppose them to be blind, but we will suppose that they have no means of producing flashes of light either by igniting combustible things or by opening and shutting opaque shutters. A number of them hear what they call the same noise. They all sense short, outstanding auditory sensa. These are very similar in quality and are connected with a common centre in the way described in the last

chapter. They agree that any pair of sensa belonging to the sense-histories of different observers shall count as neutrally simultaneous provided that one is sensibly contemporary with one member of such a group of auditory sensa and that the other is sensibly contemporary with one member of the same group of auditory sensa. What properties will neutral simultaneity, so determined, possess?

In the first place, it will be necessary slightly to extend this way of determining neutral simultaneity, so as to deal with the various auditory sensa that constitute a single noise. If we are going to allow them to have *any* neutral temporal relations to each other, we must suppose that they are all neutrally *contemporary*, or we shall get into difficulties. For suppose that any two sensa, s_1 and s_2, belonging to different sense-histories, were neutrally contemporary, as determined by the present method. This will mean that s_1 is sensibly contemporary with one auditory sensum and that s_2 is sensibly contemporary with another auditory sensum, and that these two auditory sensa belong to a single noise. Now, unless we hold that the two auditory sensa in question are neutrally contemporary with each other, we shall have to admit that two neutrally contemporary sensa can be respectively sensibly simultaneous with two auditory sensa which are neutrally successive to each other. This does not accord with the view of neutral temporal relations as a consistent extension of the sensible temporal relations that hold between sensa in the same sense-history. We must therefore determine neutral simultaneity, on the present method, as follows: Two sensa in different sense-histories are neutrally contemporary if (*a*) they are two auditory sensa belonging to the same noise; or (*b*) they are respectively sensibly simultaneous with two auditory sensa which belong to the same noise. Would such a mode of determination be satisfactory?

Let A and B be two observers at a considerable

distance apart, and let there be a bell near A and another bell near B. Let the strokes of both bells be audible to both observers. We will call them "A's bell" and "B's bell" respectively. Suppose that A's bell rings and that B hears the noise. It may happen that B's bell rings at such a date that he hears its stroke at the same time as he hears the stroke of A's bell. If so, A will hear this stroke of B's bell sensibly later than the stroke of his own bell. Call A's sensum of the stroke of A's bell a_A, A's sensum of the stroke of B's bell a_B, B's sensum of the stroke of A's bell b_A, and B's sensum of the stroke of B's bell b_B. Then by definition we have:

(1) a_A is neutrally contemporary with b_A;

(2) a_B is neutrally contemporary with b_B;

and, by the terms of the experiment, we have

(3) b_B is sensibly contemporary with b_A.

Under these circumstances we should find that

(4) a_B is sensibly later than a_A.

Now, if neutral simultaneity be just an extended application of sensible simultaneity, we should expect that (2) and (3) would together imply that a_B is neutrally contemporary with b_A. Combining this with (1), we should expect to find that a_A and a_B were sensibly simultaneous. But this contradicts the fact stated in (4). In fact, if we determine neutral simultaneity in this way, we shall find that two sensa in the same sense-history can be neutrally simultaneous respectively with two sensa in another sense-history, which are sensibly simultaneous with each other; and yet the first pair of sensa are not sensibly simultaneous with each other, but are sensibly successive. Thus neutral simultaneity, determined by this method, cannot be a mere extension of sensible simultaneity. This can only be got over if we admit that, when two people "hear the same noise," the auditory sensum of the one who is nearer the source is neutrally earlier than that

of the one who is further away from it. But, as soon as we admit this, the purely auditory determination of neutral simultaneity has been given up; for we cannot determine in purely auditory terms the neutral temporal relations between auditory sensa which belong to "the same noise." We have to introduce spatial measurement, and the notion of influences travelling out from sources with a finite velocity. The intimate linkage of Space and Time becomes evident here, as in so many places.

So far then we see that, if observers tried to determine neutral temporal relations by sound alone, they would be forced to the view that what they call the same noise is a set of auditory sensa of different neutral dates; these dates depending on the distance between the observer who senses a sensum of the group and the source of the noise. This fact was early recognised about sound for several reasons. (i) Sound travels so slowly that the difficulties pointed out above are quite obvious to ordinary observers at reasonable distances apart, and provided with no delicate apparatus. (ii) Sounds, as we have seen, are not thought of as confined to a central volume, but as being in all the space that surrounds their source. Each observer is thought of as sensing the particular part of this physical field of sound which is "where he is at the moment." It is thus natural enough to think of this physical field as travelling out from the centre and reaching different observers at different times. (iii) Again, the phenomenon of echoes makes the notion of the velocity of sound pretty obvious to anyone. An echo is qualitatively very much like the original sound with which it is obviously connected. But it is separated from it, as a rule, by a distinct sensible interval. This naturally suggests something travelling from the observer to a wall (for instance), and then travelling back to him. (iv) Lastly, we are not like the observers in our example. We can produce flashes of light by various means at

will. Now, if a number of observers count two sensa as neutrally contemporary with each other, when each is sensibly contemporary with the same flash of light that they all see, they will not, in ordinary life, get into difficulties which arise for observers who try to define neutral simultaneity by means of sound. But, of course, if they do this, they will be obliged to recognise that the various auditory sensa which they sense when they say that they all hear the same noise are not neutrally contemporary. It is, in fact, by a combination of ight-signals and sound-signals that the velocity of sound is generally measured.

The next step that naturally suggests itself is to determine the neutral simultaneity between two sensa in different sense-histories, as the relation which holds between the two when each is sensibly contemporary with some sensum of the group which constitutes a single flash of light. If we adopt this method, we shall have to begin by extending it slightly in the same direction, and for the same reasons as we extended the auditory method of determining neutral simultaneity. That is, we shall have to assume that two visual sensa belonging to the same flash are neutrally contemporary, or we shall get into difficulties. We may therefore give the following as the visual definition of neutral simultaneity: Two sensa, belonging to different sense-histories, are neutrally contemporary, if (i) they are two visual sensa of a group which constitutes a single flash; or (ii) are respectively sensibly simultaneous with two visual sensa which belong to such a group.

There is, I think, no doubt that this is the way of determining neutral simultaneity, with which we all work in practice, except in extremely delicate scientific investigations or in cases where distances of astronomical order of magnitude are under discussion. Nevertheless, we all know that no scientist would accept it as ultimately satisfactory. He would point to the facts which are alleged to prove that light travels with a finite velocity

as a conclusive objection to the definition. The assertion that light travels with a finite velocity implies, *inter alia*, that there is an extremely important sense in which the various sensa of observers in different places who see the same flash are not simultaneous but successive. The above definition of neutral simultaneity is therefore unsatisfactory, because it leads us to call sensa simultaneous, which are in some very important, but as yet undefined sense, successive.

Let us then consider this definition and the facts that are held to render it inappropriate. In the first place, there are two things to be said in its favour: It is not circular, and it does not *directly* conflict with our judgments about sensible temporal relations, as the attempted auditory definition did. It would, of course, be circular if we could not define what we mean by "the same flash" without introducing temporal relations between sensa in different sense-histories. But we can define "the same flash" without this. A number of observers may be said to see the same flash when the following conditions are fulfilled: (i) Each is aware of a *single* outstanding visual sensum of very short duration. (ii) These sensa are all qualitatively very much alike. (iii) They are all optically compresent at a common centre, in the sense defined in the last chapter. (The first condition seems to be enough to secure that we are all dealing with a single flash, and that different observers are not seeing similar but successive flashes. For, if successive flashes were being sent out, some at least of the observers would sense two or more qualitatively similar sensa which were sensibly successive.)

Again, there is nothing in our light-experiences to correspond to the case that we adduced of two distant observers hearing two bells, and one of them finding his auditory sensa sensibly contemporary, and the other finding the auditory sensa belonging to the two noises sensibly successive. We can only deal with pairs of observers separated by distances of a few miles; and

for such distances there is no conflict between sensible temporal relations and neutral temporal relations as determined by light-signals.

It is therefore possible to determine neutral simultaneity visually without committing a circle and without conflict with any judgments of sensible simultaneity that we can make. The conflict is with the facts that prove that light has a finite velocity. What are these facts and what do they prove? When people say that light travels with a finite velocity they mean that some change moves from a distant centre to the observer and that his visual sensum begins as soon as this change reaches him and goes on till it ceases to reach him. By a single flash they think of a single event at the source (*e.g.*, the opening of a shutter) and the change that travels out from this. Let us consider the facts and arguments which are supposed to prove this. We may take three typical examples. These are Fizeau's experiment, with a rotating cogwheel and a mirror; Römer's argument from the times that apparently elapse between successive eclipses of a satellite of Jupiter; and Bradley's argument from the shift in the apparent positions of the fixed stars. These three arguments are placed in order of simplicity. The first keeps the source and the observer relatively at rest for the whole time, and literally consists in producing "light-echoes," and showing that there is a time-lapse between them and the flash of which they are the "echoes." The second depends on the fact that an observer and a certain source are at different distances apart at different times of year. The last depends on the relative *velocity* of source and observer, and belongs rather to the subject of the next chapter than to the limits within which we are at present confining ourselves. I must state as shortly as possible the facts on which these arguments are based, so that we may be able to see what exactly they assume in order to reach their conclusion.

(i) *Fizeau's Experiment.*—Light is sent through a hole, in front of which is a cogwheel. When one of the teeth of the wheel is in front of the gap, light cannot pass; otherwise it can. The light travels some considerable distance, and is then reflected back along its old course, and the image is viewed from behind the cogwheel. If the passage of the light between the source and the mirror and back again be instantaneous, the image will be visible, no matter how fast the cogwheel revolves; for if *no* time has elapsed, the cogwheel cannot have moved *any* distance since the flash left it and before the light returned to it. The gap cannot, therefore, have become shut, in the meanwhile, by the rotation of the cogwheel. But if any finite time elapses between the departure and the return of the light, it must be possible to cause the original gap to be replaced by the next tooth by the time that the light returns, provided that the cogwheel has moved fast enough. In that case no image will be seen. If the speed of the wheel be now increased enough, the image ought again to be seen, since the wheel will have turned so far in the time taken by the passage of the light that the next gap will be in position to admit the reflected beam when it returns. It is found that the image can be made to disappear by rotating the wheel fast enough, that it can be made to reappear by rotating the wheel faster, and that the wheel needs to be rotated faster and faster the nearer the mirror is to the source, in order to make the image disappear. All these facts are what we should expect if the reflected sensum depends on the passage of something with finite velocity from source to mirror, and from mirror to observer, and begins when this something reaches the observer's eye, and does not end till it ceases to reach his eye.

It is clear that the result of the experiment does not bear *directly* on the question of the neutral temporal relations between two sensa of observers who see the same flash. For we are actually dealing with a single

sensum (the reflected image) of a single observer. The connexion, however, is this: It is argued that the result of the experiment shows that any visual sensum begins when something that has started from a source reaches the observer, and that this something takes a finite time to travel. The various visual sensa that together constitute a single flash are simply those sensa which begin to be sensed by various observers when something that left a source at a certain moment reaches them. If the observers are at different distances from the source, their various sensa will be correlated with different stages in this process of transmission. Hence, there is an important sense in which what is called one flash is a group of *successive* sensa. It would, therefore, be inconvenient to determine neutral simultaneity in such a way that all the sensa in a single flash would count as neutrally simultaneous.

Thus a single flash of light comes to be treated as a set of successive sensa, because different sensa in the set are held to be correlated with different stages in a certain process of transmission from the source through the surrounding Space.

(ii) *Römer's Argument.*—The earth and the planet Jupiter revolve about the sun in approximately the same plane and approximately in circles. Jupiter has a much larger orbit than the earth, and takes much longer to complete it. Thus, at certain times, the two are in the position shown below,

and at other times they are in the position shown below. The first is called a conjunction and the second an opposition.

Jupiter has satellites which revolve round it as the moon does round the earth. When a satellite moves

into the shadow on the far side of Jupiter from the sun, it is eclipsed, and becomes invisible to us. Now it is found that the number of eclipses that take place between a conjunction and the next opposition is the same as the number that take place between an opposition and the next conjunction. But there is quite a marked difference (about 33 minutes) between the total times that elapse from the first to the last of these eclipses in the two cases.

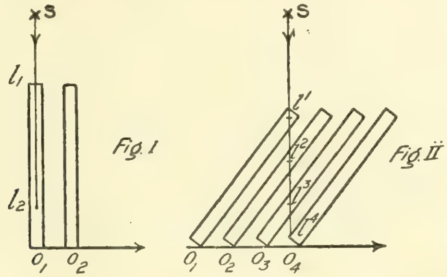
Now the eclipse of a satellite is comparable to the shutting of a shutter. The movement of the earth ensures that the observer on it is at different distances from this shutter at different times of year. He is nearer to it at the time of conjunction than he is at the time of opposition by the whole diameter of the earth's orbit. If we suppose that the visual sensum ceases to persist as soon as the shutter is closed, we can only explain the facts by supposing a periodic change in the time of revolution of the satellite. This would be extremely difficult to fit in with the facts that we believe about the laws of mechanics and the forces acting on the satellites. If, however, we assume that the visual sense-object persists after the shutter is closed, for a time which increases with the distance between the observer and the shutter, we can fully account for the divergence of 33 minutes, without needing to suppose that the periodic time of the satellite changes as Jupiter progresses in its orbit. The time-lapse between an eclipse and the cessation of the corresponding visual sense-object, which is necessary to account for the 33 minutes' discrepancy, can easily be calculated; and, if the radius of the earth's orbit be known, the velocity of light can be determined. It is found to be approximately the same as that deduced from Fizeau's experiment. Here there is no complication about mirror-images; we simply have a source and an observer which are at different distances apart at different times of year.

Once again the result of the argument does not bear directly on the question whether it is appropriate to determine neutral simultaneity in such a way that the various sensa which constitute a single flash of light shall be all neutrally contemporary. We are not dealing with two observers seeing a single flash; on the contrary, we are dealing with a single observer who sees three different flashes (if an eclipse may by courtesy be called a flash) at widely different dates in his history. There is, however, an indirect connexion. The argument is, that you must either abandon certain very well-established laws of motion, or assume that the occurrence of visual sensa depends on the motion of something from the source to the observer. The visual sense-object lasts so long as any of this something meets the eye, no matter what may have happened to the source in the meanwhile. On this assumption, you can account for the facts without abandoning the familiar laws of motion. But, as before, if you make this assumption, you must suppose that what we call a single flash is a group of sensa correlated with various stages in the process of transmission of this something. And, on that supposition, it is unsatisfactory to determine neutral simultaneity by a method which presupposes that the various sensa which belong to a single flash are neutrally simultaneous.

(iii) *The Aberration Argument.*—It is found that, if the fixed stars be observed night after night, their apparent positions undergo a periodic change. Each describes a closed curve in the course of a year. Now the apparent position of a star is, of course, the optical place of the visual sensum which is an appearance to us of the star. The direction of this place will be determined by the direction in which we have to point our telescope in order to bring this visual sensum into the middle of our visual field. Now, of course, we might suppose that all the fixed stars are describing closed curves in the time which it takes the earth to move

round the sun. But this would be a most extraordinary state of affairs, and it is not one that we readily accept. Now it happens that the facts can be quite easily explained on the same assumption as before about light.

Let S be a star, and let the line OO represent the course of a moving observer with a telescope. In the first figure we will suppose that he is pointing his telescope at the physical place of the star. At a certain moment let his position be O, and let light from the star have reached l_1, a point in the middle of the far end of his telescope. At a slightly later moment let his position be O_2. The light will then have got to l_2



in its original straight line, and will no longer be passing down his telescope at all. It is clear then that, if the moving observer points his telescope at the *physical* place of the star, he will see no star at all. Suppose now that he tilts his telescope forward by an appropriate amount in the direction of his movement. Let O_1, O_2, O_3, O_4 represent four successive positions of the telescope, and l_1, l_2, l_3, l_4, the four corresponding positions in the course of the light which is travelling from the star. It is clear from the figure that the light will pass down the telescope and meet his eye, provided that he slopes the telescope forward at an angle to his course, whose tangent is c/v, where c is the velocity of light and v is that of the observer. Now an observer on the earth is moving with it in the course of a year round a closed curve — the earth's orbit — with considerable

velocity. It is thus easy to understand that, although the physical place of a star remains constant, the optical places of the sensa by which the star appears to us will vary in the course of the year, and will repeat their variations over and over again in that period. From the speed of the earth in its orbit and the amount of the aberration of a star, it is easy to calculate the velocity of light. It is once more found to be the same, within the limits of experimental error, as that found by Römer's argument and by Fizeau's method.

This argument is of particular interest to us, not merely in connexion with the question of neutral dating, but also as reinforcing the distinction that has already been drawn on other grounds between physically and optically occupied places. We introduced that distinction originally because of facts which are found to arise when the medium surrounding an observer is non-homogeneous. We now see that the optical place of a visual appearance and the physical place of its source may be different, even when the medium is homogeneous, if the source and the observer be in relative motion.

Let us now consider what these arguments have to teach us. (i) We see that three extremely different lines of argument tend to the conclusion that visual sensa are connected with something that is transmitted from a source to an observer with a finite velocity. And they all lead to approximately the same numerical value for this velocity. Now, in each separate case, there is no doubt that the facts could be explained without taking this particular view about light, provided we made some other assumption. But, in the first place, each of these assumptions would conflict with some law of Nature which has been well established in other cases. And, in the second place, these assumptions would be quite disconnected with each other ; each would be an independent piece of "cooking." On the other hand, a single assumption as to the nature of light explains

all these very different facts, and reconciles them with the established natural laws with which they would otherwise conflict. Thus the hypothesis in question is established about as solidly as any scientific hypothesis can be. The simple-minded scientist may think that I have needlessly laboured this point; but I have deliberately insisted on it, because I know that some eminent "realist" philosophers, finding that the finite velocity of light "stains the white radiance" of their theories of perception, are inclined in private to deny it, or at least to "damn with faint praise, assent with civil leer."

(ii) We notice that the finite velocity of light is never proved *directly*; but always by the argument that, unless it be true, certain observable facts will not be reconcilable with well verified laws about *the motion of matter*. The only direct way to verify the proposition would be for two observers to stand at a distance apart, see the same flash of light, and find that their respective visual sensa were not contemporary. Now there is both a practical and a theoretical difficulty about any such experiment. The theoretical difficulty is this. The two observers would need to be provided with some means of marking, and thus comparing, the dates of their respective sensa. Suppose that the means adopted were two stop-watches. This would be useless, unless they had reason to suppose that the two watches agreed in their zero points and were going at the same rate. They might, of course, set the watches in synchronism when they are both together; but what guarantee have they that they will remain in synchronism when one has been carried a long distance away? To *assume* that they do, is to make an assumption which is contradicted by quite gross experiences. To *test* their synchronism after they have been separated, can only be done by means of light or electrical signals; and there is obviously a circle in setting two watches by light-signals and then using them to test whether two visual sensa belonging to the same flash are contemporary or

successive. The only way out of this difficulty would be if both observers could observe a certain pair of flashes, and if one of them should find that his two visual sensa were sensibly simultaneous, and the other should find that his two visual sensa were sensibly successive. But, in practice, this cannot be done, because of the great velocity of light and the fact that the only observers who can compare notes with each other are confined to the earth's surface. Thus it seems clear to me that the neutral simultaneity of visual sensa belonging to the same flash is denied wholly and solely because it conflicts with another system of dating which depends on certain alleged laws of motion.

(iii) It is evident that if we accept the view that the various sensa belonging to the same flash are not neutrally simultaneous, we shall have to admit either that two sensa which *seem* simultaneous may not really be so, or that two sensa which are neutrally successive may be sensibly simultaneous. The latter alternative would prevent neutral temporal relations from being consistent extensions of sensible temporal relations, and we shall therefore not take it, unless we are forced to do so. Now there is nothing in the Sensum theory of sensible appearance to force us to the second alternative. A sensum belonging to a certain flash and a sensum belonging to its reflected flash, seem to us to be sensibly simultaneous. If the physical theory of light be accepted, the latter is neutrally a little later than the former. But the sensible simultaneity of two sensa only means that each exactly overlaps the other in their common sensible field. Now the notion of exactness always involves a negative factor; it means that *no* part of the one sensum sticks out beyond the end of the other. And we saw, when dealing with the general theory of sensa, that there is no reason why *negative* judgments about sensa should be infallible. Thus, two sensa may often seem to be sensibly quite simultaneous, when really one begins a little later and ends a little later than the other.

We see then that the question of a neutral dating of events in different sense-histories leads inevitably to the question of motion, whether it be the transmission of those changes which are connected with sound and light, or the motion of ordinary physical bodies through Space. Thus the separation of Space and Time, with which we started, which has been wearing thinner and thinner as the argument has advanced, has now broken down altogether. This does not mean that there is no difference between temporal and spatial relations; but it does mean that it is impossible to apply the concept of a single Space to Nature without referring through Motion to Time, and that it is equally impossible to date the events of Nature in a single Time without referring through Motion to Space. And this, it will be noted, is one of the characteristic features of the Theory of Relativity.

To sum up: If I want to determine neutral temporal relations between an event which is in my sense-history but not in yours, and an event which is in your sense-history but not in mine, the only possible way seems to be to find something which is common to the sense-histories of both of us, and to determine the neutral temporal relations between the two "private" events by means of their respective sensible relations to this "public" event. At first sight this seems perfectly plain sailing, since there are events, like noises and flashes, which are admittedly "public" in a way in which headaches and toothaches are not. If it were really true that, when we say that we "hear the same noise" or "see the same flash," there is a single auditory or visual sensum in all our sense-histories, it would be easy to determine neutral simultaneity in this way. And, since it would have the same logical properties as sensible simultaneity, it would be reasonable to hold that it is really the same relation, and that the proposed method of determination is simply a *test* and not a *definition* of a new kind of relation. But, although it

is not *logically* impossible that a single sensum might be in a number of different sense-histories, closer observation of the facts makes it almost impossible to believe that a noise or a flash really is a single sensum. Moreover, it seems impossible to hold that it is even a group of *contemporary* sensa. Thus, such methods of determination, though practically useful for most purposes, owing to the considerable velocity of sounds and the very great velocity of light, are not theoretically satisfactory.

Temporal Characteristics of Physical Events.—The further development of this subject must be left to the next two chapters, but it is possible in the meanwhile to say something about the durations and dates of physical objects and events. A single flash of light or a single noise may be called a *perceptible physical event*. When a man says that he sees a flash of light, he does not mean either (*a*) merely that he senses a certain visual sensum, or (*b*) that he sees the movement, *e.g.*, of an electron at the source which is responsible for the flash. For (*a*) he admits that other people can see the same flash, whereas we have found reason to think that two people who see the same flash do not sense the same visual sensum. And (*b*), so far from admitting that he saw the movement of the electron, he would say that this is invisible, and that he only believes it to have taken place on the authority of a scientific theory which he does not himself understand. Thus, to see a flash means something more than to sense a visual sensum, and something, partly more and partly less, than to perceive the motion of an electron. An angel might perceive the motion of the electron and see no flash, whilst a man sees the flash and does not perceive the motion of the electron. Seeing the flash involves sensing the sensum and also something more. It involves the excitement of traces connected with similar experiences in the past. These may or may not actually produce the explicit perceptual judgment that other observers are sensing

similar sensa which are optically in the same place, and that some movement has happened in that place. But, whether these judgments actually arise or not, the observer will tend to behave in a way in which it would be reasonable to behave *if* he had explicitly made these judgments. If such judgments be not true in a particular case, we say that the observer is mistaken in his belief that he has seen a flash of light, even though he has sensed a short, bright visual sensum. Thus a man who "sees stars," because he has hit his head against a post, senses a bright visual sensum, but would be deceiving himself and others if he said that he had seen a flash of light.

A perceptible physical event, like a flash or a noise, may therefore be defined as a certain group of sensa having certain similarities to each other and certain neutral spatial relations. Nearly always they will be, in some sense, compresent at a certain place in Space. We have seen that, as a rule, they will not all be neutrally simultaneous, but that their neutral dates will depend upon the positions of the various observers who sense them. To perceive such a perceptible event means (*a*) to sense a sensum belonging to such a group; and (*b*), in consequence of the traces left by similar experiences in the past, either explicitly to judge that it is a member of such a group, or to act as it would be appropriate to act if one had explicitly made this judgment.

(*a*) *Dates of Perceptible Physical Events.*—Now, since a perceptible physical event consists of a number of sensa of different neutral dates, it is obvious that the question: "What is *the* date of a certain perceptible physical event?" can only be answered in a more or less Pickwickian manner. To give any answer to it we must notice the two following facts: The neutral dates of the sensa in such a group are none of them earlier than the date of a certain physical movement, such as the opening of a shutter. If we include in the

flash not only actual sensa but the sensa of possible observers, the dates of the various sensa would approach the date of this movement at the source as their lower limit. This date might, therefore, be defined as "the date at which the perceptible physical event *begins*." The second point to notice is that, where a group of sensa have later and later neutral dates as the observer is further and further from the source, the sensa in question are fainter and fainter. Thus the dates of the sensa which constitute a single noise approach a limit where we are dealing with an observer so remote that he can only just sense a sensum of the group. This does not give an absolutely sharp date which may be taken as "the date at which the perceptible physical event *ends*," because the question of the different acuteness of different observers comes in. Still it is clear that in this way we could define approximately the date at which such an event ends. The duration of a perceptible physical event may then be defined as the time that elapses between its beginning and its end.

(*b*) *Relative Dates of Act of Perceiving and Event Perceived.*—Next we see that, although the beginning of an act of sensing may be regarded as contemporary with the end of the sensible field that is sensed throughout the whole of it, there is not the same simple relation between the date of an act of perceiving and the date of the physical event perceived by it. This is obvious, since there is nothing that can appropriately be called *the* date of a perceptible physical event. We may reasonably identify the date of an act of perceiving with that of the act of sensing on which it is based. So that, in general, all we can say is that an act of perceiving is later than the beginning and earlier than the end of the physical event that it perceives. It is very common to suppose that an act of perceiving must be contemporary with the event perceived. This is, of course, a mere mistake, due to a confusion between an act of sensing, whose object is a sensum,

and an act of perceiving, whose object is a physical event.

There is one more confusion to be pointed out before we leave this subject. It might be said : "Does not a physical event, such as a flash of light, persist *for ever* once it has started?" I answer that the movement that is transmitted from the source and is correlated with the various visual sensa of the group, may very well go on for ever. But this movement, of whatever nature it may be, *is* not the flash of light. A flash of light is a *perceptible* object; the movement in the ether is not perceptible—by us at any rate. It is merely silly to say that a certain perceptible event lasts for ever, because a certain imperceptible event, with which it is closely connected, does so.

(*c*) *Scientific Events.*—This naturally brings us to the question of the dates and durations of *imperceptible* physical events. We know that perceptible physical events, such as flashes of light, are supposed to be intimately connected with movements of electrons and changes in the ether which we cannot perceive. These are much more important theoretically to the scientist than perceptible events. The epistemological relation between the two is the following: It is by observing and noting the relations between perceptible events that we infer the existence of these imperceptible events, which, following Whitehead, I will call *scientific events.* Instead of stating the laws of Nature as direct relations between perceptible events, we analyse these relations into the relative product of two different kinds of relations, viz., (*a*) those of scientific events to each other, and (*b*) those of scientific events to perceptible events. This process seems to be indispensable, if we are to deal satisfactorily with Nature at all. The relations between perceptible events are very complex, and few simple and invariable laws can be stated about them. On the other hand, the relations of imperceptible events to each other and to perceptible events are reasonably simple,

and laws of very wide range can be stated about them. We can then use these hypothetical laws to predict what perceptible events will be perceived under assigned perceptible conditions. In so far as the predicted events actually take place, our hypothesis about imperceptible events and their laws is strengthened. It is very easy for a scientist, who constantly deals with scientific events and sees their great practical and theoretical importance, to fall into the mistake of supposing that they alone are "real." This is a great error. The actual position is this: The existence of sensa is absolutely certain, and those positive sensible properties which they seem to have they certainly do have, if the Sensum theory be accepted at all. The existence of *some* perceptible physical events is practically certain, if we are prepared to accept the existence of other observers and to believe what they tell us about their sensa. But, in any particular case, an observer who thinks that he perceives a certain physical event may be mistaken. For he may sense a sensum of a certain kind and mistakenly suppose that it is one of a group of connected sensa, when really it is "wild" and isolated. Lastly, since imperceptible physical events are only assumed in order to fill the gaps between the various sensa of single perceptible events and to connect different perceptible events with each other, it is clear that our certainty that there are such and such imperceptible events cannot logically exceed our certainty that there are such and such perceptible ones.

There is a connecting link between purely perceptible events, like flashes of light, and purely scientific events, like the movements of electrons and ether-waves. This link is the *unperceived parts* of perceptible events. We defined a flash as a certain group of visual sensa, and we said that its duration was the time that elapses between the earliest and the latest of these sensa. But, it must be admitted that the really important point about

perceptible events is not the actual sensa in the group, but the possible sensa. Actually only a few of the sensa in such groups are sensed by anyone, and it may quite well happen that only one of them is sensed. The perceptual judgment does not assert that other sensa of the group *are* sensed, but only that they *would* be by any observer sufficiently like ourselves placed in any suitable position. Thus the cash value of the statement that perceptible events persist, even when no one happens to sense any sensum of the group, is that *whenever* a suitable observer is present at any position in a certain spatio-temporal region, he *will* sense a member of the group. We are not content with this merely hypothetical assertion. We assume that if *any* observer at *any* position of a certain spatio-temporal region will sense a sensum of a certain group, this must be because something independent of all observers is going on at *all* positions in this region. This assumption rests partly on our passion for spatio-temporal continuity. When there is a close connexion between events in different places and of different dates, we feel that the gaps between them *must* be filled in somehow. And this conviction is strongly reinforced if we find that *any* observer who takes up his position *at random* within the spatio-temporal region in question equally senses a member of the group.

We must notice, moreover, that the presence of an observer is found to be irrelevant to most chains of physical causation. If I put a kettle on the fire and watch both, the perceptible event of the fire burning is followed after a certain time by the perceptible event of the kettle boiling. If I and all other observers go away for a time and then return, we find that the kettle has boiled after the same lapse of time. These and millions of other experiences show that the gaps between the sensa belonging to a perceptible event are filled by something that produces just the same effects as if we were present. Thus, even at the level

of common-sense, a perceptible physical event is thought of as a group of sensa connected by events that go on in the absence of observers. Common-sense is very vague as to the nature of these unperceived parts of perceptible events. I think that it generally supposes in a rather half-hearted way that they are of the same nature as the parts that are actually sensed. How far such a view can be maintained cannot be decided until we have dealt with the physiological conditions of sensa. But, at any rate, we can say that it seems essential to suppose that *something* bridges these gaps ; and science professes to determine more and more accurately the nature of this something. Whether it has the properties of *sensa* or not, it certainly has the properties of *scientific events*, subject of course to the possibility of scientific theories being wrong on points of detail.

In the last chapter I said that scientific objects are conceived to have shapes and to occupy places in the movement-continuum in the same literal way in which visual sensa are immediately sensed to have shapes and to occupy sensible places in their fields. In fact, the concepts of what I will now call *Scientific Space* and scientific physical objects are constructed together in an inseparable union. They are constructed *on the analogy of* sensa and their fields *out of* data derived from the sense-experiences of many observers through various senses and at various times. Exactly similar remarks apply, *mutatis mutandis*, to the concepts of what I will call *Scientific Time* and scientific events. Scientific Time is conceived by analogy with a sense-history ; scientific events are conceived to have dates in Scientific Time as sensa have dates in the sense-history of the observer who senses them ; scientific objects are conceived to have duration in Scientific Time as sense-objects have duration in a sense-history. There is one difference, however. For reasons already stated, it is impossible that sensa should literally occupy places in scientific space, though it may not, of course, be im-

possible to construct a space-like whole of more than three dimensions, in which sensa of all kinds, and scientific objects, literally have places. If so, I suppose that Scientific Space would be one kind of section of such a quasi-space, and *e.g.*, a visual field would be another kind of section of the same quasi-space. But, if such a construction can be made at all, I, at any rate, am not capable of doing the trick. On the other hand, it is not obviously impossible that sensa should literally have dates and durations in the same Scientific Time as scientific events; for, as we have seen, temporal relations are much more pervasive than spatial relations. The scientific dates of sensa would seem to be the dates at which certain scientific events happen in the brain of the observer who senses these sensa. Unless there be some positive inconsistency between the temporal relations of such scientific events and the sensible temporal relations of the corresponding sensa, there seems no reason to reject the naïve view that the temporal relations between sensa in our own sense-history, with which we become acquainted through sensation and memory, are literally the same as the temporal relations between the corresponding scientific events in our brains. Whether this view can be held, is a question which must be reserved for a later chapter.

Duration of Physical Objects.—We have now said all that can be said with profit about the dates and durations of physical events before dealing with motion and the union of Space with Time. It remains to say something about the durations of physical objects or "things." A thing, as we have seen, is simply a long event, throughout the course of which there is either qualitative similarity or continuous qualitative change, together with a characteristic spatio-temporal unity. A sense-object, as defined earlier in the chapter, is an example of such a long event; though, for reasons which will appear in a moment, it would hardly be called a

"thing," and it is certainly not "physical." Thus the dividing line between events and things cannot be very sharply drawn in theory. Nevertheless, we can draw a rough practical distinction, and it is useful to do so, in order not to depart too far from common speech.

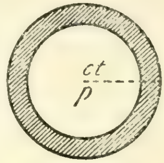
(*a*) *Perceptual Objects*.—A flash of light would be called a perceptual event, but not a perceptual thing or object. This is because each person who sees the flash senses a single short sensum, and not a series of sensa in successive fields which join up with each other to form a sense-object of decent duration. This is true, although, as we have seen, the flash itself as a perceptible event has considerable duration, which may extend to thousands of years. Thus one point about a perceptible object is that it must be capable of being perceived for a long time by the same observer. And this means that its appearance to him must be not merely a *sensum* but a *sense-object*. Again, a perceptible thing is always understood to combine a number of connected qualities which can only be perceived by different senses. An observer might see a mirror-image for an hour at a time, but he would never say that he was seeing a physical object, so long as he knew that it was a mirror-image. For he would know that, if he went to the place where it is optically present, he would sense no correlated tactual sensa, and that there would be no relevant scientific objects there.

Of course, as I have already hinted, these criteria are not theoretically satisfactory. What we count as a perceptible object may be moving so fast that we sense only one short sensum in connexion with it. Conversely, an observer who moved in the right direction with the velocity of light would continually sense sensa connected with a single flash, so that he would be aware of a sense-object of considerable duration, and might therefore be inclined to say that he was seeing a perceptible thing and not merely a perceptible event.

Still, the criteria that we have just laid down work in a great many cases and will do for our present purpose.

We can now improve the definition of a perceptual object which we gave in the last chapter, where we deliberately overlooked for the moment complications due to time. We still cannot give a perfectly satisfactory definition, because we have not yet dealt properly with the movement of physical objects and observers and the consequent displacement of visual sensa in the movement-continuum. We will assume for the present that we are confining ourselves to a resting object and resting observers, and we shall not attempt to remove this restriction until the next chapter. Suppose that a scientific event of the kind which is connected with a single flash of light were to happen at a certain moment at a certain place in scientific space. Suppose that observers were dotted about in all directions and at all distances around this place. Then it is true that the place in question would be optically occupied by visual sensa from all *directions* for a very long time. But it would be optically occupied only for a moment by visual sensa from a given *distance*. At any given moment the sensa which occupied the place would occupy it from places on a certain sphere surrounding it, and at a later moment it would be occupied only by sensa from places on a larger sphere. It would never be occupied at once by sensa from places on two such spheres. If there were a persistent optical object, instead of a mere flash, at the place, this place would be optically occupied at a given moment from many different distances as well as from all directions. We might regard a persistent optical object as a continuous series of successive flashes. Each flash is itself a series of successive sensa belonging to different fields, and the later a sensum is in its flash the further off is the place from which it is present at the luminous centre. Thus there are two temporal series to be considered: (1) The series of flashes which together make

up the history of the persistent optical object; and (2) the series of successive sensa which together make up a single flash. It is obvious that an early sensum belonging to a later flash and a late sensum belonging to an earlier flash may be simultaneous with each other. The former will be optically present at the centre from a near place, and the latter will be optically present at the centre from a more remote place. Thus the centre is optically occupied by sensa from different *distances* at the same moment. Imagine for simplicity a visible object of very small spatial dimensions, which we can treat as a point. Suppose it lasted for a time T, and that a time t has now elapsed since it began to exist. Then the places *from* which sensa are present *at* this point at the moment t are all the points contained in the volume between a pair of spheres with the point as centre and ct and $c\ (t+\mathrm{T})$ as radii. (Here c is the velocity of light.) The diagram will make this plain.



At this moment sensa from the first flash in the history of the object will be present at P from places on the outer sphere, and sensa from the last flash in its history will be present at P from places on the inner sphere. Sensa of intermediate flashes will be present at P from places in the volume contained between the two spherical surfaces. Thus the thickness of this solid shell of places, from which sensa are contemporaneously present at P, is characteristic of the duration of the optical object. From places within the smaller sphere there are no longer any sensa present at P, and from places outside the larger sphere there are not yet any sensa present at P. The "shell" will continually spread out from the centre, but it will always remain of the same "thickness," and this thickness is characteristic of the duration of the optical object.

So far, we have confined our attention to the *places* from which sensa are present at a given place at a *given*

moment. But we can equally well regard the whole situation from another point of view. We can consider the *moments* at which sensa are present at a given place from a *given place*. In the case of a flash each observer senses just one sensum, which is optically present at the place where the flash is said to be. In the case of a persistent optical object all the observers will be aware in course of time, not merely of a single sensum, but of a sense-object. And the duration of this sense-object would commonly be identified with that of the optical object. The sense-object in this case is a group of successive visual sensa in a single sense-history, one of which belongs to each of the successive flashes into which the history of the persistent optical object can be analysed by Extensive Abstraction. It is clear that we must distinguish between (1) the *duration of an optical object from a place*, and (2) the *total duration of an optical object*. The former is simply the duration of the sensible object which is the appearance of the optical object to an observer at that place. But an optical object, however short its duration from any one place, has an enormously great duration, when you take into account *all* the sensa which belong to it from all places. Its *total duration* is the time that elapses between the earliest and the latest visual sensum which belongs to it. And this, even in the case of a momentary flash, may amount to millions of years. A flash, in the limit, has only duration of the second kind ; a persistent optical object has both kinds of duration.

We can now define a *persistent complete optical object*, subject to the limitations about motion which we have already indicated. Such an object is a group of visual sensa of various dates, correlated with each other, and having the following properties : (1) There is a certain closed contour in Scientific Space (the "place occupied by the optical object"), such that every member of this group of sensa is optically present at some part of its surface from somewhere. (2) Every part of this contour

is optically occupied from somewhere by some member (or members) of the group. (3) At any moment after the optical object has started to exist, any part of this central contour is occupied by sensa of the group from all the places within a certain *volume*. This volume is bounded by two closed surfaces, both of which contain the place occupied by the optical object. After the optical object has completed its history, the thickness of this volume is a measure of the duration of the object from any point. (4) From any point a certain part of the central contour is occupied by a series of successive sensa, forming a sense-object in the sense-history of an observer who stays at this point. The duration of this sense-object is the duration of the optical object from this place.

To define a *non-persistent complete optical object*, *i.e.* a complete optical event, or "flash," we leave clauses (1) and (2) standing, and modify clauses (3) and (4) as follows: In (3) substitute "on a certain surface" for "within a certain volume." In (4) substitute "a single sensum" for "a series of successive sensa," and omit the rest of the clause. Finally, a mirror-image of a chair or a pin would be a persistent incomplete optical object, whilst a mirror-image of a flash would be a non-persistent incomplete optical object.

We said in the last chapter that an ordinary perceptual object, like a penny, as understood by common-sense, is really a *compositum* consisting of a number of correlated constituent objects of various kinds, all occupying a place in the movement-continuum in their various appropriate Pickwickian ways. This place, moreover, is conceived to be literally occupied by correlated scientific objects; and the difference between science and common-sense is largely a difference in the amount of knowledge which the two claim to have about these scientific objects. It is obvious that some of the constituents of a perceptual object may cease to persist while others remain. Again, a place where

a perceptual object has once been, may continue to be haunted from certain places by its ghost, in the form of its optical constituent. The compositeness of a perceptual object infects the notion of "its" duration with an incurable vagueness. We can make accurate statements about the durations of its constituents, and we can make accurate statements about the durations of the correlated scientific objects, but the perceptual object of common-sense is too much a mixture of non-homogeneous constituents to be worth treating very seriously as a whole.

We saw that an observer can very well be mistaken in thinking that he perceives a physical event of a certain kind, because this implies a reference beyond the sensum which he senses to other sensa, actual and possible, of other observers. *A fortiori*, we can be mistaken in supposing that we perceive a certain physical thing; and this can happen even when we are quite right in thinking that we perceive a physical event or a series of them. Such mistakes take various forms, and contain various amounts of error. (i) We may mistake a partial for a complete optical object, *i.e.*, we may think that a certain place is optically occupied from all directions when really it is occupied only from one or from a restricted range of directions. This happens in optical illusions which really deceive us. (ii) If we make this mistake we shall almost certainly make the further mistake of supposing that the place in question is also occupied by correlated tactual and other constituents, that it is a centre for sound and radiant heat, and that it is occupied literally by scientific objects specially correlated with our visual sensa. Actually the most relevant scientific objects will be at some remote place. (iii) We may make very grave mistakes about time. We practically always think that physical things have endured and remained in the same place longer than our visual perceptions really justify us in believing. If an ordinary man sees a star in a certain

optical place, he assumes that it must have been there at least up to the time when he ceases to see it. This is of course unjustified. My visual sensa are indeed optically present at this place at the time when I sense them, and for as long as I go on sensing them. But, in saying that the *star* is there at that time, I am asserting much more than this. I am asserting that other types of constituent object are also there, and that the place is now occupied by correlated scientific objects and events. This may happen to be true, but it is not justified by my visual perception alone. The star may have blown up or moved elsewhere since the light left it. The first statement implies that there is now no centre occupied by scientific objects correlated with my present visual sensa. The second implies that there is still a centre occupied by events of this kind, but that it is no longer at the place where the optical object is present. The facts of aberration show that such divergences between the place of a perceptual event and that of the thing with which it is connected, may arise through mere movement of the observer.

(*b*) *Scientific Objects.*—It is admitted that ordinary perceptual objects, like pennies and chairs, begin to exist, last for so long, and then come to an end. In the chapter on Time and Change in Part I, I tried to explain what exactly is meant by saying of any object that it began to exist, lasted so long, and came to an end. Now perceptual objects are supposed to be connected with scientific objects in the way described earlier in the present chapter. And the total scientific object specially connected with any perceptual object is believed to be a very complex whole of related parts. Such structures have more or less stability, once they are formed; but they do begin to exist and come to an end under suitable conditions. We shall have to distinguish between scientific objects of various *orders*. The sort of scientific object which is specially connected with a perceptual object, like a chair, may be

called a *first order* object. It is supposed, as we know, to consist of a great many molecules arranged in a pattern in space. These may be called *second order* objects. Each molecule is supposed to consist of a number of atoms, characteristically arranged in space and moving in characteristic ways in time. These atoms are *third order* objects. Finally, each atom is supposed to be an arrangement of positive and negative electrons, with characteristic types of motion. These are *fourth order* objects ; and it is of course possible that they are themselves complicated structures composed of fifth order objects.

Such a hierarchy represents real facts about Nature. The simplest way to look at it is the following : Many agents, such as the presence of a sufficiently prosperous profiteer on the seat, will break up a chair without affecting the molecules of cellulose of which it is composed. Other agents, such as heat, will break up the cellulose molecules, but leave the atoms of carbon, hydrogen, and oxygen of which they are made, unaltered. A very few agents will, with great difficulty, break up the atoms themselves into their constituent electrons. So far as I know, no agent yet employed will break up an electron, though it is possible by heroic methods to knock pieces off the nucleus of an atom. Thus the orders in the hierarchy of scientific objects are the stages where certain disintegrating agents, which have previously been effective, cease to be so. Chairs really are permanent under a great variety of conditions, cellulose molecules under a greater variety, carbon atoms under a still greater range, and electrons under all variations that have been tried.

Now, for our present purpose, the important thing to notice is that scientific objects of different orders need different minimal spaces and durations to live in. This is generally recognised in regard to space, though it is stated in a rather misleading way, *e.g.*, that "molecules are divisible and electrons are not." It is equally true

of time, and it is one of Whitehead's great merits to have pointed this out clearly. I will first explain what is meant by this statement as regards space. If you divide up the space which is occupied by a chair into two parts, neither of these parts will be occupied by a chair, though one may be occupied by a leg and another by a seat. Again, you could divide up the space occupied by a chair into partitions, each of which was occupied by a cellulose molecule. If you further subdivided one of these divisions you would find that some of your subdivisions were occupied by a hydrogen atom, some by a carbon atom, some by an oxygen atom, and some by nothing at all. When a person says that a molecule is divisible in space, whilst an electron is not, what he means, over and above the fact that one has been experimentally split up and that the other has not, is roughly the following: If you take a space containing one and only one molecule and nothing else, you can divide it into a set of exhaustive and mutually exclusive partitions, such that there is a positive difference of quality between the contents of some of these partitions and the contents of others. (*E.g.*, the contents of one may have the "hydrogen quality," that of another the "oxygen quality," and so on. Of course, some of your partitions may have no contents at all.) If you take a space containing one electron and nothing else, then either (1) *all* sets of exhaustive and mutually exclusive partitions into which you can divide the space are occupied by contents of the same quality, or (2) you can divide the space into two mutually exclusive and exhaustive partitions, one of which is empty whilst the other has the property (1). What is called "indivisibility" is really rather *homogeneity of quality* for all spatial subdivisions below a certain maximum. Whether in fact an electron answers to this definition is, of course, a matter for empirical investigation.

Now, as Whitehead has pointed out, we have the same distinction among objects as regards division of

their history into successive slices. There are many types of object whose characteristic qualities need a certain minimum of duration to inhere in. *E.g.*, memory is one of the outstanding features of the sort of thing that we call a "mind." It is, therefore, clear that the very notion of a "momentary mind" is nonsense. Now the same is true of any scientific object which is partly characterised by some special type of motion. Suppose that a certain kind of atom consisted of a nucleus and an electron rotating about it at a certain characteristic rate. Such an atom would need at least the duration of one complete rotation to display its characteristic properties. The history of such an atom is a "pattern" in time, just as the momentary arrangement of electrons and nucleus is a pattern in space. If the duration of one complete rotation be sliced up into adjacent successive parts, the contents of the parts will differ in quality from the contents of the whole.

On the other hand, there may well be objects which are temporally homogeneous. This would mean that, however you choose to divide up their history, the contents of all the slices are the same as each other and as the whole in quality. Many types of scientific object then have a characteristic minimum duration as well as a characteristic minimum extension.

Now science regards the ultimate scientific objects as being spatio-temporally homogeneous. And it assumes that these ultimate scientific objects never begin or end. Thus the ultimate scientific objects are regarded as eternal in the sense of existing throughout all time. The only ultimate scientific changes are the groupings and regroupings of such objects according to a single set of fundamental laws. Whether this assumption be true, and whether it be self-evident, I do not profess to know. But I believe we may assert (as I have pointed out elsewhere, and as Mr Keynes has independently and much more clearly shown in his *Treatise on Probability*) that, without some such assump-

tion, it is impossible to justify the confidence which we feel in the results of "well-established" inductions. I do not propose to pursue this subject further here.

In the next chapter I shall say what I can about Motion, and, in the next but one, I shall discuss the concept of Space-Time, from which Scientific Space and Scientific Time are two abstractions of different types.

The following additional works may be consulted with advantage:

B. A. W. RUSSELL, *Lectures on the External World*, Lectures III. and IV.

A. N. WHITEHEAD, *Principles of Natural Knowledge*, Part IV.

S. ALEXANDER, *Space, Time and Deity*, Book I.

A. A. ROBB, *Absolute Relations of Time and Space.*

CHAPTER XI

"Oh, how glorious and resplendent,
Fragile Body, shalt thou be!"
(*Hymns Ancient and Modern.*)

Sensible and Physical Motion

In the last chapter I touched incidentally on the sensible motion of sensa within their own fields. Both in it and in the chapter before I talked of the motion of our bodies, and said that the concept of physical Space is based on such motions, interpreted spatially by analogy with our visual fields. I propose now to go considerably more into detail about these matters; to consider exactly how the concepts of physical Space and Motion are connected, on the one hand with our bodily movements, and on the other with the positions and movements of our sensa in their fields; and finally to work up to the concept of physical Space-Time. We shall find that the consideration of our own bodies and of the bodies of other observers who can communicate with us about their experiences fills a gap in our concept of physical objects, and is an essential factor in the development of the concept of physical Space.

General Remarks about Change and Motion.—When we say that something changes, or, more particularly, that it moves, we imply a certain identity and a certain difference. There must be enough identity for us to be able to say that we are dealing with the same object, in spite of the movement or other change. And there must be some difference between one part of the history of the object and others, or we

should not say that it had changed or moved. Change is a more general concept than movement, since movement is simply change of position in space. We will, therefore, begin with change in general.

In ordinary life we distinguish between an object and its history, and we are inclined to think that the former is logically prior to the latter. We say, *e.g.*, that there is a certain object, such as a penny, and that it may either rest or move, keep bright or tarnish, and so on. These events, we say, "happen to" the object, and its history is just all the events that happen to it. You might, we think, have an object without a history, but you could not have a history without an object. I believe this to be a profound mistake, which arises from taking "history" in too narrow a sense. An object, separated from its history, is clearly not the kind of thing that could possibly exist. Every object that is not merely momentary has a history of some kind, and no merely momentary object could really exist. "Object," apart from "history," is therefore as much an abstraction as "history," apart from "object." Of course some histories are very tame, *e.g.*, that of a penny which keeps in one place and never varies in its other qualities. Others are more exciting, *e.g.*, that of a penny which moves about, gets bent and defaced, and is finally dropped into the collection-plate. Now we are inclined to identify history with exciting, *i.e.*, variable, history. We then identify the object with the tame tracts of its history; and forget that these are history at all, because they are so uniform. But really all that literally exists is strands of history, some tamer and some more exciting.

Now it is conceivable that there might have been succession but no history. If so, there would have been neither *an* object nor a plurality of objects. Let us consider a fragment of the whole course of Nature, lasting for an hour. Let us imagine it cut up into successive slices, each lasting for a second. Theoreti-

cally there are three possibilities. (i) We might find that the contents of any adjacent pair of seconds had no particular resemblance either in whole or in part. And we might still find the same result if we took shorter and shorter divisions. In that case we could hardly talk of history at all ; there would merely be a perfectly chaotic hail of events. (ii) We might find that there was considerable qualitative resemblance between the *whole* contents of any adjacent pair of seconds, and that this resemblance increased as we took shorter and shorter sub-divisions. But we might have to compare the contents of each second *en bloc*. We might not be able to divide it into clearly distinguishable co-existing parts. In that case we should say that there is *a* history (of the world as a whole), but that there is not a number of distinct *strands of history*. We could then talk of *an* object, which endures and perhaps changes, viz., the universe ; but not of a number of distinct objects. (iii) We might find, as we actually do, that the content of each second is distinguishable into different co-existing parts, and that a certain part of the content of one is hooked on to a certain part of the content of the next by close qualitative resemblance. Under this head I include resemblance of shape and position, as well as resemblance of colour, temperature, etc. We should then say, not only that there is a history of the world as a whole, but also that there are various distinct strands of history. Each strand would be called the history *of* such and such an object, but this does not mean that there is another existent, viz., "the object," beside the strand itself. It is only because there are such strands that we can talk of a plurality of objects. The world as a whole would have a history, partly because it is composed of such strands of history. But its history is more than the sum total of a number of distinct strands lying side by side. If there be causal and other regularities which hold throughout the whole period under discussion, there

will be characteristic relations between the strands, and the history of the world as a whole would have more unity and complexity than is implied by the simple statement that it is composed of such and such parallel strands.

Whenever we talk then of "objects," the fundamental fact is the existence of distinct strands of history. A given object is a certain strand, pervaded by a certain special unity and continuity, which characterise it and mark it out from strands of other kinds. To say that a certain object has not changed in any respect is to say that all the successive slices of a certain strand are qualitatively indistinguishable from each other. An unchanging object is thus a *completely uniform* strand of history. To say that a certain object has moved, but has not otherwise changed, is to say that the positional qualities of successive slices of a certain strand are progressively different. A moving object is therefore a *positionally non-uniform* strand.

Now it happens, of course, that there are many distinct strands which are so much alike in the characters of their slices, and in the type of unity that pervades them, that they are called histories of objects of the same kind. Yet some of these strands may be positionally uniform, whilst others are positionally non-uniform. An example would be given by a resting and a moving penny. Again, a strand which has enough unity and continuity throughout to count as the history of a single object may yet for some part of its length be positionally uniform and for others be positionally non-uniform. An example would be a penny which sometimes keeps still and sometimes moves. I think that it is partly in consequence of such facts that we tend to separate objects from their histories, and to treat their histories as something more or less external, which may or may not "happen to" them. A given penny really is a certain definite strand of history, positionally uniform if it be a resting penny, positionally non-uniform if it

be a moving penny, and so on. But you can always find plenty of other strands of history sufficiently like this one in their non-positional qualities to be called histories of pennies, and yet uniform where this history is positionally non-uniform. You tend to identify the first penny with a uniform history, such as the second penny, and to regard the non-uniform part of the first penny as something that "happened to" it, but was not a part of it. The real fact, however, is that the first penny *is* the first strand and nothing else, and the second penny *is* the second strand and nothing else. Of course the general characteristic of "being a penny" is common to both, since it is the general type of qualitative character which pervades all such strands; but this is a universal, not a particular existent; and when people talk of "objects," and say that they rest or move, they are certainly not primarily talking about universal characteristics but about particular existents.

It is evident then that every object has a time-dimension as well as any space-dimensions that it may have. There is nothing mysterious about this; it means no more than that every existing object, whether at rest or in motion, is a strand of history with some duration. The question whether it is a changing or an unchanging object is simply the question whether successive slices of the strand, normal to the time-dimension, are exactly alike or progressively different in quality. The notion of an object with nothing but spatial dimensions is an abstraction. You can divide up the object into thinner and thinner slices normal to its time-dimension, and these slices will approximate, as you make them thinner and thinner, to purely spatial figures. In the limit each will be a purely spatial figure, in general of three dimensions. But these *are* not the object, nor are they literally even *parts* of it. The object is the whole four-dimensional strand of history. And these momentary spatial figures are "parts" of the object only in the Pickwickian sense in

which plane sections of an ordinary solid are "parts" of the solid. A person who refuses to identify an object with its whole history must either identify it with a *momentary section* of that history or with a *uniform slice* of it. If he does the former, the object is a mere abstraction, incapable of existence. If he does the latter, his restriction to the uniform part of the whole strand of history is clearly arbitrary.

If it should happen that all the successive momentary sections of an object have the same shape, you can call this *the* shape of the object. But, if they have different shapes, there is nothing that can be called *the* shape of the object. A penny and a mist are both objects; but, whilst you can talk of the shape of the former, you cannot talk of the shape of the latter.

Motion and Rest in Visual Fields and Sense-histories.—After these general remarks about the nature of objects and their motion or rest, we can consider the various types of motion and rest which happen within our visual fields and sense-histories.

(*a*) *Motion and Rest of Visual Sensa.*—A single sense-field lasts for a finite, though short, time. Spatially it is of three dimensions. It is therefore a four-dimensional spatio-temporal whole. In sensing it, we thus sense directly a four-dimensional whole with three spatial dimensions and one temporal. A sensum is an outstanding part of the total content of a sense-field. It has some duration, which cannot be greater than that of the sense-field, and it has spatial extension. It is therefore in general a four-dimensional object. Now, as we have seen, a visual sensum may shift its position in its own field or not. If it does, it is affected with sensible motion, otherwise it is sensibly at rest. Thus *all* visual sensa are four-dimensional objects, and those that are affected with sensible motion are positionally non-uniform objects. Just as we cannot see at once an object of more than a certain size, so we cannot sense by one act an object

that exceeds the duration of a Specious Present, whether it be uniform or non-uniform. In sensing a resting sensum we are aware in one act of a positionally uniform four-dimensional object of short duration; in sensing a moving sensum we are aware in one act of a positionally non-uniform four-dimensional object of short duration. Thus, sensible motion is the way in which the positional non-uniformity of a four-dimensional object presents itself to us when this non-uniformity is "sharp" enough to be noticeable within the duration of a single sense-field.

(*b*) *Motion and Rest of Visual Sense-objects.* — Our successive visual fields join up with each other to form a single sense-history, as already described. This is simply a four-dimensional whole, of the same general nature as a single visual field, but of greater duration. It cannot, of course, be sensed as a whole, though some of its earlier slices may be remembered while its latest slice is being sensed. Now, when a certain resting sensum has occupied a certain position throughout the whole of one field, similar sensa may occupy exactly similar positions in a series of successive fields. Just as the fields join up to give one sense-history, of which they are successive slices, so these resting sensa join up to give a single sense-object, of which they are successive slices. This will be a positionally uniform sense-object, and may be described as a sense-object which *rests in the space of the observer's sense-history*.

Now it may happen that there is a series of more or less similar sensa in a series of successive fields, but that they occupy progressively dissimilar positions in their respective fields. And it may be that the thinner two fields are and the nearer they are together, the less is the dissimilarity between the positions of the sensa of this set which belong to these fields. On these conditions the sensa of the set still join up to form a sense-object of which they are successive slices. But this sense-object is positionally non-uniform, and may be

described as a sense-object which *moves in the space of the observer's sense-history*. Often there is no sensible non-uniformity in the individual sensa of such a group, although they join up to form a positionally non-uniform sense-object. On the other hand, it often happens that each of the component sensa of a moving sense-object is itself affected with sensible motion in its own field. It is reasonable to suppose that, even in the former case, the component sensa are really not quite positionally uniform objects, but that their departure from uniformity is not "sharp" enough to be sensed as movement within the sense-field.

Now, it is very important to notice that the movement of sensa in their fields and of sense-objects in the spaces of their sense-histories is the ultimate empirical basis of the concept of absolute motion. The sensible motion of a sensum in its field really is something absolute; it does not simply *consist in* the fact that this sensum alters its spatial relations to other sensa in the field, though, of course, it *involves* this as a necessary consequence. This is quite clear, from the following example: Suppose I am looking at the sky, and a shooting star darts across. I am aware of a field; and within this are sensa which are the appearances of the other stars, and a sensum which is the appearance of the shooting star. The latter is affected with sensible motion, whilst the former are not. Now, if the sensible motion simply consisted in a change of relative position within the field, it would be perfectly symmetrical, and it would be impossible to say that the shooting star sensum sensibly moves and that the other sensa do not. But it is quite clear that in fact we do sense an intrinsic peculiarity of the shooting star sensum which is not present in the others. Thus, sensible motion and rest are something absolute and intrinsic, not merely relational; and I take it that this fact is at the basis of the concepts of absolute motion and rest. It does not, of course, follow that the

concept thus formed really is *applicable* beyond sense-fields and sense-histories. It may well be that the absolute motion or rest of a sense-object in the space of my sense-history is connected with merely relative motion between my body and other physical objects. This does not alter the fact that the motion of the sense-object in the space of my sense-history is itself absolute, and not a mere change of relation to other contents of the history. We shall consider this question at a later stage in the chapter.

Correlations between the Motions of Visual Objects and the Kinæsthetic Sensations of an Observer.—The best way to approach this complicated subject seems to be by taking special cases as illustrations. Taking a single observer and a single physical object, we can begin by distinguishing four cases which constantly happen: (A) The observer stands still, and (i) watches a resting physical object, or (ii) watches a moving physical object. (B) The observer moves bodily, and (i) watches a resting physical object, or (ii) watches a moving physical object. These four cases must be distinguished from each other by certain differences in our sensible experiences, and I shall begin by pointing out the peculiarities of each in turn.

(A) There are two kinds of kinæsthetic sensation, one connected with walking, and the other with turning the head. I will call them respectively *translational* and *rotational* kinæsthetic sensations. The A-cases are all alike in the fact that the observer feels no translational kinæsthetic sensations.

(i) When a resting observer watches a resting physical object he finds that, once having turned his head so as to sense a field with a visual appearance of this object in the middle of it, he must henceforth keep his head still if he wants to go on sensing fields with similar sensa at their centres. That is, in order that the physical object may appear in his sense-history as a

resting sense-object, he must henceforth keep free from rotational kinæsthetic sensations. If at any moment he chooses to start turning his head, the physical object will still continue for a time to appear in his visual sense-history. But the visual sensa by which it appears will occupy progressively dissimilar places in his successive fields. Moreover, they may be affected with sensible motion within their fields. Thus, in this case, the physical object still appears, for a time at least, as a visual sense-object in the observer's sense-history. But its appearance is now a positionally non-uniform, *i.e.*, a moving, sense-object.

There are also certain points to be noticed about the shapes, etc., of the successive sensa in this sense-object. While the observer keeps his head still, the successive sensa will be indistinguishable in shape, unless, of course, physical changes are going on in the object. But when he moves his head, the successive appearances will differ in shape; they will be more and more distorted as he turns his head more, and as they occupy more eccentric positions in his successive fields. Thus, when he turns his head, the sense-object by which the physical object appears in his sense-history is not only positionally non-uniform; it is also non-uniform as regards shape. There is another difference between the successive sensa, which I will just mention here and deal with more fully later. They do not differ merely in the fact that each is a distortion of the original central sensum. Very often there is something in the later sensa to which nothing corresponded in the earlier ones, and conversely. This is the sensible basis of the fact which we express by saying that, as we turn our heads, "fresh parts of the object come into view, whilst others which were formerly visible cease to be so."

A final and very important point to notice is that, in the present case, by exactly reversing the series of rotational kinæsthetic sensations I exactly reverse the

series of sensa, and end up with a field like that from which I started, with a sensum like the original one in its centre. I can do this as often as I like, and always with the same result. Again, I can move my head from its initial position in a great variety of ways, which are distinguished for me by characteristic differences in my rotational kinæsthetic sensations. Each such way will involve a non-uniform sense-object of the kind described; and each, on reversal, will bring me back to a field like that with which I started. But there are characteristic differences of detail between the various non-uniform sense-objects which correspond to the various series of rotational kinæsthetic sensations.

(ii) When I stand still and watch a moving physical object, I find that I *must* keep turning my head if I want to keep the successive appearances of the physical object in the centres of my successive fields. And I must do this in a perfectly definite way. Moreover, there is a difference between the sense-object which I sense in this case and in the last. In the last case, if I keep my head still, I sense a *completely* uniform sense-object. In the present, the sense-object never is completely uniform; it is not even completely uniform in position. What we should find would be this: There would be a steady increase, a steady decrease, or the one followed by the other, in the sizes and depths of the sensa. There will be distortion in their shapes. There will be variations in brightness. And, finally, the later sensa will have parts to which nothing corresponds in the earlier, and conversely.

Suppose now that, at a certain moment, I stop moving my head. From that moment the successive appearances of the physical object will begin to occupy dissimilar positions in my successive fields. Very probably each will have sensible motion in its own field. And the distortion of later sensa, and the addition of new and dropping of old features, will be greatly accelerated. In fact, the physical object will

henceforth appear as an extremely non-uniform sense-object, both positionally and in other respects. Very soon it will cease to appear at all in my sense-history, *i.e.*, the later parts of the sense-history will be fields containing no sensa connected with this physical object. When this is so, I could, as a rule, start again at will to sense a field with an appearance of this physical object at its centre. In order to do this, I shall have to turn my head to a definite extent, independent of my choice. And, when I do at length sense another field with a sensum of the required kind in the middle of it, I shall find that this sensum differs in shape, brightness, depth, etc., from the one that was in the middle of the last field which I sensed before I stopped turning my head.

(B) The B-cases resemble each other, and differ from the A-cases, in that the observer experiences translatory as well as rotational kinæsthetic sensations.

(i) If a man walks, and wants to keep his eye on a resting physical object, he will find that he must continually turn his head as he walks. And the amount of rotational kinæsthetic sensation needed is correlated with the amount of translational kinæsthetic sensation experienced. Provided he turns his head properly, the physical object will appear in his sense-history as a partly, but only partly, uniform sense-object. It will not be uniform in depth or brightness. There will also be distortion and revelation of new parts. But the sensa will be at the centres of his successive fields. If he walks, and keeps his head and eyes fixed, the physical object will appear in his sense-history as a moving sense-object, and possibly the constituent sensa may have sensible motion in their respective fields. The non-uniformity in respect of shape will be very much greater than when he keeps his eye on the physical object, and soon this will cease to appear at all in his sense-history. After it has disappeared he can again sense a field with a sensum of the group

at its centre, provided he turns his head properly. The amount of rotational kinæsthetic sensation needed for this purpose will be completely determined by the nature and amount of translational kinæsthetic sensation which he has experienced since he ceased turning his head. Lastly, the sensum which will occupy the middle of his present field will never be exactly like that which occupied the middle of the field which he was sensing when he stopped turning his head. There will be differences in shape, depth, brightness, etc. ; and there will be parts to which nothing corresponded in the last sensum, and conversely.

It is obvious that, on the visual side, there is a close analogy between B (i) and A (ii), *i.e.*, between the visual experiences of a moving observer watching a resting object and those of a resting observer watching a moving object. There is also a partial resemblance between the rotational kinæsthetic sensations, since both of them are obliged to keep moving their heads in a certain way in order to keep the appearances of the physical object in the centres of their successive fields. The difference is that in A (ii) the rotational kinæsthetic sensation needed is absolutely independent of the observer's volition, whilst in B (i) it is indirectly dependent on his volition. It is primarily dependent only on the amount and kind of his translational kinæsthetic sensations ; but these in turn are dependent on his will, since he can walk as he chooses. This gap, however, is bridged by the case of observers whose bodies are carried about in trains, motor cars, etc. Their movements do not involve translational kinæsthetic sensations, and here the analogy between B (i) and A (ii) becomes practically complete. Such facts as this analogy lie at the basis of the concept of the relativity of physical motion.

(ii) When an observer moves about and keeps his eye on a moving physical object he will find that the nature and amount of kinæsthetic sensation needed are

determined partly, but only partly, by his translational kinæsthetic sensations. He will sometimes have to turn his head more quickly, and sometimes less quickly than if he were walking in the same way and keeping his eye on a resting physical object. If he were to retrace his steps, and then walk over his old course again, it would be useless to repeat the same head-movements which he made on the previous occasion. If he did this, it is very likely that the physical object would no longer appear in his sense-history at all; and, even if it did so, it would certainly not appear in the form of a sense-object whose successive sensa occupied the centres of his successive fields.

There is a very important point to notice about these B-cases. In them the observer has both translational and rotational kinæsthetic sensations. Now these fall into pairs of correlated series in the following way: The successive appearances of a physical object can be kept at the centres of one's successive fields in an infinite variety of different ways, all of which involve different combinations of translational and rotational kinæsthetic sensations. Take first a resting physical object. (α) Its successive visual appearances can be kept in the centres of one's successive visual fields by suitably turning the head and henceforth moving neither the head nor the body. (β) A similar result (though not an identical one) can be produced by walking in innumerable different ways, and at the same time continually turning the head in correlated ways. Lastly, (γ) there is one and only one way of walking without turning the head which will produce similar results, though, of course, this one way may be pursued at different rates. This is what we call "walking straight up to the object." (α) and (γ) are two extreme cases of the huge group included under (β). It must be noticed that the various combinations of correlated rotational and translational kinæsthetic sensations are not absolutely equivalent in their results on the sense-object by which the physical object

appears in the observer's sense-history. The (α)-method gives a completely uniform sense-object. Each of the (β)-methods gives a somewhat different sense-object. All these sense-objects are non-uniform in shape and depth; for different component sensa will have different depths in their respective fields. Moreover, there is always that difference between successive sensa which we describe by saying that we "see fresh parts and lose sight of some which we saw before." Lastly, the (γ)-method gives a sense-object which is uniform, in the sense that there is no distortion between the successive sensa which constitute it. But each of these sensa has a larger size and a smaller depth than the one before, whilst there will be a progressive increase in brightness. In spite of this, there may be the difference which we should express by saying that the earlier sensa "reveal parts of the physical object which cease to be revealed by the later ones."

Somewhat similar remarks apply to the correlation between rotational and translational kinæsthetic sensations in watching a moving physical object. But there are certain differences. (α) Its successive appearances *cannot* be kept in the centres of our successive fields if we neither walk nor turn our heads. (β) If we choose to do both, there are innumerable combinations of the two which will produce the required kind of sense-object. But the rotational kinæsthetic sensations which have to be combined with a given set of translational sensations for this purpose are not the same as they would be if we were looking at a resting object. In fact, no general rule of correlation can be laid down without bringing in an additional factor, viz., the motion of the physical object itself. (γ) There is one and only one way of keeping the successive appearances of a moving physical object in the centres of our successive fields without continually turning our heads, and that is, of course, by walking parallel to its line of motion at a suitable pace. The particular series of kinæsthetic sensations

needed for this purpose varies, of course, with the motion of the particular physical object which is being watched. By the (γ)-method, and by it alone, does a moving physical object appear to us as a completely uniform sense-object.

There is thus a close resemblance between the cases A (i) and B (ii) (γ). So far as the visual object is concerned, they are precisely alike. The difference is that in A (i) a completely uniform sense-object requires complete absence of both kinds of kinæsthetic sensation, whilst in B (ii) (γ) it requires a characteristic series of translational kinæsthetic sensations. The gap here is to some extent bridged, as in the analogy between A (ii) and B (i), by the fact that an observer's body may be carried parallel to another physical object without effort of his own. This happens, *e.g.*, when an observer in a moving train keeps his eye on a certain window of a carriage, moving at the same rate and in the same direction on a parallel line. Here we have another sensible fact which lies at the basis of the concept of the relativity of physical motion.

(*b*) *Summary of Facts elicited in the last Sub-section.* We have been discussing the sensible experiences, both visual and kinæsthetic, which make an observer say sometimes that he stands still and watches a resting body, sometimes that he stands still and watches a moving body, sometimes that he moves and watches a resting body, and sometimes that he moves and watches a moving body. The most important general conclusion that emerges is that there is a mixture of arbitrariness and compulsion in all such cases, and that it is the particular character of the mixture which causes us to make now one and now another of these four types of statement.

(i) I can always, if I choose, sense a series of visual fields, each of which contains an appearance of an assigned physical object at its centre. (ii) I can always, if I choose, sense a series of fields in which successive

appearances of the assigned physical object occupy progressively more dissimilar sensible positions. But (iii), once I have decided which kind of sense-object I want to sense, conditions are *imposed* on my kinæsthetic sensations, which I must simply accept. And these imposed conditions vary from case to case. Sometimes I must keep my head and body still if I want to sense a completely uniform sense-object; sometimes I must move bodily to secure this result. If the latter, I cannot move just as I like; only one way of moving will secure the result in a given case, and the right way will vary from occasion to occasion. Then (iv) there are various mixtures of rotational and translational kinæsthetic sensations which will cause the physical object to appear as a *partially* uniform sense-object with its successive sensa at the centres of my successive fields. But (v) the sense-object will not be uniform in depth, shape, brightness, etc. And (vi) not every mixture of translational and rotational kinæsthetic sensations will secure even this result. If I arbitrarily choose to experience a certain series of translational kinæsthetic sensations, the amount and speed of the rotational kinæsthetic sensations needed will always be partly and sometimes wholly determined by the former series. Similar remarks apply, *mutatis mutandis* if we arbitrarily choose a certain series of rotational kinæsthetic sensations. (vii) Sometimes when we deliberately confine ourselves to rotational kinæsthetic sensations, *i.e.*, when we deliberately stand still and merely turn our heads, we find that as often as we completely reverse the series a qualitatively unchanged appearance of the given physical object occupies the centre of our final visual field. On other occasions we find that, if we have once turned our heads and thus ceased to sense an appearance of a certain physical object at the centre of our field, mere reversal of the original series of rotational kinæsthetic sensations will not suffice to restore a similar field. In such cases the amount and kind of rotational

kinæsthetic sensation needed for the purpose are independent of our choice, and vary from one object to another. (viii) When, in spite of our best endeavours, the physical object fails to appear in our visual sense-history as a completely uniform sense-object, the kind of non-uniformity in depth, shape, brightness, etc., which it displays is independent of our choice. It is determined *partly* by the particular mixture of translational and kinæsthetic sensations which we have chosen out of the whole set which will keep the successive appearances in the centres of the successive fields. As a rule, it is not *wholly* determined by this, but is partly determined by another factor which is quite independent of us. This other factor is what we come to know as "the physical motion of the body at which we are looking."

It is this mixture of arbitrary choice and subsequent external compulsion which is at the basis of our distinction between "objective physical motion and rest," and "subjective sensible motion and rest." I shall now go into this important matter a little more fully, taking some important special cases which we have so far touched on only incidentally.

(*c*) *Successive Sensible Appearances of Co-existing Physical Objects.*—We have already seen that, when a physical object moves away from us while we stand still and keep our eyes on it, it never appears in our sense-history as a *completely* uniform sense-object, although its successive appearances are in the centres of our successive fields. I am not at present concerned with the non-uniformity of the sense-object in respect to depth or brightness. Nor am I now concerned with that kind of non-uniformity which may be described as "distortion" of the successive appearances as compared with the appearance in some standard field of the sense-history, *i.e.*, with the kind of variation which takes place in the successive appearances of the upper surface of a penny as it moves away from us while we keep our

eyes on it. What I want to discuss is that kind of change which we describe by saying that, as time goes on, we see parts of the object which we could not see before, and cease to be able to see parts of it which we could see before.

As far as our visual sensa are concerned, there is no particular difficulty in describing such experiences. We sense a series of sensa which have enough continuity with each other to count as successive slices of a single sense-object. But, although closely adjacent sensa of the series are barely distinguishable in quality, those at some distance apart differ in the following way: The earlier has some parts to which nothing corresponds in the later, and the later has some parts to which nothing corresponds in the earlier. The real problem is this: These sensa are *successive;* when the last is present the first is past. But we suppose that the part of the first to which nothing corresponds in the second, and the part of the second to which nothing corresponds in the first, are appearances of *co-existing* parts of the physical object. Why do we assert physical co-existence on a basis of sensible succession? Since the spatial parts of physical objects are themselves physical objects, and the spatial parts of sensa are themselves sensa, we may generalise the problem as follows: Under what conditions do two *successive* sensa justify us in asserting the existence of two *contemporary* physical objects?

This question is, of course, roughly equivalent to a very famous one discussed by Kant in the *Analytic of Principles* of his *Critique of Pure Reason.* I think that Kant hit on one very important part of the answer, but that other important factors are involved beside the one which he stresses. Moreover, the Sage of Königsberg did not number clearness of exposition among his many merits, so that it will be well worth while to discuss the whole question afresh. Let us take a very simple concrete example. From where I am sitting,

if I look straight in front of me, the middle of my visual field is occupied by an appearance of a certain picture. The rest of the field consists almost wholly of a cream-coloured background, which is an appearance of the wall. In this field there is nowhere an appearance of a door. If I turn my head enough to the left I sense a field whose general background is much as before. But, in the middle of it, is an appearance of a door, and nowhere in it is there an appearance of the picture. From where I sit it is impossible for these two physical objects to be represented by simultaneous visual appearances in a single field. Nevertheless, I judge them to co-exist, although their appearances are always successive.

Now, first of all, what does my judgment of co-existence really profess to assert? It does not, I think, mean that the part of the history of the picture which appears to me when I look in one direction, and the part of the history of the door which appears to me when I look in the other direction, are contemporary. If physical objects exist and endure, they must be strands of history, just as sense-objects are, *i.e.*, they must be extended in time. And a sensum is presumably an appearance of a short slice of the history of a physical object. Now, apart from complications about the velocity of light, it is reasonable to suppose that successive sensa are appearances of successive slices of physical history; and I think we always do assume this in the absence of special reasons to the contrary. Thus the judgment that the picture and the door co-exist, although their appearances are successive, does not mean that the successive appearances reveal contemporary slices of their histories. What it means is this: The history of the picture has gone on while I turned to the door; and, when the door appears to me, there is a slice of picture-history contemporary with the slice of door-history which now appears to me, and practically indistinguishable in quality from the slice of picture-history

which appeared to me when I last looked toward the picture. Conversely, the door-history extends backwards from the slice which is now appearing to me; and there is a slice of it which is contemporary with the slice of picture-history which appeared to me when I formerly looked at the picture. So what we are really asserting is that the picture-history extends forward for some time with practically no qualitative variation after the last slice that has appeared to me, and that the door-history extends backwards for some time with practically no qualitative variation before the first slice that appeared to me.

Now, I have already said that I do not profess to be able to *prove* that such assumptions are *ever* true. If anyone says that the existence of long strands of physical history of almost uniform character does not follow logically from the *mere* existence at certain times of picture-sensa and at other times of door-sensa, I heartily agree. I can only answer that we all do, in fact, assume that sensa are appearances of short slices of things which last longer than themselves, and that we can neither refute this assumption, get rid of it in practice, nor stir a step without it. What we can do, however, is to state the special conditions under which we hold that successive sensa are appearances of co-existing physical objects (in the sense defined above), and show that, subject to the general assumption just mentioned, these conditions are reasonable.

I find that over a long period of time I sense a practically uniform picture-sense-object, whenever I look in a certain direction. Moreover, I can look away and then look back again after all kinds of different intervals, and I still find a similar sense-object. Exactly similar remarks apply, *mutatis mutandis*, to the sense-object by which the door appears to me. Now, theoretically, there are four possibilities: (i) My looking in a certain direction is a sufficient as well as a necessary condition for producing a field with a picture-sensum in the middle of it. (ii)

The occurrence, at a certain moment, of a field with a picture-sensum at the middle of it, is a necessary and sufficient condition of my turning my head at that moment in a certain direction. (iii) There is a certain event which (α) causes me to turn in the given direction whenever it occurs, and (β) produces the picture-sensum at the same time. (iv) The head-turning, and the production of the sensum when I have turned, are the results of two causally independent series.

We will first give familiar examples of these various possibilities. Suppose that on a certain day I pass a certain building several times at various intervals, and that on each occasion a brick falls on my head as I pass. It might be (i) that my passing shakes down a loose brick, which would not otherwise have fallen. Or (ii) that whenever I see that a brick is about to fall, I am so much interested that I rush to the spot, and that nothing else ever takes me there. Or (iii) that I go to the place when and only when a workman who is working there calls me, and that he throws down a brick when and only when I get there, because he is a "class-conscious proletarian" and regards me as a "lackey of the bourgeoisie." Or (iv) it might be that my journeys to the place and the falling of the bricks belong to causally independent series. Now I might be able to cut out the first three alternatives by reflecting on the facts. I might know that I am not heavy enough to shake bricks down by passing underneath. I might know that I had not gone to the place because I saw that a brick was going to fall, and I might know that no workman had called me or thrown the bricks at me. I might be able to explain why I had passed there on each occasion without needing to refer to anything going on at the place whatever.

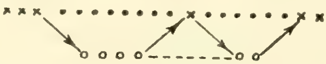
Supposing that this is so, only one explanation of the facts would be reasonable, viz., that a fairly steady stream of bricks has probably been falling for most of the day. It is almost incredible that each of my visits

to the place should happen to coincide with the fall of a brick, granted that the causes of the visits and of the falls are quite independent, unless many more bricks fall than the few that I happen to "stop." Now let us apply this argument to the sensible appearance of the picture and of the door. It is certain that merely to look in a given direction is not sufficient to produce one particular sensum in the middle of my visual field; for at other times I can look in the same direction and sense no such sensum (*e.g.*, if someone has moved the picture). It is also certain that the occurrence of the sensum does not make me turn my head in that direction; on the contrary, I often turn my head simply in order to see whether I shall again sense the same kind of sensum as before. And, in general, I know why I turn my head on each occasion, and can see that my act is completely determined by causes which have no discoverable connexion with the causes which produce the sensum in the middle of my field when I do turn. I am therefore forced to conclude, either that there is a pretty continuous strand of very similar picture-sensa, of which I sense the particular one which happens to be occurring when I turn my head, or at least that there must be a pretty steady stream of similar physical events, each of which is sufficient to produce a sensum of the required kind whenever my eye is turned in the right direction. Which of these two alternatives is to be accepted does not much matter for the present purpose, and the question must be left to the next chapter. On either alternative we are justified in concluding that there is a persistent and practically uniform "picture-object," slices of which fill up the gaps between my successive picture sensa. On the same grounds I am justified in supposing that there is a persistent and practically uniform "door-object," slices of which fill up the gaps between my successive door-sensa.

Now let us suppose that I start by looking at the picture, and then turn my eyes several times between

the picture and the door, ending up finally with the picture. We will suppose that I do this at different rates on different occasions, also that I sometimes dwell for a time on one of the objects without moving. Let us represent picture-sensa by little crosses, door-sensa by little circles, and the lapse of time by a direction from left to right. Then my sensible experience may be represented by the diagram below.

Now let us represent the physical events which appear as picture-sensa by dots, and those which appear as door-sensa by little lines. Then the argument from causal independence, applied to both objects, justifies me in filling out my sensible experience as indicated below.



A slightly more dangerous argument would justify me in extrapolating to some extent, *i.e.*, in assuming that the history of the door and that of the picture extend backwards for some distance before my earliest door- and picture-sensa. It would also justify me in supposing that the history of the door extends forward for some distance after my last door-sensum. For, unless there be some special reason to think otherwise, it is highly improbable that I should happen to have looked first in the door- or the picture-direction just when there first began to be door or picture events. And it is highly improbable that door events ceased to happen just when I happened to turn my head in the picture-direction for the last time. Like all extrapolations, this argument is weaker than an intrapolation, and its probability is quickly diminished as it is extended further before the first sensum of one series or after the last sensum of the other series.

The argument for co-existence is now quite straightforward. There is a slice of picture-history between my first and last picture-sensum. And there is a slice of door-history between my first and last door-sensum. But my first door-sensum is after my first picture-sensum and my last door-sensum is before my last picture-sensum. Hence the interpolated picture-history completely overlaps the interpolated door-history, as the second diagram shows. I believe this to be the truth underlying Kant's rather confused argument in the *Analytic of Principles*; but that is a purely historical question in which I take no particular interest.

There are, however, at least two other criteria of physical co-existence in face of sensible succession. One of these can be dealt with only when we have considered our knowledge of our own bodies. The other may be mentioned at once. I am not obliged to stay in one place. While I sit in my chair at the table it is true that the picture and the door can only appear successively in my sense-history. But, if I move backwards to the other side of the room, I can sense a single field with a picture-sensum at the middle, and a door-sensum to the left. These sensa co-exist, and they are extremely like the corresponding sensa in my successive fields when I was nearer the wall. They are smaller, and have greater depth; otherwise there is very little difference. As I approach the wall on which the picture is hanging, keeping my eye on it, I first sense a series of fields with both the door and the picture-sensa in each of them. As I go on, the door-sensum is more and more to the extreme left of its field, and more and more distorted. At last there comes a point where the field does not contain any appearance of the door. The two kinds of sensa can now only be sensed successively. Now the co-existent sensa were presumably appearances of contemporary slices of two overlapping strands of physical history. And the subsequent successive sensa are so much like the former simultaneous ones, that it

is reasonable to suppose that the same pair of strands of physical history continue, and continue to overlap in time, although contemporary slices can no longer appear in my sense-history.

Similar remarks apply to looking at a physical object and gradually feeling its surface. It is true that the tactual sensa are successive, and yet that I take them as informing me about the shape of the physical object at *some one moment*. But we find that we can make the tactual sensa follow each other in various series at will, provided we initiate suitable series of kinæsthetic sensations. And we can repeat any of these series as often as we like. Meanwhile, the visual appearances keep constant, and we sense a completely uniform visual sense-object. In whatever order we sense our tactual sensa, they are connected with a part of the visual appearance at the time. It is difficult to resist the conviction that we are dealing with a uniform strand of physical history, and that each of our tactual sensa reveals a bit of some slice of it. True, the slices revealed by successive tactual sensa are presumably successive; but then the uniformity of the visual sensa-object suggests that they are all alike in their spatial characteristics. Hence, what we learn by touch about *different parts of successive slices* may be put together to tell us about the *whole of any one slice*. Here, again, there are certain facts about our experiences of our own bodies which reinforce this interpretation.

(*d*) *Single Observer Watching two Physical Objects in Relative Motion*.—In the last sub-section we were really dealing with the case of one observer who watches two physical objects which are at rest relatively to each other and to his body, but which cannot both be seen at once. Let us now consider the case of an observer who watches two physical objects, which are in motion relatively to each other. As we have already seen, the observer will always be able to make one of these physical objects appear as a uniform sense-object,

whose successive sensa are at the centres of his successive fields, provided he moves suitably. We can therefore simplify matters by supposing that one of the bodies appears in the observer's sense-history as a completely uniform sense-object. Let this body be A. It may be that at first he will sense a series of fields in which both A and the other body B appear as sense-objects. If so, he will notice that B does not appear in the form of a uniform sense-object. Each sensum of the sense-object by which B appears, will very likely have sensible motion in its own field. Again, successive B-sensa will occupy more and more eccentric positions in their respective fields and will be more and more distorted. Thus A and B appear at first as two sense-objects which overlap in time, *i.e.*, as two overlapping strands in the observer's sense-history. But, if we take successive pairs of contemporary slices of the two strands, we shall find a progressive variation in their respective sensible distances apart. Sensum a_r and sensum b_r in the field f_r have a certain sensible distance d_r. This is slightly greater than d_{r-1}, the sensible distance between a_{r-1} and b_{r-1} in the field f_{r-1}. And it is slightly less than d_{r+1}, the sensible distance between a_{r+1} and b_{r+1} in the field f_{r+1}. In fact, if you take the two sense-objects together as forming a kind of composite sense-object of a higher order in the observer's sense-history, it has the peculiar kind of non-uniformity which I have just been describing. And this kind of non-uniformity is characteristic of the relative motion of sense-objects.

Now as time goes on the sensa of the B-sense-object will occupy more and more eccentric positions in their respective fields, till at length no more sensa of the B-kind appear in the observer's sense-history. After this, he will still be able to sense appearances of A and of B, provided he turns his head ; but he will no longer be able to sense them in a single field : they must be sensed successively or not at all. Let us now compare and contrast this with the cases discussed in the last sub-

section. (1) Obviously the later stages of this case bear a certain resemblance to the last; *i.e.*, in both, the observer can only sense appearances of the two physical objects successively. One important difference is that this situation has developed out of one in which he could sense appearances of both objects together. And it has developed *independently* of the observer; it is not due to any changes of bodily position that he has made. In the previous case, if he *started* by being able to sense appearances of the two objects in the same field, he *went on* being able to do so, unless he deliberately moved nearer to the two objects. (2) It is true that, in the present case, if the observer chooses to walk backwards quickly enough, he can again sense fields in which both A and B appear. But, whereas in the former case he could continue to sense the two appearances together by merely walking a certain distance backwards and *stopping* there, he will now find that he must *keep on* walking backwards if he wants to keep on sensing fields in which both the objects appear. It is thus clear that in this case there is a lack of reversibility, due to the operation of some external condition, which is not present in the former cases. The externally imposed condition is evidently something of the nature of a continuous *process*, with a rate and direction of its own, which, if it is to be compensated for at all, must be compensated for by another appropriate continuous process in the observer's body. The interpretation of this process as *movement* is rendered almost inevitable by the fact that, so long as A and B are appearing under the form of two sense-objects with contemporary slices in each of the successive fields of a sense-history, there is sensible relative motion between these sense-objects, as described above. (3) Finally, the irreversibility of the present, as compared with the reversibility of the last case, shows itself in another way. When I dealt with two resting physical objects which I could see only successively, I could always pass from the field containing an appearance of

A at its centre to the field containing an appearance of B at its centre, and back again, by a mere reversal of my rotational kinæsthetic sensations. And the *amount* of turning needed was quite independent of the *rate* at which I turned, or the time for which I *dwelt* on one of them before turning to the other. With the relatively moving physical objects this complete reversibility breaks down. The position here is as follows: If I turn from A to B on one occasion, a reversal of the process will indeed bring me back to A. But, if I now repeat the process, the amount of turning will always be greater than before, and it will be greater the longer I have dwelt on A. Again: If I turn too slowly, I shall not be able to pick up an appearance of B at all; and, if I turn quickly enough to do this, then the quicker I turn the less amount of turning will be needed. Lastly, the minimum quickness needed will be correlated with the swiftness of the relative motion between the sense-objects of A and B, when both these co-exist in my sense-history.

(*e*) *Rotation.*—For the sake of completeness I must say something about rotation, and for the sake of brevity I shall say but little. It will be fairly easy for the reader to work out the details for himself by analogy with what has already been said. I have so far assumed that we were looking at objects which either rested altogether or moved with a purely translatory motion in space. Let us now consider the experiences of an observer who stands still and watches a rotating physical object which is translationally at rest. He will be able to keep its successive appearances in the centres of his successive fields without needing to have either translational or rotational kinæsthetic sensations. But the sense-object, which is the appearance of the rotating physical object in his sense-history, will be far from uniform. In the first place, each of the sensa may have sensible rotation (a quite peculiar and characteristic sense-quality) in its own field. Then, although closely

successive appearances will be very much alike, there will always be a part of the later to which nothing corresponds in the earlier, and conversely. In this respect the sense-object which is the appearance of a rotating body bears some resemblance to the sense-object by which a moving, but non-rotating, body appears in the sense-history of an observer who follows the body with his eye by turning his head.

There is, however, an important difference. After a time the series of sensa will begin to repeat itself in the same order, and it will do this again and again. We may say, then, that a rotating body, which keeps in the same place and is looked at by a resting observer, appears in his sense-history as a *positionally* uniform, but *periodic*, sense-object. Now it is possible for a non-rotating body to appear as a periodic sense-object, and for a rotating body to appear as a non-periodic sense-object. But in each case the observer will have to "walk round" the body; and, as he does so, suitably turn his head at each moment. "Walking round" a body appears in the sense-experience of the observer as a peculiar series of kinæsthetic sensations. If he wants a rotating physical object to appear in the form of a completely uniform sense-object, he must walk round at a perfectly definite rate, which depends on circumstances over which he has no control. Thus, again, we are forced to the conclusion that there are external processes of change, connected with changes in our visual sense-histories; and that certain definite series of kinæsthetic sensations are the signs of processes of change in our own bodies which are "equivalent to" these, in the sense that they compensate for them and give a uniform sense-object.

(*f*) *Summary of Results of the present Section.*—The upshot of our discussion on the correlations between visual motion and rest and the kinæsthetic sensations of a single observer seems to be as follows: (1) In dealing with a single physical object we can generally

arrange at will whether it shall appear in the form of a positionally uniform or a positionally non-uniform (*i.e.*, moving) sense-object. But (2) in order to do this, we must sometimes initiate series of kinæsthetic sensations, and must sometimes refrain from doing so. Sometimes a physical object will appear in my sense-history as a uniform sense-object, if and only if I refrain from starting a series of kinæsthetic sensations. If so, it will appear as a non-uniform sense-object when I do initiate any such series. And the nature of the non-uniformity will depend wholly on the nature of the series which I choose to carry on. (3) Sometimes a physical object will appear in my sense-history as a uniform sense-object if and only if I initiate a certain series of kinæsthetic sensations. If so, the appropriate series is fixed for me. If I do not carry out one of the group of appropriate series, the physical object will appear as a non-uniform sense-object, whose particular non-uniformity depends *partly*, and *only* partly, on me and my kinæsthetic sensations. Having made up my mind whether I want a physical object to appear as a uniform or a non-uniform sense-object, I have to conform to conditions which are imposed on me. And these conditions vary from one case to another. (4) Now a series of kinæsthetic sensations in me is presumably an appearance of a certain process of change in my body. I know that this process is *one* condition which produces non-uniformity of sense-objects in my sense-history; for in many cases I do sense a uniform sense-object so long as I *refrain* from having kinæsthetic sensations, and it becomes non-uniform so soon as I *start* to have such sensations. Conversely, I know that in many other cases sense-objects have the same kind of non-uniformity when I have *no* kinæsthetic sensations, and that this non-uniformity can be eliminated if I *start* a suitable series of kinæsthetic sensations. It therefore seems reasonable to suppose that the other set of conditions, to which I have to conform, is another

process of the same general character as that in my own body which is revealed to me by my kinæsthetic sensations. In fact, it seems probable that the positional uniformity or non-uniformity of the sense-object by which a certain physical object appears to me, depends in general on the co-operation of *two* sets of physical processes, one in my body and the other in the physical object; and that the latter process is of the same general character as the former, which is revealed to me by my kinæsthetic sensations. (5) Of course it remains a question whether these processes should be regarded as *motions*, and, if so, in what Space and what Time they happen. For the present all that we can do is to make the following tentative suggestion: Two different physical objects often appear as two temporally overlapping sense-objects throughout a long tract of my sense-history. One may be positionally uniform and the other not; if so, one of the sense-objects will be in sensible relative motion to the other. Let A be the physical object which appears as a uniform sense-object α; and let B, the other physical object, appear in my sense-history as the non-uniform sense-object β. From what has gone before, I conclude that the uniformity of α depends on certain processes (or, in the limiting case, on the absence of such processes) in my body and in A. Similarly, the positional non-uniformity of β depends jointly on certain processes in my own body and B. Since the process in my body is common to both, it seems certain that there must be a difference between the A-process and the B-process; for otherwise there is no apparent reason why α should be uniform and β non-uniform. Thus a *difference* between the processes in A and B is correlated with sensible *relative motion* between α and β, the two sense-objects by which A and B appear in this tract of my sense-history. Conversely, if A and B had both appeared as uniform sense-objects, a similar argument would show that there is no reason to assume that there is any difference

between the relevant physical processes in A and B. Thus sensible *relative rest* between α and β, the sense-objects by which A and B appear in this tract of my sense-history, is correlated with *identity* of the processes in A and B.

This, I think, is about as far as we can go without entering into further detail about the human body as a physical object, and our knowledge about it. When we have done this, we shall find that the general conclusion (4), and the more special conclusion that the physical processes on which the uniformity or non-uniformity of visual sense-objects depends are of the nature of motions, are greatly strengthened. We will, therefore, make this the subject of our next section.

The Human Body as a Physical Object. — Human bodies may be, as we are told that they are, "temples of the Holy Ghost"; in which case it must be admitted that the Third Person in the Trinity sometimes displays a strange taste in temples. But, whatever else they *may be*, they certainly *are* physical objects as much as chairs or tables. Nevertheless, they do occupy a peculiar position among physical objects. In the first place, each is connected in a perfectly unique way with an observing mind, which looks out at the rest of the world from its body. Secondly, each of these minds has a peculiar knowledge of its own body, which it does not have of any other body in the universe. A given mind perceives every other body except its own in exactly the same way as it perceives a chair or a potato. It perceives its own body, partly in this way, and partly in a quite different way, viz., by organic sensations. Lastly, the minds connected with various human bodies can and do constantly communicate with each other, so that observer A learns that observer B perceives B's body in the same way in which A perceives his own body. A also learns that B can no more perceive A's body in this way than he himself can perceive B's body in this

way. I believe that these peculiarities of human bodies and of our knowledge about them are essential factors in founding the common-sense and scientific notions of physical objects, and in developing the concepts of physical Space, Time, and Motion.

(*a*) *A Solitary Observer's Perception of his own Body.*—(1) I do not know very much about my own body directly by sight, but I do know something. I cannot see my own head at all, though by means of a mirror I can see an incomplete optical object in a different place, and I now conclude on various grounds that it is very much like the optical constituent of my head. I can see the front of my trunk from a little below the chin; can see my hands and feet often quite distinctly; and can see less distinctly the upper parts of my arms. The greater part of the visual appearance of that fraction of my body which does appear in the visual field is very vague and distorted.

There are two important points to notice about the visual appearances of my trunk. (i) Although they are so fragmentary, they are almost invariably present in my visual sense-history. To sense a visual field with no such sensa in it, I have to follow the advice given to the "happy band of pilgrims," and "look upward to the skies," in a most unnatural and uncomfortable way. In fact, my own trunk appears to me as a highly uniform and highly persistent visual sense-object. Whenever I carry on a series of translatory kinæsthetic sensations the greater part of the contents of my later fields bears no resemblance to that of my earlier fields. But the visual appearances of my body are present with little variation throughout. (ii) The other peculiarity is that all the visual appearances of my trunk have a very small visual depth in all the fields. They are at the extreme "front" of each field, and the visual appearances of all other physical objects are "behind" them at various greater depths in the field.

Now, with other objects that appear in my visual

sense-history, I have to initiate a certain series of translatory kinæsthetic sensations before I can sense any correlated tactual sensa. As this series goes on, the visual depths of the successive sensa, which together make up the sense-object, steadily decrease in each successive field. But, as I have said, the visual appearances of my own body have a practically constant minimal depth in all my successive visual fields. Thus, when I walk up to a resting physical object, there are two sense-objects which co-exist throughout this tract of my sense-history. One is the sense-object by which the distant physical object, to which I am walking, appears. This is positionally non-uniform, in so far as the successive sensa that belong to it have progressively diminishing depths in their respective fields. There are also correlated variations in size, brightness, etc. The other is the sense-object by which my own body appears in my sense-history. This is practically uniform, since all its successive sensa have minimal visual depth. Thus, successive pairs of contemporary sensa, one from one sense-object and the other from the other, have progressively smaller visual distances apart. So the series of translatory kinæsthetic sensations, experienced in walking up to an external physical object, is associated with sensible relative motion between the sense-object which represents the external body and the sense-object which is the appearance of my own body in my visual sense-history.

(2) My tactual sensations of my own body are peculiar. (i) As I have said, most physical objects which appear in my visual sense-history can only be touched after an appropriate series of translatory kinæsthetic sensations. If this series be reversed, we soon cease to be able to sense any tactual sensa correlated with our visual sensa. But we do not need to walk in order to touch our own bodies; and, having once touched them, we do not cease to be able to do so by walking away. In fact, all other tactual sense-objects

are rigidly bound up with series of translatory kinæsthetic sensations; but the tactual sense-object which represents my body is indifferent to all such series. This must be correlated with the fact that translatory kinæsthetic sensations make no difference to the depths of successive visual appearances of our own bodies, whereas they do make a difference to the depths of the successive visual appearances of nearly all other physical objects. My trunk is the *only* physical object which appears throughout the *whole* of my visual sense-history as a positionally uniform sense-object; and it is the only physical object which I can touch whenever I like, *i.e.*, which I need not walk up to and cannot walk away from.

(ii) The tactual sensa which I sense when I touch my own body are characteristically different from those which I sense when I touch any foreign body. Suppose that in each of two successive visual fields of my history there is an appearance of my hand. In the first, let this be in visual contact with an appearance of my table, and in the second let it be in visual contact with an appearance of my leg. Apart from minor qualitative differences there will be the fundamental difference that, in the second case, I "feel my leg being touched" as well as "feel my leg with my finger." This peculiar experience of "double contact," as it is called, helps me to distinguish the surface of my own body from those of all other physical objects. It also helps the solitary observer to fill out the very fragmentary knowledge of his own body which he would have if he were confined to visual appearances alone. He can feel a closed surface, marked out by the characteristic of double contact; and can gradually explore its contours. Only a very small part of these tactual sensa will be correlated with his visual sensa. But I can start with a visual appearance of my hand visibly in contact with a visual appearance of some part of my trunk, and can gradually move my hand so that its successive appearances in

successive fields are nearer and nearer to the extreme edge of the appearance of my trunk. At length I shall no longer be able to see my hand ; but the characteristic tactual sensa will still be sensed, and they will be continuous with those earlier ones which *were* correlated with the visual appearance of my hand visibly in contact with the visual appearance of part of my trunk. Finally, as I go on moving my hand, it may become visible again ; and its visual appearance will again be in visible contact with the extreme edge of a visual appearance of part of my trunk. My own body is thus known to me by tactual exploration as a closed surface which resists my efforts to penetrate it, like any other physical object. But it is marked out from the other closed surfaces that I feel by the qualitative peculiarity of the tactual sensa, and by the fact that I do not have to walk up to it and cannot walk away from it.

(3) We come finally to a most important peculiarity of our sense-experience of our own bodies. I am constantly getting mild tactual sensations from the whole surface of my body without actively exploring it with my hand. These come from the contact of my clothes, from air-currents, and so on. In each Specious Present they form a mass which is the largest part of what I will call the *somatic field*. These somatic fields are, in the main, extremely alike over long periods of time ; they thus join up with each other to form an extremely uniform *somatic sense-object*. Within each somatic field certain characteristic sensa stand out ; *e.g.*, at one time I may itch in one place, and at another time I may feel a burn at another place, and so on. Now literally "inside" the somatic fields there are from time to time outstanding bodily feelings, like headaches and toothaches and stomach-aches, which enliven my somatic history and prevent it from being perfectly tame and uniform. Again, my kinæsthetic sensations are sensible events with places in my somatic fields. Thus a peculiarity of my body is that I have sense-perception

of events which happen in its *inside*, as well as of events on its outside. Of course, events in the inside of my body appear to me in a very peculiar way, viz., by kinæsthetic sensations, bodily pains, etc. But the insides of *other* bodies do not appear to me in sense-perception in any way whatever, unless, of course, I cut them open or "turn them inside out." And if I do this, I am not perceiving their insides while they are inside, but am only perceiving new outsides, which for various reasons I take to be exactly similar to *former* insides.

(*b*) *Several Intercommunicating Observers watching each other's Bodies.*—If I were and had always been a completely solitary observer, these facts about my body would not help me very much to form the concept of physical objects, having insides as well as outsides, occupying positions in physical Space, and moving about in it as physical Time elapses. I should rather be inclined to stress the differences between my own body and all other objects that appear to me, and leave the matter there. But I am not in this solitary situation. The important fact is that there are other people like myself, whose bodies I can see and touch, and with whom I can exchange notes by verbal communication and gestures. I am convinced that this fact plays a vitally important part both in the development of the general concept of physical objects, and in the development of the connected concepts of physical Space, Time, and Motion.

Any other human body is perceived by me in exactly the same way as I perceive a stone or a chair. If I look at it, it appears as a characteristic visual sensum in the middle of my visual field. I can then approach it and sense correlated tactual sensa. And there is no essential difference in the experiences which I have in this case and in that of an ordinary inorganic object. Similarly, I perceive the motion or rest of another human body in precisely the same way as I perceive those of any other external object. But I recognise that other human

bodies are connected with minds like my own; and, although I can only know their bodies from the outside, they tell me that they know them from the inside, and that they know mine only from the outside. I understand what they mean, because of my own experiences, described in the last sub-section. I thus come to recognise that there are plenty of other bodies beside my own, having internal processes; although I cannot *perceive* these processes in any body except my own. So the fact that I cannot perceive such processes elsewhere ceases to be any reason for supposing that they do not *exist* elsewhere. I know that they happen in *my* body, although other people tell me that *they* cannot perceive them; and I am therefore ready to believe that they happen in *other men's* bodies, though *I* cannot perceive them; since they tell me that *they* can do so.

The logical position is therefore as follows: (i) I know what is *meant* by internal processes from my own sense-experiences of pleasures, pains, kinæsthetic sensations, etc. (ii) I believe that there are other *instances* of bodies with such internal processes, from communication with other minds; though I cannot myself perceive these processes in the other instances. (iii) I then *extend* this conception that bodies have "insides," in which all kinds of interesting events happen, from human bodies to others, which, so far as I know, are not connected with minds. (iv) This is reasonable, because they appear to me in exactly the same way as do all human bodies except my own; and I already know, from the instances of other human bodies, that the non-appearance of internal processes to my senses is quite compatible with the fact that such processes are going on. I thus conceive that *all* my sense-objects are appearances of physical objects, which have an inner history of their own, and are seats of internal processes in the way in which human bodies are the seats of those processes which appear to the minds connected with

them as headaches, toothaches, kinæsthetic sensations, etc. How far in *detail* the analogy is to be pressed is of course another question, which can only be gradually answered by empirical investigation. I propose now to apply these general considerations, first to the general concept of physical objects, and then to the more special concept of physical motion and rest.

(*c*) *The Human Body as the typical Physical Object.*—Intercommunication with other human minds, and observation of the appearances of their bodies, fill out the general concept of physical objects in the following ways:

(1) Any of the sense-objects by which other physical objects appear to us is liable to sudden interruptions. The visual sense-object comes to an end in darkness, or when we shut our eyes or turn our heads away. And the tactual sense-object exists only when we are at or near a certain place. But, in spite of these interruptions in the sensible appearances of other men's bodies in my sense-history, the minds connected with these bodies tell me that their somatic history has gone on all the time with very little change. Thus, in the case of human bodies, I have reason to believe that their inner history is much more permanent and continuous than their appearances in my sense-history. I extend this conclusion by analogy to non-human bodies, which appear in the same kind of way in my sense-history. This argument is strengthened by the fact that I know that my own somatic history is going on steadily at times when other men tell me that my body has ceased to appear in their sense-histories.

(2) I know that I can initiate noises, bodily movements, etc., and that when I do so they are preceded by special series of events in my somatic sense-history. Other people tell me that they hear noises, see movements, and so on, at the centre which is the optical place of the visual appearances of my body. Similarly, when I hear noises or see movements connected with

the place occupied by the optical constituent of another man's body, he will tell me that he has been "making" the noises or movements. This means that he produced them by initiating an appropriate series of sensible events in his somatic history. Thus we arrive at the general conclusion that many changes in the visual appearances of A's body in B's visual sense-history are connected with changes in A's somatic sense-history. Now the latter are appearances to A of physical events within his own body. Thus, in the case of a human body, we reach the notion that the place which is *optically* occupied by its optical constituent is *physically* occupied by certain events which produce changes in this optical object, or at any rate in parts of it. This is the crude beginning of the notion of scientific events and their connexion with sensible appearances. We extend this result in the usual way to those places which are optically occupied by complete optical objects which are constituents of non-human bodies. That is, we conclude that these places are physically occupied by certain events which are responsible for the changes that take place from time to time in the complete optical object.

(3) The comparative constancy of my somatic sense-history, combined with the fact that no one can "see" the whole surface of my body at once, supports the view that *successive* visual sensa often justify a belief in *co-existing* physical objects, or parts of one physical object. No one can see my face and the back of my head at the same time, though there may be an appearance of each of these in successive visual fields of the same observer. But I know that my somatic history includes "face-feelings" and "head-feelings" in *each* of its successive fields. Thus, although the observer's visual sensa were successive, and presumably revealed non-contemporary slices of my body-history, yet there is reason to suppose that *each* of these slices (and all that came between them) included a part corresponding

to the appearance of a head, and a part corresponding to the appearance of a face.

These seem to be the main factors which our perception of our own bodies and our intercommunication with other observers supply to the concept of physical objects in general. The human body is *the* physical object *par excellence;* with an "inside" which is continually, if inadequately, perceived by its own mind through bodily feelings; with an outside which is perceived on and off by other observers through their visual and tactual sensations; and with internal processes, which reveal themselves to its own mind as kinæsthetic and other bodily feelings, and reveal themselves to other minds as movements and other changes in its visual and other appearances. Each observer reaches the notion of human bodies as complete physical objects by combining his own experiences of the inside of his body with what other observers tell him about their experiences of the outside of his body. He then extends the general conception, thus formed, to non-human physical objects, which cannot tell him about their own insides.

(*d*) *The Human Body and the Concept of Physical Motion.*—In the section on the correlations between kinæsthetic sensations of a single observer and the motion or rest of visual sense-objects in his sense-history, we made no special assumption as to the nature of the physical objects which he was watching. They might be other human bodies, or they might be inorganic bodies, like pennies or chairs. Even so, we reached the following results, of which I will remind the reader: (i) That this observer might reasonably conclude that the positional uniformity or non-uniformity of the visual sense-object, by which a certain physical object appears in his sense-history, depends in general on the co-operation of two processes, one in his own body and the other in the physical object which he is watching. The one in his own body appears to him

in the form of a series of kinæsthetic sensations in his somatic sense-history. And it is reasonable to think that the other is of the same general nature. (ii) That this observer might reasonably hold that a certain identity between such processes in two physical objects A and B involves relative rest between them, and that differences between the two processes involve relative motion between A and B.

Now these conclusions, which are rendered highly plausible by the mere correlations between a solitary observer's kinæsthetic sensations and the motion or rest of his sense-objects, are greatly strengthened when the physical objects which he watches are the bodies of other observers who can communicate with him.

(1) Suppose that observer α watches B, the body of observer β, and that at the same time observer β watches A, the body of observer α. The correlations between the kinæsthetic sensations and the visual sense-objects of each observer are of exactly the same kind as if he were watching an inorganic body. But, in the present case, the observer and the observed can compare notes about their kinæsthetic sensations and their visual sense-objects. Let us first suppose that α does not have to keep turning his head in order to keep his eye on B, and that B appears to him as a completely uniform visual sense-object. Then β will tell α that he, too, does not need to keep turning his head in order to keep his eye on A, and that A appears in his sense-history as a completely uniform visual sense-object. If they now compare their translatory kinæsthetic sensations, they will find either that they are absent in both, or, if present, are of precisely the same character.

Let us next suppose that α finds that he has to keep turning his head in order to keep his eye on B. B will then appear in α's sense-history as a partly, but only partly, uniform sense-object. The nature of its non-uniformity has already been fully described. Now β will also find, and will tell α that he finds, that he must

keep turning his head in order to keep his eye on A, and that A appears in his sense-history as a partly, but only partly, uniform sense-object of the kind already described. In this case α and β will find, on comparing notes, that they both experience a series of rotational kinæsthetic sensations, and that there is an *analogy* between them. But, on the other hand, they will always find that there is a *difference* between their translatory kinæsthetic sensations. This will sometimes take the form that one and only one of them has such sensations at all (I am leaving out of account for the sake of simplicity observers who are carried about without effort in trains or motor-cars). There is one other important point which they will discover on comparing their experiences. The appearance of α's head in β's sense-history will be a *rotating* visual sense-object, and so will be the appearance of β's head in α's sense-history. Thus each will discover that, of his two kinds of kinæsthetic sensation, one is correlated with a *rotationally* non-uniform sense-object by which his head appears in the sense-history of the other observer, and the other kind is correlated with a *positionally* non-uniform sense-object, by which his body appears in the sense-history of the other observer.

(2) So far, we have confined ourselves to two observers α and β respectively watching B and A, the bodies of the other. Let us now take an observer γ, who watches the bodies A and B of the two observers α and β, who can communicate with him and with each other. As we have said before, if γ keeps up a suitable series of kinæsthetic sensations, he can always make A appear in his sense-history as a completely uniform sense-object, each of whose successive constituent sensa is at the middle of its field. We will suppose that γ does this. He may then find either (i) that B appears as a completely uniform sense-object, or (ii) that B appears as a positionally non-uniform sense-object. Each of the component sensa in this may have sensible movement

in their fields. And, even if they do not, successive pairs of contemporary A- and B-sensa will have progressively different sensible distances in their respective common fields in γ's visual sense-history.

Now, in case (i), α and β will tell γ that, on comparing notes with each other, they find no difference in their translational kinæsthetic sensations, which may, of course, in the limiting case both be non-existent. In case (ii), α and β will tell γ that, on comparing notes, they do find a difference in their translational kinæsthetic sensations. If one of them has no such sensations the other will have them. Moreover, each of them will tell γ that the body of the other appears to himself as a non-uniform sense-object. And γ's body C will appear in β's, though not in α's, sense-history as a non-uniform sense-object.

Now these communicated experiences (1) and (2) leave no doubt at all that the positional uniformity or non-uniformity of the sense-object, by which one human body appears in the sense-history of another observer, depends jointly on those physical processes in the two bodies which are revealed to their respective minds in the form of kinæsthetic sensations. Moreover, they show clearly that *uniformity* in the sense-object depends on a certain *identity* of quality and quantity in the two processes, whilst positional *non-uniformity* in the sense-object depends on certain qualitative and quantitative *differences* between the two processes. Lastly (2) shows that relative motion of the sense-objects by which two human bodies appear in the sense-history of a third observer depends on a *difference* between these two processes in the two human bodies, whilst relative rest of two such sense-objects depends on an *identity* of character between the two processes.

We now extend this conclusion in the usual way to physical objects which are not connected with minds that can communicate with us. We assume that, in *all* cases, the uniformity of a sense-object in the sense-

history of an observer depends upon a certain identity between that physical process in his own body which appears to him as a series of kinæsthetic sensations, and another physical process of the same general type, which happens in the physical object of which this uniform sense-object is the visual appearance in the observer's sense-history. And we assume that, in *all* cases, the positional non-uniformity of a sense-object in the sense-history of an observer depends on differences between the physical process in his body which appears to him as a series of kinæsthetic sensations, and another physical process of the same general type, which happens in the physical object of which this non-uniform sense-object is the visual appearance in this observer's sense-history.

(*e*) *Several Intercommunicating Observers watching the same Physical Object.*—One more very important fact remains to be described. Suppose that two observers, α and β, are watching a certain physical object O, and that a third observer γ is watching their bodies, A and B. It may happen that O appears in α's sense-history as an uniform sense-object, and that it appears in β's sense-history as a positionally non-uniform sense-object. If this be so, the observer γ will always notice that the sense-objects by which A and B appear in his sense-history are in relative motion to each other. And, as usual under these conditions, there will be a difference in the translational kinæsthetic sensations of α and β. If we generalise this from human bodies to all physical objects we reach the following conclusion : It is possible for any physical object to appear at once as a uniform sense-object in the sense-history of one observer and as a non-uniform sense-object in that of another observer. But, if it does so, it will always be found that there is some difference between those physical processes in the bodies of the two observers which appear to them as series of their kinæsthetic sensations.

This result, which can actually be observed, might

also have been deduced from what has gone before. If the physical object O appears as a resting sense-object in α's visual sense-history, this implies a certain *identity* of character between the relevant physical processes in A and in O, according to the argument of the last sub-section. If O appears as a moving sense-object in β's sense-history, this implies a *difference* between the relevant physical processes in B and in O, on the same principles. It follows at once that, under these circumstances, there must be a difference between the relevant physical processes in A and in B. And this should appear to α and to β as a difference between their kinæsthetic sensations. That such a difference is *actually* found supports the conclusions of the last sub-section, since they are here used as hypothetical premises from which it follows that such a difference *ought* to be found.

In the next chapter I propose to apply the results of this one to the notions of sensible and physical Space-Time, and so to end my treatment of the spatio-temporal aspects of Nature and their sensible and perceptual basis.

The following additional works may be consulted with advantage :

G. F. STOUT, *Manual of Psychology*, Bk. III. Part II.
W. JAMES, *Principles of Psychology*.
KANT, *Critique of Pure Reason* (*Analytic of Principles*).
SCHOPENHAUER, *World as Will and Idea*, Vol. I. Bk. II.

CHAPTER XII

"And nu bit and for Godes naman halsath ælcne thara the thas boc rædan lyste thæt he for hine gebidde, and him ne wite gif he hit rihtlicor ongite thonne he mihte. Forthæmthe ælc mon sceal be his ondgites mæthe and be his æmettan sprecan thæt he spreeth and don thæt thæt he deth."—KING ALFRED, *Preface to his Translation of Bœthius.*

Sensible and Physical Space-Time

WE have at length reached a position where it becomes possible to deal with the concept of physical Space-Time, from which, as we shall see, the concepts of physical Space and of physical Time are abstractions of two different kinds. We shall thus finally work back, from a wholly different starting-point, to the position which we reached at the end of Part I.

Let us first take a backward glance over the country that we have crossed, and see how the universe looks from our present standpoint. We shall then be able to see what part of our journey from crude sensation to the refined concepts of mathematical physics remains to be completed; and, having done so, we can try to complete it.

(*a*) *Statement of the Present Position.*—The situation, so far as it has now developed, is roughly as follows: There is a world of physical objects, some of which, like my own body, are connected with observing minds which can communicate with each other. Others, so far as we know, are not connected with minds; but in their general character they are very much like those which are. Correlated with each human body there is a general sense-history, which is split up into several special sense-histories, visual, tactual, auditory, somatic,

and so on. We can sense *temporal* relations between sensa in our different special sense-histories, just as we can sense temporal relations between different sensa in the same special sense-history. But we cannot sense *spatial* relations between contemporary sensa in our different special sense-histories, though we can sense such relations between contemporary sensa of the same special history. These spatial characteristics are much more marked in the visual sense-history than in any of the others.

My somatic sense-history contains sensa which are appearances of internal states and processes of my own body. In my other special sense-histories are various sense-objects, some uniform for a time, others non-uniform. There are correlations between certain sense-objects in my different special histories which lead me to regard them as different kinds of appearances of the same external physical object. All these remarks about me and my sense-histories apply equally, *mutatis mutandis*, to other observers and their sense-histories ; as I learn by intercommunication.

Between sensa in the histories of different observers neither spatial nor temporal relations can be sensed by either of the observers or by any third observer known to us. But there are correlations between certain sense-objects of different observers which lead us to say that the same physical object is appearing to all of them. When this is so, there is generally a certain external place which all these sensa may be said to "occupy" in some Pickwickian and definable sense, such as optical occupation. Again, there are certain methods, discussed in the last chapter but one, by which some sensa of different histories are grouped together as "neutrally simultaneous," and others are grouped apart as "neutrally successive."

Then there are the very elaborate correlations between the uniformity or non-uniformity of sense-objects in the visual histories of observers, and certain events

in their somatic histories called "kinæsthetic sensations." We have been studying these in the last chapter. We came to the conclusion that the positional uniformity or non-uniformity of the sense-object by which a certain physical object appears to an observer, depends upon certain physical processes in the external object and the observer's body; and that these processes in one's own body appear to oneself as kinæsthetic sensations. A more careful study of these correlations revealed two further closely connected points. One is that the positional uniformity of a sense-object depends on an identity of character between these two physical processes, and that positional non-uniformity is correlated with certain differences between them. The other is that relative rest between two sense-objects in a sense-history depends on a similar kind of identity between two such physical processes in the bodies which appear as these two sense-objects, whilst relative motion between two sense-objects is correlated with similar kinds of difference between two such physical processes in the bodies which appear as these two sense-objects. Sensible motion and rest are *absolute*, but they seem to depend on *relations* of identity and difference respectively between physical processes in the body which appears and the body of the observer to whom it appears.

(*b*) *Statement of the Remaining Problem.*—These, then, are some of the facts which have so far been elicited, and some of the highly probable inferences which have been made from them. The next thing is to state clearly the problem which still remains. The rest of the problem is to make, if possible, a further synthesis by analogy with what we already know. Can we treat the world of physical objects and events as forming a whole which is analogous to a single sense-history? That is: Can we regard scientific objects as analogous to sense-objects; can we suppose that they have spatial relations to each other, such as we can *sense* only between sensa in a single sense-field; and can we suppose that

they endure, and have temporal relations to each other, such as we can *sense* only between sensa within a single general sense-history? Lastly, can we suppose that physical objects rest and move in this spatio-temporal physical whole, as sensa do in their fields, and as sense-objects do in our sense-histories? This, I think, is the real problem about physical Space, Time, and Motion. It is the problem of constructing a single, neutral, public Space-Time of physical objects and events, on the analogy of the many personal private space-times of the various observers' sense-histories.

Now it is not, of course, a question of just making such suppositions in the abstract. Our only possible justification for supposing anything of the kind is that it provides a scheme which summarises all the known correlations between sensa, and is, at the same time, familiar to us because of its analogy to our own sense-histories with which we are directly acquainted. It is theoretically possible that no such supposition would do justice to the actual correlations among sensa. It is still more likely that no supposition which made the structure of physical Space-Time *exactly* analogous to that of an individual sense-history would account for the known facts. Again, if the physical world can be consistently regarded as a spatio-temporal whole with considerable, though not complete, analogy of structure to an individual sense-history, it is probable that this can be done in a number of alternative ways, all of which will synthesise the known facts equally well. Even if up to a certain date human beings had only happened to think of one view of the structure of physical Space-Time, there is no reason to doubt that, if they thought more carefully and paid less attention to certain traditional points of view, they would be able to devise dozens of alternative structures for physical Space-Time equally capable of doing justice to all the known correlations among sensa. No doubt the physical world has a certain absolute intrinsic structure; and this structure

exhibits itself, in part at least, in the correlations between sensa of the same and of different observers. But we have to treat this structure piecemeal in the sciences of geometry, chronometry, kinematics, dynamics, and electro-magnetics, and by making suitably correlated modifications in the axioms of these various partial sciences we can express the same absolute structure in innumerable different and equally satisfactory ways. If, so far, very few alternative schemes have been proposed, this is due to nothing more recondite than lack of scientific imagination and the imperfection of our technical mathematical and logical apparatus.

It is, nevertheless, an interesting and important inquiry to see how far we can do justice to the known facts by supposing that the structure of the physical world is analogous to that of our sense-histories, and to see what is the minimum difference of structure between the two which we must postulate. For, after all, our physical concepts have their roots in our sense-histories.

It is evident that it might be possible to regard the physical world as forming a spatio-temporal whole analogous in general outline to a single sense-history, and yet that we might have to postulate differences of detail. I do not mean by this simply that the *contents* of the two might be different. It is perfectly certain that they will be. The ultimate contents of a sense-history are the sensa of the observer whose sense-history it is. The ultimate contents of physical Space-Time are scientific events. Even if it should be possible to regard scientific events as composed of sensa (which is far from certain), each scientific event will be composed of sensa from the histories of many different observers, and also presumably of many more sensa which do not belong to the history of any observer. Thus, even on this hypothesis, the ultimate contents of physical Space-Time will be *groups of correlated sensa*. But, beside this difference which there certainly must be between physical Space-Time and any sense-history, there may well be

a difference of *structure* between the two, *e.g.*, the kind of difference which there is between a Euclidean and a hyperbolic space. A sense-history and the physical world are both four-dimensional spatio-temporal wholes, and we must therefore talk of their *geo-chronometry* rather than their geometry. What I am saying then is that, although a sense-history and the physical world may be so far analogous in structure that we can say that both have a geo-chronometry of some kind, yet the geo-chronometries of the two may differ in detail.

The reader must beware of supposing that a Space-Time is an entity which exists in its own right, side by side with its contents. It is often convenient to talk as if this were so, and it does no harm, provided we recognise that it is always an abbreviated expression, and understand clearly what it is an abbreviation for. Having got rid of the absolute theories of Space and of Time, we must not introduce them again for Space-Time. Many really eminent writers on the Theory of Relativity have expressed themselves in a most unfortunate way, which suggests to innocent readers that they think of Space-Time as a particular existent, with properties of its own, which acts on matter like a cue acts on a billiard-ball. When we talk of the properties of physical Space-Time we are simply enumerating certain very general structural characteristics of that spatio-temporal whole which is the physical world. The only existent under discussion is this whole, which is composed of scientific events bound together in a characteristic unity by spatio-temporal relations.

An analogy will perhaps make this clearer than much discussion will do. The French and British armies are two elaborately organised hierarchies. Their contents are different; since the former is composed of Frenchmen, and the latter of Englishmen and Scotsmen and a few items from the Celtic Fringe. There is a great analogy between the organisations of the two,

which renders it reasonable to call them both armies. But there are also considerable differences in detail. If a military writer set out to describe in general terms the structure of the French army and that of the British army, he would be studying something akin to two systems of geo-chronometry. He could do this without referring to particular French and English soldiers, such as Jacques Bonhomme and Tommy Atkins. He could even talk intelligibly of the "effects" which these two types of organisation "produce" on French and English soldiers of various temperaments. But, if this led him to suppose that the organisations whose structure he is describing were substances that existed side by side with the soldiers, he would be talking nonsense; and it would be the same kind of nonsense as is talked by people who imagine Space-Time to be an existent substance which pushes and pulls bits of matter about. It must, therefore, be clearly understood that, when we talk of the geo-chronometry of Space-Time, we are simply describing certain very general and abstract structural features of that whole which is the physical world.

Since the geo-chronometry which is to be ascribed to physical Space-Time depends entirely on the correlations between our sensa, we must not be surprised if opinions about it alter with the growth of scientific knowledge. For one view might fit all the facts that were known up to a certain date, and a different view might be needed to fit both them and certain new facts which were discovered later. This is exactly what has happened in the change from Newtonian to Relativistic dynamics and kinematics.

(*c*) *The Concept of an Idealised Sense-history.*—If we want to see how closely the geo-chronometry of the physical world can be approximated to that of a single sense-history, we must begin by considering what is the geo-chronometry of a sense-history. But, before doing this, it will be well to remove in thought certain

limitations, which are, in fact, present in all our sense-histories, but which seem rather to depend on *de facto* limitations of our powers of sensing and remembering than on anything characteristic of the structure of sense-histories as such. (1) We can think of a sense-history as stretching back indefinitely into the past, although in fact we can only remember a certain distance back, and although presumably the history does not extend backwards beyond our birth. (2) We can remove in thought the limitation of a finite Specious Present. We can regard the fact that only a very thin slab can ever be sensed at once, and that the whole history is a series of such slabs, as contingent. That is, we can regard the whole history as a continuous four-dimensional strand. (3) We can remove in thought those limitations which our finite powers of seeing, hearing, etc., impose on the extension of each of our actual sense-fields. We can, *e.g.*, imagine the spatial limits of our visual fields indefinitely extended ; as they would be if we could see everything, however distant from our bodies. (4) We can also remove the limitation which is imposed by the fact that we cannot see all round us at once. (5) So far we have been conceptually extending our sense-histories by removing certain limits imposed by sensation and memory. It now remains to proceed in the opposite direction. We cannot sense fields of no duration. But we can sense events of shorter and shorter duration. We can thus conceive any slab of a sense-history as cut into thinner and thinner slabs. In the end we can conceive of slabs of *no* duration, and can imagine the whole sense-history analysed into an infinite series of such instantaneous slices, just as we can conceive a cylinder as analysed into an infinite series of parallel plane circular sections. Such momentary slices are not of course existents, and they are not literally parts of the sense-history ; but they can be defined by Extensive Abstraction, and a Pickwickian meaning can be given to the statement that the sense-

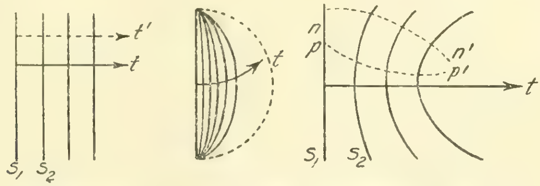
history is composed of them. These momentary slices will be *purely* spatial, whereas the sense-history as a whole and any finite real part of it are spatio-temporal. We may call each of these momentary sections a *momentary sense-space* in the given sense-history. By further applications of Extensive Abstraction within a single momentary sense-space, we could evidently define *momentary sense-planes*, *momentary sense-lines*, and *momentary sense-points*.

It is pretty evident that, if the physical world be analogous to a sense-history at all, it will be analogous to an idealised visual sense-history, extended conceptually in the ways described. And I think there is very little doubt that this is the original of the concept of the physical world as a whole in Space and Time. We must now consider more in detail the geo-chronometry of an idealised visual history. In the section that follows I am more than usually indebted to Whitehead, and I shall be contented if I provide the reader with "first aid" to the study of Whitehead's two great works on the philosophy of Nature.

The Geo-chronometry of an Idealised Visual History.—The idealised visual history is a four-dimensional spatio-temporal whole, formed by the continual addition of successive slices, which are idealised fields. Each of these slices has duration, and the duration of the whole history is the sum of the durations of the successive slices up to and including the last that has become. Now we can regard all these successive fields as normal to a certain straight line in the history, just as successive circular slabs of a cylinder are all normal to its axis. This common normal to all the fields may be taken as the time-axis of the history. By Extensive Abstraction we then reduce the temporal thickness of the successive slabs to zero, and we thus get a series of momentary three-dimensional spaces, all normal to the time-axis of the history.

Now the geo-chronometry of the history might,

apart from all wilder alternatives, be either Euclidean or elliptic or hyperbolic. According to which of these alternatives is realised, the geometry of its momentary spaces will be Euclidean or elliptic or hyperbolic. On either of the two latter alternatives the successive momentary spaces will not be parallel to each other. In elliptic geometry (which is analogous to the geometry of the surface of a sphere) there are no parallels, for all co-planar straight lines intersect each other twice. In hyperbolic geometry there are parallels and there are non-intersecting co-planar straight lines which are not parallel. And the common normals to a given straight line are not parallel to each other, though they do not intersect each other. It is only on the Euclidean alternative that the momentary spaces will be parallel. The three alternatives may be very roughly illustrated in two dimensions and on a Euclidean plane by the three diagrams below.



(i) Euclidean Case (ii) Elliptic Case (iii) Hyperbolic Case

(It must, of course, be remembered that what appears in these diagrams as *lines* normal to the time-axis represent three-dimensional *spaces* in the four-dimensional sense-history. Also that the *curves* in diagrams (ii) and (iii) are attempts at representing non-Euclidean *straight lines* on a Euclidean plane.)

We may perhaps dismiss the elliptic alternative at once. If the geo-chronometry of a sense-history were of this type, its time-axis, like all other straight lines in this geometry, would be a closed curve, like a great circle on a sphere. Whilst I see no theoretical impossibility in the time of Nature being of this kind, I think that

there is no evidence to support the suggestion. If it were so, the course of Nature would continually repeat itself in cycles. These might, of course, be of enormous duration; and so the fact that we have no empirical evidence for this alternative cannot be counted as evidence against it; we may make a present of the suggestion to the Dean of St Paul's and the Neoplatonists.

We will therefore confine ourselves to the Euclidean and the hyperbolic alternatives. On the Euclidean alternative there would be an infinite number of equally permissible time-axes for the sense-history, and these would all be parallel to each other. The line t' in (i) is an example. On the hyperbolic alternative, so far as my very limited knowledge of four-dimensional hyperbolic geometry may be trusted, I should say that there could only be one time-axis for the sense-history. It is true that there are plenty of straight lines in the history, parallel to t. The line pp' in (iii) is an example. But none of them will be normal to the momentary spaces which are normal to t, and therefore none of them could be taken as time-axes. Again, there are plenty of lines beside t which are normal to all the momentary spaces. The line nn' in (iii) is an example. But none of them are *straight* lines, and therefore none of them can be taken as time-axes. They are, in fact, curves called *horocycles*, and horocycles are to hyperbolic straight lines much as small circles are to great circles on the surface of a sphere. I do not think that the uniqueness of the time-axis suffices to show that the geo-chronometry of an idealised sense-history *could not* be hyperbolic; but we shall see later that the Space-Time of Nature could hardly be supposed to have one single unique time-axis, even apart from the Theory of Relativity. Hence, we had better work out the geo-chronometry of the idealised sense-history on the Euclidean hypothesis, since we want it only as a basis for the geo-chronometry of physical Space-Time.

There is a more positive reason for rejecting the hyperbolic alternative for the idealised sense-history. In the Euclidean case, since the normals to the time-axis are parallel to each other, and since Euclidean parallels are everywhere equidistant from each other, any slab of the sense-history, bounded by two such normals, has the same thickness throughout (*see* Fig. (i) above). In the hyperbolic case the normals diverge from each other on both sides of the common time-axis. The result is, that it is only on the Euclidean alternative that a Specious Present would have one definite limited duration. On the hyperbolic alternative sensa, far from the centres of a field, could last for enormous stretches of time, remaining in a single Specious Present. This seems to be contrary to fact. So, on every ground, it seems reasonable to take the geo-chronometry of the idealised sense-field as of the Euclidean type.

We can now advance to the very important conception which Whitehead would call the *timeless space* of the idealised sense-history. When we talk of objects resting or moving in a space, we clearly cannot be thinking of a *momentary* space. For both rest and motion involve lapse of time. We must, in fact, be thinking of some kind of space which lasts for the whole time under consideration, and does not change as the time flows on. This is what Whitehead means by a *timeless space*. We have now to define such a space for the idealised sense-history.

Let us imagine a completely uniform sense-object which lasts throughout the whole of the sense-history. As we slice the history up into thinner and thinner sections we shall, *ipso facto*, be slicing this sense-object into thinner and thinner sections, all exactly alike and all occupying precisely similar positions in these fields. Finally, by Extensive Abstraction, we shall reach a series of successive momentary spaces, each containing a momentary section of the uniform sense-object. All these momentary sections will be exactly alike, and

exactly similarly situated in their respective momentary spaces. If, now, we imagine the spatial dimensions of the uniform sense-object reduced more and more, so that, finally, it is the history of a mere point, it is clear that the object reduces to a line parallel to the time-axis of the sense-history. Each point in this straight line is in one of the momentary spaces of the history, and each of the momentary spaces contains one of the points. And these points are in corresponding places in their respective momentary spaces. Thus any straight line in the sense-history which is parallel to the time-axis, is the history of a sense-object of punctual spatial dimensions, which rests in a single "place" throughout the duration of the history.

We may therefore say that every *straight line*, parallel to the time-axis of a sense-history, is a *point* of the timeless space of the history. The timeless space of the history thus consists of the whole bundle of straight lines in the history which are parallel to its time-axis.

We have now to define the *straight lines* of the timeless space. To do this, let us imagine a sense-object which is positionally non-uniform and of punctual spatial dimensions. It is evident that it will consist of a series of points, one in each of the successive momentary spaces. But these points will not occupy corresponding positions in their respective momentary spaces, since the object is positionally non-uniform. Thus the whole assemblage of them will be a curve of some kind in the sense-history. It will, in general, be a tortuous curve; and it will, of course, *never* be a straight line parallel to the time-axis, for that would be the history of a positionally *uniform* punctual object. Again, it will, of course, never be a line in any one momentary space, for it would then not be the *history* of any enduring object whatever. Now, through each of the points of this curve, there goes one and only one straight line parallel to the time-axis of the history. And each of these lines, as we have seen, is one point in the timeless space of the history. It

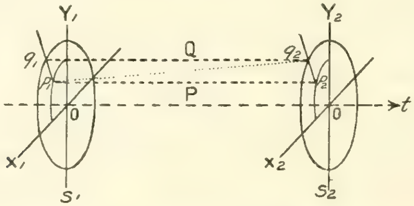
follows that the assemblage of all these lines is the course traced by the moving object in the timeless space. Such an assemblage of parallel straight lines will form a *surface* in the sense-history, which will not in general be flat. But, if the moving object happens to describe a straight line in the timeless space of the history, this surface will flatten out into a plane parallel to the time-axis. The easiest way to see this is the following: It is admitted that the *points* of the timeless space of a sense-history are *straight lines* in the history, parallel to its time-axis. Now a straight line is uniquely determined by two of its points. Now the only figure in the sense-history, which is uniquely determined by two straight lines parallel to the time-axis, is the plane which contains them both, and is, of course, itself parallel to this axis. It is thus evident that a *straight line* in the timeless space of a sense-history is a *plane* in the sense-history, parallel to its time-axis.

It remains to define the *planes* of a timeless space. A plane in the timeless space will be a figure uniquely determined by a straight line in that space, and a point which is in the space but not on the straight line. Now, we have already seen that a straight line in the timeless space is a plane in the history, parallel to its time-axis; and that a point in the timeless space is a straight line in the history, parallel to its time-axis. The fact that the point is outside the line in the timeless space is identical with the fact that the corresponding line is outside the corresponding plane in the sense-history. It follows at once that a *plane* in the timeless space of a sense-history is a three-dimensional region in the history, uniquely determined by a plane, parallel to the time-axis, and a straight line, also parallel to the axis but not contained in this plane. This is an unlimited region, which plays a corresponding part in a four-dimensional manifold to a plane in an ordinary three-dimensional space.

We have thus defined the points, straight lines and

planes of the timeless space of a given idealised sense-history in terms of certain special types of figures in the latter. These definitions are wholly due to Whitehead. It will be noticed (1) that the timeless point is something more complex than the momentary point, since it consists of a whole series of the latter; (2) some straight lines in the sense-history are also momentary straight lines in one of the momentary spaces; but no straight line in the history is also a straight line in the timeless space. At best, it can only be a *point* in the latter; (3) a timeless straight line is a *set* of straight lines in the sense-history, of a certain kind. Once the timeless concepts have been defined, the geometry of the timeless space can be worked out. It will be of the same character as the geometry of the momentary spaces of the history. For there is a one-to-one correspondence (though never an identity) between the timeless points, straight lines and planes, as defined above, and the momentary points, straight lines and planes of any one of the momentary spaces.

As a visual sense-history is a four-dimensional whole, it is not possible completely to illustrate all this on paper. But we can help the reader to understand the four-dimensional case by imagining a sense-history which has only three dimensions, two spatial and one temporal. The momentary spaces will then be planes at right angles to the paper, and we can illustrate the relations between sense-history, momentary spaces, and timeless space in the drawing given below.



In this picture S_1 and S_2 are two momentary sections of such a sense-history. The dotted line p_1q_2 is the straight line in the sense-history which represents the

history of a point-object, moving along a certain straight line in the timeless space of the history with a certain uniform velocity. The first momentary section of this object is the momentary point p_1 in the momentary space S_1. The last section of it is the momentary point q_2 in the momentary space S_2. Intermediate sections are momentary points in intermediate momentary spaces.

The dashed line p_1p_2 is the point P in the timeless space of the sense-history. The dashed line q_1q_2 is the point Q in the timeless space of the history. P would have represented the history of the punctual sense-object if the latter had stayed in its original position. Q would have represented the history of this object if the latter had always been in the position which it finally occupies. The plane $p_1q_1q_2p_2$, which is determined by the two straight lines P and Q, is the timeless straight line in the timeless space of the history which the moving punctual object traverses. It is uniquely correlated with the momentary straight lines p_1q_1 in S_1 and p_2q_2 in S_2, which might be called the "instantaneous directions of motion of the moving object at the two moments t_1 and t_2." These are of course similar, in the present case, since the object is moving all the time in one direction in the timeless space.

The angle between the dotted line p_1q_2 and the dashed line p_1p_2 depends on the velocity of the moving point-object in the timeless space. The histories of *all* moving points which traverse this particular line in the timeless space will be straight lines in the plane $p_1q_1q_2p_2$, but their directions in this plane will depend on the velocity with which the object traverses the line. If the velocity be non-uniform, they will, of course, no longer be straight lines; but they will still be plane curves in this plane. Naturally we cannot illustrate timeless *planes* in our diagram; for we can only get them in connexion with a four-dimensional sense-history, whose momentary sections are not planes, as in the diagram, but three-dimensional spaces. Also, there are no *momentary*

planes in our diagram, except the timeless spaces themselves.

(*d*) *Physical World-lines and their Mutual Relations.*—It is evident that such an idealised sense-history as we have just been describing would be a kind of "world," with a time, a timeless space, and objects which move or rest in the latter as the former flows on. The question now is: How far can the world of physical objects and events be regarded as forming a spatio-temporal whole, analogous in character to an idealised sense-history? If the analogy be complete, the physical world will have one time-*direction* (though many parallel *time-axes*), and one timeless Space, which will be of the Euclidean type. In this Space all physical objects will rest or move as the one physical Time flows on.

We must be prepared to recognise at once that it is by no means obvious that any such view of the structure of the physical world will fit the known facts. After all, why should the physical events and objects which are connected with a number of *different* sense-histories form a spatio-temporal whole which is exactly analogous in structure to a *single* sense-history? Even if there should be a certain analogy, we have not the slightest right to expect it to extend to every detail; *i.e.*, we have no right to be surprised if the geo-chronometry of physical Space-Time should not be exactly like that of the idealised sense-history. We shall see in a moment that most of the apparent paradox of the Theory of Relativity is due to the fact that it disappoints our simple-minded expectation that the geo-chronometry of physical Space-Time shall be exactly like that of a single idealised sense-history. But, on reflection, we see that this expectation is absolutely groundless, and that it would be rather a queer coincidence if the geo-chronometries of two such different wholes were exactly alike.

After these general preliminaries, let us see how far the analogy can be carried. A physical object is a succession of scientific events, just as a sense-object is

a series of successive sensa in a sense-history. A punctual sense-object, whether positionally uniform or non-uniform, is a line of some kind in its sense-history. If it be positionally uniform, and therefore rests in the timeless space of the sense-history, it is a straight line, parallel to the time-axis; if it moves, it is a curve of some kind on a surface generated by lines parallel to the time-axis, and so on. If then a punctual physical object can be regarded as analogous to a sense-object, we must suppose that it (or its history, if you prefer it) is a curve of some sort in physical Space-Time. We will call such a curve a "world-line," following Minkowski. All other material particles must equally be regarded as curves in physical Space-Time. We must next consider the intrinsic characters and mutual relations of world-lines, for the whole question of whether it is worth while to talk of a physical Space-Time at all depends on the nature of these.

Suppose that B, the body of observer β, appears as a resting sense-object in the visual sense-history of another observer α. We know that A, the body of α, will appear as a resting sense-object in the visual sense-history of β, provided that α's and β's kinæsthetic sensations are alike. The complete symmetry between α's experiences in connexion with B, and β's experiences in connexion with A, suggests that there is some great similarity in the world-lines of A and B. (Or rather in the world-lines which would represent their histories if they were reduced to punctual spatial dimensions.) It seems reasonable to suppose that, in such cases, we are dealing with pairs of intrinsically similar and similarly situated world-lines in physical Space-Time. We can conceive of groups of observers whose bodies form sets of similar and similarly situated world-lines. We will call these *sets of relatively resting physical objects*. We know that, if a certain body appears as a sense-object which moves in the timeless space of any *one* member of the set, it will appear as a sense-object which moves in the

timeless space of *each* member of the set. If it happens to be the body of an observer, we know further that his translatory kinæsthetic sensations will differ from those of all members of the set. Moreover, all the bodies of the set will appear to this observer as sense-objects which move absolutely, but rest relatively to each other, in the timeless space of his sense-history. It seems reasonable to suppose that the world-line of this observer's body is in some way different from those of the set in question. There might be an intrinsic difference in the nature of the curve, or some kind of difference in its situation or direction in physical Space-Time. A geometrical illustration of the first kind of difference would be given by a straight line and a hyperbola; an illustration of the second kind of difference would be given by two non-coplanar straight lines, or by two coplanar straight lines at an angle to each other.

We can now extend these suggestions in the usual way from the bodies of observers to physical objects in general. We can suppose that a set of relatively resting particles is a set of similar and similarly situated world-lines, and that any particle which moves relatively to this set is a world-line which differs, either intrinsically or in its situation in physical Space-Time, from the members of this set.

(*e*) *Straight and Tortuous World-lines.*—World-lines might be curves of many different kinds; some might be intrinsically very complex (like highly tortuous curves in ordinary space); others might be intrinsically very simple (like ordinary straight lines). It will be remembered that a punctual sense-object, which rests in the timeless space of its sense-history, is a straight line parallel to the time-axis of the history. Punctual sense-objects, which move in the timeless space of the sense-history, may be straight lines (though they need not be); but they are never parallel to the time-axis. We must see how far there is analogy to this in physical Space-Time.

If any analogy at all can be drawn between a sense-history and the physical world, we must assume (1) that at least some particles are *straight* world-lines; (2) that at least some of these straight world-lines are permissible directions for time-axes for physical Space-Time; and (3) that, by taking certain particles as having the characteristics (1) and (2), and by using suitable criteria of simultaneity, we can account for all the known general rules of spatio-temporal correlation among physical events. We will now see how far the analogy can be carried on this assumption.

A straight world-line which is a permissible time-axis for physical Space-Time will be analogous to the time-direction of a sense-history. If the whole physical world is to be analogous to a single sense-history, every momentary physical event must have one and only one straight world-line passing through it, parallel to the given time-direction. The whole of such a bundle of parallel world-lines may be called a *physical reference frame*. From what has been said in the last section it is clear that every line of such a bundle is a point in the timeless space of the frame, and conversely. Each line of the bundle is, in fact, the history of a hypothetical particle, which rests at a certain place in the timeless space of the frame as the time of the frame flows on. The *place* of any momentary point-event in the timeless space of the frame will be the particular line of the bundle which passes through this point-event. The *date* of this event in the frame will be its particular position on this line.

Particles which move uniformly in straight lines in the timeless space of this frame will be world-lines which (1) are straight, and (2) are contained in a certain plane parallel to the time-axis, but (3) are not themselves parallel to it. Particles which move non-uniformly but rectilinearly in the timeless space of the frame will be world-lines which (1) are not straight, but (2) are contained in some plane parallel to the time-axis. This

plane in Space-Time is, of course, the straight line in the timeless space of the frame along which the particle moves. Particles which move non-uniformly and non-rectilinearly in the timeless space of the frame will be lines which (1) are not straight, (2) are not plane, but (3) are confined to a surface generated by straight lines parallel to the time-axis of the frame. Finally, the momentary spaces of the given frame will be sections of physical Space-Time, normal to the time-axis of the frame. Momentary events in the same momentary space will be contemporary with respect to the frame.

(*f*) *The Point of Separation between the Traditional View and the Special Theory of Relativity.*—There is thus a complete analogy between a physical reference frame and an idealised sense-history, on the assumptions which we are at present making. On these assumptions every event in Nature has its place and date in such a frame. But now there arises a question to which there is nothing analogous in a sense-history. The question is this: Are all straight world-lines permissible time-axes for physical Space-Time, or are some of them permissible and others not? And, if the latter be true, what distinguishes those which are, from those which are not permissible?

In a given sense-history there is one and only one time-*direction*. This is because the simultaneity or successiveness of sensa in the same sense-history is actually *sensed*, and we have therefore no choice as to which we shall group together as simultaneous, and which we shall group apart as successive. The successive slabs of the sense-history are *given* to us in the form of sense-fields, and the only possible time-direction is that of their common normal. The only choice allowed to us is that we could take any straight line in the sense-history, parallel to the time-direction, as a permissible time-axis, assuming that the geo-chronometry of the sense-history is Euclidean.

If there were an exact analogy between physical

Space-Time and an idealised sense-history, there would be one and only one direction in physical Space-Time which could be taken as the time-direction. If this were so, there would be one and only one frame of reference in which all the events of Nature could be consistently placed and dated. The only latitude allowed us would be that any frame which rested in the timeless space of the first would itself be a permissible frame. For this would merely amount to taking another world-line, parallel to the original one, as our new time-axis.

Now this is exactly the assumption which the classical mathematical physics did make. It assumed that there was one and only one fundamental frame of reference in which all the events of Nature could be consistently placed and dated. The timeless space of this is the "stagnant ether," and the one permissible time-direction is the history of any particle of the ether or of any particle that rests in it. No straight line which makes an angle with the one outstanding time-direction will be a possible time-axis; the sections of physical Space-Time normal to such a line will not be momentary spaces, and the whole bundle of lines parallel to such a line will not form the points of a timeless space.

Now there is nothing antecedently absurd in such a view. Temporal and spatial characteristics *are* different, for all observers; and therefore it might well be that there is one and only one outstanding direction in Space-Time which can be taken as a time-direction. Moreover, it is certain that the assumption is not *far* wrong; since it is the assumption of the traditional physics, and this has proved capable of dealing with all the more obvious spatio-temporal correlations of physical events in a single spatio-temporal scheme. We can, in fact, at once reject the opposite extreme view, viz., that *all* straight world-lines are equally permissible as time-axes. For this would be inconsistent with the admitted difference between spatial and temporal characteristics for all observers, and with the

very great measure of success which has attended the diametrically opposite assumption, that there is only one direction in Space-Time which can be taken as a time-axis.

(*g*) *The Hypothesis of a limited Range of Time-directions.*—The only alternative worth discussing is that all straight world-lines whose directions *lie within certain limits*, and only these, are permissible time-directions. The traditional physics makes physical Space-Time *exactly* analogous in structure to a single idealised sense-history. The present suggestion makes it considerably different in principle, though not necessarily very different in practice. Nothing but the observable correlations between physical events, as betrayed by correlations between sensible events in various sense-histories, can decide between these alternatives.

A little reflection shows that there is a certain incoherence in the traditional view, as regards mechanical phenomena. It is admitted that axes which move uniformly in straight lines in the timeless space of the supposed fundamental frame will do equally well for placing events for mechanical purposes. And such axes will be represented by straight world-lines which make an angle with those which represent the fundamental frame. If there were only mechanical phenomena to be considered, it would be natural to suppose that all such world-lines would do equally well as time-axes, and that all the corresponding frames would do equally well for placing and dating physical events. The only reason for thinking that there must be one fundamental frame connected with a certain unique direction in Space-Time, was the notion that any pair of events must be either simultaneous or successive, and that they could never be both. It was thought that the phenomena of light, electricity and magnetism, would show us the one fundamental frame, which was merely concealed in mechanical phenomena by the particular form which the laws of motion happen to have. Thus the traditional view

holds that there is only one permissible time-direction, which can and must be used for *dating all* physical phenomena. But it allows you to *place mechanical* phenomena by reference to any axes which move uniformly and rectilinearly in the timeless space of the fundamental frame.

Now the experiments on which the Special Theory of Relativity is based, show that this supposed difference between mechanical and electro-magnetic phenomena is a pure myth. Electro-magnetic phenomena fail to reveal any unique fundamental frame. Their laws remain of exactly the same form if you refer the events to axes which move uniformly and rectilinearly in the space of one fundamental frame, *provided* that you take the straight world-line which represents these moving axes as a permissible time-direction, and use it for dating your electro-magnetic events.

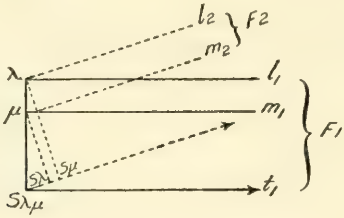
The Special Theory of Relativity may, in fact, be summed up in the following statement: There is a whole set of different directions in Space-Time, equally permissible as time-directions for dating physical events. But all the permissible time-directions are confined within certain limits. Corresponding to any one of these will be a timeless space, whose points are the world-lines parallel to it. Every physical event has a unique place and date in any one such frame. Its place in the timeless space of any frame is determined by the line, parallel to the time-direction of the frame, which passes through it. Its date in the frame is determined by its position on this line. The laws of all physical phenomena have precisely the same form, no matter which of these frames is used for placing and dating them.

All the characteristic features of the Special Theory of Relativity follow at once from this supposition as to the geo-chronometry of physical Space-Time, as I will now show in brief outline.

(1) There is nothing that can be called *the* timeless

Space of Nature. There will be as many different timeless spaces as there are different permissible time-directions.

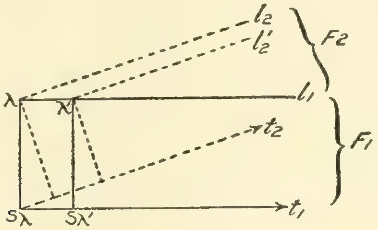
(2) Two events which are contemporary in one frame will not be contemporary in another, unless they happen to occupy the same place in the timeless space of the first frame. The figure below will make this clear.



Call the two frames F_1 and F_2. Since they differ, they will consist of two bundles of parallel world-lines, inclined to each other. Since the two events are not to be at the same place in the timeless space of F_1, they will be on two different world-lines of the bundle, say l_1 and m_1. Since they are to be contemporary in F_1, they must both be in some one momentary space of F_1. This will be a section of Space-Time, normal to the time-direction of F_1. Call this momentary space $S_{\lambda\mu}$. Then the points λ and μ, in which the lines l_1 and m_1 cut $S_{\lambda\mu}$, will represent our two events, which are simultaneous in the frame F_1, but spatially separated in its timeless space. Now let λ lie on the line l_2 of the frame F_2, and let μ lie on the line m_2 of the frame F_2. In this frame, instead of being in a single momentary space $S_{\lambda\mu}$, they are in the two successive momentary spaces S_λ and S_μ. They are therefore successive in F_2, though simultaneous in F_1. Moreover, their distances apart in the two timeless spaces are not the same. In the former, it is the distance between l_1 and m_1; in the latter, it is the smaller distance between l_2 and m_2.

(3) Conversely, two events which are in the same place in the timeless space of F_1 will not be in the same

place in the timeless space of F_2, unless they happen to be also contemporary in F_1. The diagram below will show this.



The two events are on a certain line l_1, parallel to t_1, since they are in the same place in the timeless space of F_1. Since they are not to be contemporary in F_1, they must be in different momentary spaces $S\lambda$ and $S\lambda'$ of F_1. The two events will be represented by the two points λ and λ', in which the line l_1 cuts these two momentary spaces respectively. In F_2 the two events λ and λ' are necessarily on two *different* lines, l_2 and l'_2, parallel to t_2, the time-direction of F_2. They are therefore at *different* places in the timeless space of F_2. Moreover, their temporal separation is different in the two frames. In F_1 it is represented by the line $\lambda\lambda'$, in F_2 by the shorter line between the two dotted normals to t_2, which represent the momentary spaces of F_2, in which the two events are respectively situated.

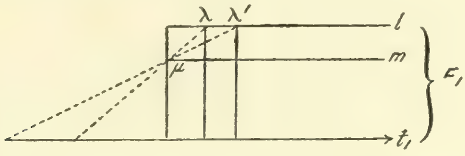
(4) We have still to consider some implications of the fact that not *all* straight world-lines are permissible time-axes, but only those whose directions lie within a certain limited range in physical Space-Time. Take any straight world-line t, which is a permissible time-axis, and consider any other non-parallel straight world-line p. There will be one and only one plane in Space-Time which is parallel to t and contains p. In this plane take a line t', parallel to t. Then p and t' will cut each other at an angle. This plane will be a straight line in the timeless space of the frame of which t is the time-axis. The line p will represent a particle moving

along this straight line in the timeless space with a uniform velocity. As we saw in the last section, the greater the velocity of this particle the greater will be the angle between p and t'. Now we know that, if the angle between p and t' exceed a certain size, p will not be a permissible time-axis. This would imply that there is *no* frame in whose timeless space the particle, whose history is the line p, rests. This would be contrary to the complete relativity of physical rest and motion. There is thus a certain maximum possible relative velocity, whose magnitude is determined by the size of the angle in Space-Time within which all permissible time-directions lie. If a straight world-line make a greater angle than this with any permissible time-direction, it cannot be the history of an actual particle or physical process. Such a world-line will, of course, cut each momentary space of any one frame at a point; but you cannot take these successive momentary points as sections of the history of any *one* object, though of course each may be a section of the history of a *different* object. Now this notion of a certain maximum relative velocity is characteristic of the Special Theory of Relativity, which, on empirical grounds, identifies this velocity with that of light *in vacuo*.

(5) We cannot, so far as I can see, determine anything about the actual magnitude of the angle of the four-dimensional cone in physical Space-Time, within which all permissible time-directions lie. The tangent of its half-angle will indeed be the velocity of light. But we must beware of supposing that, because c, the velocity of light in centimetres per second, is a very large number, therefore the half-angle of the fundamental cone must be very nearly a right angle, and therefore that there is a very wide range of possible time-directions. For the numerical value of the velocity of light obviously depends entirely on the units that we choose for measuring distance and duration. The largeness of c may simply mean that the centimetre

is a very small space-unit, or that the second is a very large time-unit; it tells us nothing about the size of the angle of the fundamental cone.

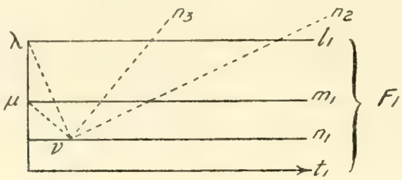
(6) It follows at once from what has just been said that, whilst all the points in any timeless space are straight world-lines, there are many straight world-lines which are not points in any timeless space. It follows also that some pairs of momentary point-events are *intrinsically* separated spatially, *i.e.*, occupy different positions in *all* timeless spaces, whilst others are not, *i.e.*, they occupy the same place in some timeless space. The diagram below will make this clear:



Let λ and λ' be two momentary point-events at the same point l of the timeless space of the frame F_1. Let μ be another momentary point-event at the point m of the same frame, and let λ, λ', and μ, all have different dates in this frame. Draw the straight world-lines $\lambda\mu$ and $\lambda'\mu$. If both fall within the fundamental cone, both are permissible time-directions. If so, λ and μ will occupy the same place in the timeless space of the frame corresponding to $\lambda\mu$, and λ' and μ will occupy the same place in the timeless space of the frame corresponding to $\lambda'\mu$. But it may happen that $\lambda'\mu$ falls inside the cone, whilst $\lambda\mu$ falls outside it. If so, $\lambda\mu$ is not parallel to a possible time-axis, and therefore is not a point in any timeless space. Hence the momentary point-events λ and μ will have an *intrinsic* spatial separation. It will be noticed that the question whether two momentary point-events, which occupy different places in the timeless space of a certain frame, are intrinsically separated in space or not, depends on whether their dates in the frame are much or little separated. λ and μ, which are intrinsically

separated in space, are much nearer together in date than λ' and μ, which are not spatially separated in *all* timeless spaces.

(7) Almost exactly similar remarks apply, *mutatis mutandis*, to temporal separation. This is sometimes intrinsic and sometimes not. The diagram below will explain how this happens.



λ and μ are two momentary point-events, which are simultaneous in the frame F_1, and occupy the two points l_1 and m_1 respectively in the timeless space of this frame. ν is a third point-event, which differs both in place and in date from both λ and μ in the frame F_1. Join $\lambda\nu$ and $\mu\nu$ by straight world-lines. Draw the straight world-lines n_2 and n_3, normal to $\lambda\nu$ and $\mu\nu$ respectively. If both n_2 and n_3 be permissible time-directions, $\lambda\nu$ and $\mu\nu$ will both represent momentary spaces, one in the frame corresponding to n_2, and the other in the frame corresponding to n_3. If so, λ and ν will be contemporary in one of these frames, and μ and ν will be contemporary in the other. Their temporal separation is therefore non-intrinsic. But it may happen that, whilst n_2 falls inside the fundamental cone, and is therefore a permissible time-direction, n_3 falls outside it, and therefore is not a permissible time-direction. If so, $\lambda\nu$ will be a momentary space, and $\mu\nu$ will not. It will follow that μ and ν are *intrinsically* separated in time, *i.e.*, that there is no frame in which they are simultaneous. Here, again, the difference depends on the fact that λ and ν are further apart in the timeless space of F_1 than are μ and ν. Hence, two point-events, which are successive in a certain frame,

are *intrinsically* successive if they be near enough together in the timeless space of the frame. If they be far enough apart in the timeless space, they will not be intrinsically successive, *i.e.*, it will be possible to find a frame in which they are simultaneous.

All these seven consequences of the view that more than one, but not all, directions in physical Space-Time are permissible time-directions, are characteristic results of the Special Theory of Relativity; and, as this certainly fits the facts better than the traditional views, we may assume that physical Space-Time has this particular kind of structure, at least to a very high degree of approximation. Thus the physical world as a whole is not completely analogous to a single idealised sense-history, since the latter has only one possible time-direction, whilst the former has several. Instead of being surprised at this difference, we ought rather to be impressed by the remarkable amount of similarity which exists between the structures of two such wholes.

(*h*) *The Facts underlying the above Theory of the Geochronometry of Physical Space-Time.*—If the above view of the structure of physical Space-Time is to be verifiable, as it is to a high degree of approximation, we must have some empirical means of (i) distinguishing straight from tortuous world-lines, and (ii) distinguishing those straight world-lines which are permissible time-directions from those which are not. We find that we can unify the facts by assuming that the history of any particle which rests relatively to the fixed stars is a straight-world line, and that the history of any particle which moves in a straight line with respect to the fixed stars, and with uniform velocity as judged by clocks set by the method of light-signals described in Part I, Chapter IV, is another straight world-line inclined to the first. And the history of a wave of light is the limiting kind of straight world-line which we can take as a permissible time-direction. It is important to notice that, although any one permissible reference-

frame for physical Space-Time is strictly analogous, on the present theory, to an idealised sense-history, yet we have to treat the two from rather different standpoints. The temporal relations between events in the sense-history are cognised directly by sense and memory. Certain events are *given* simultaneously and others are *given* in succession. Moreover, the sense-history has an *intrinsic* unit of duration in the constant sensible duration of all the successive Specious Presents. In dealing with the physical world we have to set up *criteria* for the simultaneity or succession of physical events; and it is not until we have done this that we can say which physical events are to be put into the same momentary space and which into different momentary spaces of a given frame. Moreover, there is no intrinsic standard of equality of physical duration, and so we have to set up some criterion for equality of time-lapse. Until we have done this, we cannot decide whether the motion of a certain particle in the timeless space of a certain frame is uniform or not. And, until this has been decided, we cannot say whether the history of this particle is or is not to be regarded as a straight world-line, inclined to the time-direction of the frame in question.

(*i*) *The Difference between the Special and General Theories of Relativity.*—The traditional physics and the special Theory of Relativity agree in making the geo-chronometry of physical Space-Time Euclidean. Or, to put it more accurately, the geo-chronometry of the *one* permissible frame on the traditional theory is Euclidean, and that of *each* of the *many* permissible frames on the special Theory of Relativity is also Euclidean. This amounts to saying that, on both views, all straight world-lines are Euclidean straight lines. This implies that the geometry of the one timeless space of the traditional theory and of the many timeless spaces of the special Theory of Relativity is Euclidean.

Now in both theories we have taken the history of a particle which rests or moves relatively to the fixed stars with a uniform rectilinear velocity, as judged by properly adjusted clocks, to be a straight world-line. Similarly, on both theories, we have taken the history of a wave of light to be a straight world-line. But, even on the traditional theory, it would have to be admitted that the universality of gravitation prevents the history of any actual particle from being an exactly straight world-line, on this definition, if the geo-chronometry of physical Space-Time be Euclidean. For, however far a particle may be from the fixed stars and from all other bodies, it is, even on traditional views, subject to gravitational forces, though these may be practically negligible. We have now to add to this the newly discovered fact that light, and all other forms of radiant energy, are themselves affected by gravitational fields. Thus it turns out that, *if* the geo-chronometry of physical Space-Time be Euclidean, it must be admitted that the history of *no* particle or process that we could possibly meet with is, in fact, a straight world-line. Thus both the traditional physics and the Special Theory of Relativity are in the odd position of holding that the geo-chronometry of physical Space-Time is Euclidean, and that therefore all straight world-lines are Euclidean straight lines, and then admitting that the history of no actual particle or process is a Euclidean straight line. The universal force of gravitation thus appears as a hypothesis to account for this universal divergence. It must be admitted that this hardly inspires confidence.

Now the Euclidean hypothesis is only one of three possibilities; the other two being the hyperbolic and the elliptic, as described earlier in this chapter. These three types of hypothesis agree in the important respect that any manifold which has either of these three structures is *homaloidal*. This means roughly that the structure of any finite region of the manifold will be

the same as that of any other, no matter where that region be situated within the whole. It is only on these three hypotheses that this is true. Obviously then, the next step would be to suppose that the geo-chronometry of physical Space-Time is not Euclidean, but is, nevertheless, homaloidal. We might then suppose that the histories of actual particles and processes in gravitational fields are straight world-lines, though these are not Euclidean, but hyperbolic or elliptic, straight lines. If this view of the structure of physical Space-Time would account for all gravitational phenomena, without our having to introduce gravitation *ad hoc* as a special but universal force, it would obviously be reasonable to adopt it.

Now we can deal with gravitational fields on such a hypothesis, so long as we confine ourselves to regions of physical Space-Time which are not occupied by physical events. For here we are concerned with regions for which the analogy to Laplace's equation

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=0$$

holds. This analogy, as we saw in Part I, is the vanishing of the Modified Riemann-Christoffel Tensor throughout the region. But, when we are concerned with regions occupied by physical events, we require an analogy, not to Laplace's, but to Poisson's equation

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}=4\pi\rho,$$

where ρ is the density of the "filling" of the region.

Now the analogy to this is not the *vanishing* of the Modified Tensor, but the equating of it to another tensor, which expresses the "filling" of the region under discussion. And we must remember that, under the heading of "occupied regions" of physical Space-Time we have to include not merely those which contain matter in the ordinary sense of the word, but also those

which contain only radiant energy of any kind, since this also gravitates.

It is evident then, that if we want to explain gravitational phenomena by reference to the spatio-temporal structure of Nature, we cannot do this by ascribing a homaloidal structure to physical Space-Time. We must assign different values to the Modified Tensor for different regions; since some regions are physically occupied and others are not, whilst of those which are physically occupied, some are more densely filled than others. The vanishing of the *Unmodified* Tensor, everywhere and everywhen, would imply that physical Space-Time is homaloidal and Euclidean; the vanishing of the *Modified* Tensor only, everywhere and everywhen, would imply that physical Space-Time is homaloidal, though not Euclidean; but, since it is certain that neither of these alternatives is compatible with explaining gravitational phenomena in terms of the structure of physical Space-Time, any such theory must assume a *non-homaloidal* structure for physical Space-Time. The only property which remains common to all regions of physical Space-Time is that the square of the spatio-temporal separation of any pair of adjacent events is a homogeneous quadratic function of the differences between the values of their four corresponding co-ordinates in any frame.

Now it does seem to me immensely important that we should not slur over this last transition. The passage from one to another view of the structure of physical Space-Time, so long as this structure is assumed still to be homaloidal, is of no particular philosophical importance. But the jump from a homaloidal to a non-homaloidal structure ought not to be taken lightly. It does involve, so far as I can see, the definite abandonment of a certain concept of Nature, which has so far been universally held. This is, roughly speaking, the concept of Space and Time as inert indifferent "containers," distinguishable from the material which

happens to occupy them. This view appears in a very crude form in the Absolute theories of Space and Time. But it survives, and can be restated, in the Relational theories and in the Special Theory of Relativity. The cash value of the distinction between physical Space-Time and its contents is that the sum total of physical events has a certain spatio-temporal structure which is the same always and everywhere, and is independent of qualitative differences between events. One region of Space-Time is differentiated from another only by qualitative differences in the filling of the two regions. Now any such view vanishes altogether on the General Theory of Relativity. It has been said that the Special Theory broke down the distinction between Space and Time, and that the General Theory broke down the distinction between both and Matter. The first part of the statement seems to me very loose, since the distinction between spatial and temporal isolation remains for every observer. The Special Theory breaks down, not the *distinction*, but the isolation of space and time. But, in a very real sense, the general theory does break down the *distinction* between Space-Time and events.

Now I do not make this an objection to the General Theory. All theories are but ways of unifying the observable facts under concepts ; and any theory that succeeds in doing this is permissible. I only want the reader to be quite clear that there is here a radically new way of looking at Nature. I think it will always be possible to unify the same facts by the more usual scheme of a homaloidal Space-Time and suitable fields of force. In so far as this fits in better with our traditional way of looking at things, this is to be preferred. But I should suppose that its advantages are only temporary ; that they will vanish as we become more familiar with alternative concepts ; and that our preference for homaloidal Space-Time, *plus* material and fields of force, has no greater ultimate significance than our preference for beginning dinner with *hors d'œuvres*

and ending it with coffee over taking it in the opposite order.

The following additional works may be consulted with advantage:

A. N. WHITEHEAD, *Principles of Natural Knowledge*, Chaps. IX. to XIII.
,, ,, *Concept of Nature*, Chaps. V. to IX.
,, ,, *Mathematical Concepts of the Material World*. (*Proc. Roy. Soc.*, vol. 205.)
,, ,, *The Principle of Relativity*.*
H. MINKOWSKI, *Raum und Zeit*.
H. WEYL, *Space, Time, and Matter*.
A. S. EDDINGTON, *Report on the Relativity Theory of Gravitation*.
,, ,, *Space, Time, and Gravitation*.
A. A. ROBB, *A Theory of Time and Space*.
,, ,, *Absolute Relations of Time and Space*.
S. ALEXANDER, *Space, Time, and Deity*, Bk. I.
B. RIEMANN, *Ueber die Hypothesen welche der Geometrie zu Grunde liegen*.
D. M. Y. SOMMERVILLE, *Non-Euclidean Geometry*.
E. H. NEVILLE, *The Fourth Dimension*.

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* This most important work appeared while the present book was in the press. Whitehead argues that Space-Time must be homaloidal; and he deduces the characteristic results of the General Theory of Relativity from a modification of the traditional law of gravitation, and not from supposed variations in the structure of different regions of Space-Time.

CHAPTER XIII

"... . quam sedem Somnia vulgo
Vana tenere ferunt, foliisque sub omnibus haerent.
Multaque praeterea variarum monstra ferarum,
Centauri in foribus stabulant, Scyllaeque biformes,
Et centumgeminus Briareus, ac bellua Lernae
Horrendum stridens, flammisque armata Chimaera.
. . . .
Et ni docta comes tenues sine corpore vitas
Admonuit volitare cava sub imagine formae,
Irruat, et frustra ferro diverberet umbras."

(VIRGIL, *Æneid*, VI.)

The Physiological Conditions of Sensations, and the Ontological Status of Sensa

AT the end of Chapter VIII we said that the Critical Scientific Theory of physical objects and our perception of them left two main problems on hand. One was to clear up the meanings of physical *place*, *shape*, *size*, *date*, *duration*, etc., and to establish their cash value in terms of those corresponding characteristics of our sensa, on which they must ultimately be founded. This task I have performed to the best of my ability in the last four chapters. The other problem was to elucidate the very obscure statement that external physical objects and our own bodies "jointly produce in us the sensa by which these external bodies appear to us." Probably any solution of this problem will be found to favour (if not actually to require) some particular view as to the nature of sensa and their ontological status in the universe. So this book will fitly end with an attempt to define the meaning and estimate the truth of the above statement.

Almost every phrase in this statement bristles with ambiguities. (1) The notion of "joint" production will be found to be far from clear, and its possible alternative meanings will have to be analysed. (2) We shall have to raise the question whether the conditions jointly produce sensations, or sensa, or both. (3) The word "production" is highly ambiguous, even when we have settled what we mean by "*joint* production." It may mean a kind of creation out of nothing, or a process of ordinary causation, or a process of selection out of a mass of pre-existing material.

These questions are not, of course, independent of each other. It is pretty certain that any answer that is given to one of them will cut out certain answers to the rest, and will favour certain other answers to them. But we must start by treating each question separately, and then try to view the results of our separate discussions as a whole.

Without prejudice to the conclusions that we may reach when we discuss question (2), we shall find it best to start by saying that processes in external bodies and in our own jointly condition *sensations*, rather than that they jointly condition *sensa*. On our view a sensation is a complex whole, in which an objective factor (the sensum) and a subjective factor (the act of sensing) can be distinguished. Whether either of these can exist apart from the other we do not at present either assert or deny. But this at least is certain; all the sensa of whose existence I am directly aware are constituents of *my* sensations, and all the sensa of whose existence other observers tell me are constituents of *their* sensations. Hence any evidence that I may think I have that certain physical and physiological processes are necessary and sufficient to produce *sensa* is *prima facie* evidence that they are necessary and sufficient to produce *sensations*. It may be that they can only produce sensations by producing sensa, but this question must be left aside for the present. So, to start with,

we shall talk about the production of sensations, and shall leave it an open question whether this involves the production of sensa.

The Notion of Joint Production.—I think that the view of educated common-sense is that there are certain events, very definitely localised in Time and Space, which happen in my brain and are the necessary and sufficient conditions of the occurrence of each of my sensations. If I sense a practically uniform sense-object, it is thought that there is a practically uniform process in some part of my brain, which lasts as long as the sensation, and is its necessary and sufficient condition. Some, but not all, of these brain-events are supposed to be due to external physical events, such as the striking of bells, the lighting of matches, etc. Others are supposed to be due to internal causes. It is held that, even when a sensation is due to some external cause, such as the striking of a bell, this is never a sufficient condition. Something must be transmitted from the external object to the sense-organ, and something must be transmitted from the sense-organ to the brain. Otherwise the brain-event, which is supposed to be the necessary and sufficient condition of the occurrence of the sensation, will not happen, and so the sensation will not be produced. I propose first to introduce some necessary technical terms for stating the common-sense view; then to clear up certain ambiguities in the notion of necessary and sufficient conditions; and then to ask in what sense, if any, there is reason to believe that certain definitely localised brain-events are the necessary and sufficient conditions of each of my sensations.

(*a*) *Originative, Transmissive and Productive Conditions.*—On the ordinary view, the production of a sensation by an external physical event requires the fulfilment of at least three types of condition. Let us take the case of hearing a certain stroke of a certain bell. (1) The

bell must be struck, or I shall not hear any sound characteristic of it at the time. This may be called the *originative* condition. (2) Unless there be air or some other material medium between my body and the bell I shall hear nothing, even though the bell be struck. There are excellent reasons, some of which have been mentioned in Chapter X, for holding that something travels with a finite velocity from where the bell is, through the medium, to my body. This may be called an *external transmissive condition* for my sensation of sound. (3) We have reason to think that, even though the originative and the external transmissive conditions for the occurrence of a sensation be fulfilled, no sensation will happen unless a certain nerve be intact, leading from the sense-organ to the brain. And it is generally held that the process in the nerve is transmissive in character. The evidence for this is fairly good. (α) If the nerve be cut at any point, no sensation of the kind will henceforth be experienced. Its integrity is therefore a necessary condition. (β) It is possible to note the time when an external stimulus acts on a sense-organ, and to get the patient to press a button as soon as he can after getting the sensation. If this button stops a clock, and the clock be delicate enough, there will always be a lapse of time between the two events. This, of course, does not conclusively prove that there is any lapse of time between the reception of the stimulus and the occurrence of the sensation, since the observed lapse might simply be the time between having the sensation and pressing the button. We have direct experimental evidence that a process, which takes time, travels along motor-nerves to muscles. So far as I am aware, we have no direct experimental evidence that a process which takes time travels up a sensory nerve from the stimulated organ to the brain. Still, it is reasonable to suppose that this is so, and it is in fact always assumed. On this assumption, we may say that there is an *internal transmissive condition* which is

necessary if I am to have here and now a sensation of the sound characteristic of this bell.

A transmissive condition might be defined as follows: It is a process which is practically uniform in character, and is *immanent*. This means that it is divisible into successive slices which are qualitatively very much alike. They differ only in date and place, and the nearer they are together in date the nearer they are together in place. And the character of each slice is the necessary and sufficient condition of the character of the next slice.

(4) Now, at a certain stage, viz., when the process has reached a certain part of the brain, it is supposed that a *transeunt* causal relation supervenes. This means that there is a certain brain-event, which is continuous with the immanent process, and is the necessary and sufficient condition of an event of an entirely different kind, belonging to a different "substance" or strand of history. This event is a sensation, which is, of course, an event belonging to that substance or strand of history which we call the observer's *mind*. Even if the transmissive process in the body should continue beyond the point at which the sensation occurs (as it no doubt does when the sensation is followed by a motor-reaction), we should say that the sensation belonged to an entirely different series from the later events in the transmissive process in the body. If the internal transmissive process ends up in the brain, we say that a certain slice, which ends it, is the *productive condition* of the sensation. If the internal transmissive process continues after the sensation has been produced, we must say that the productive condition of the sensation is a certain intermediate slice of this process.

It seems to be commonly supposed that the slice of the internal transmissive process which is the productive condition of the sensation must be extremely thin in time, *i.e.*, that it cannot stretch back from the date at which the sensation begins for any appreciable time. We shall

see in the next sub-section that this belief is based on tacit assumptions, which are far from self-evident and cannot be proved.

It is held that all sensations have originative and productive conditions, even though the sensation be "hallucinatory." If I "see stars," this sensation is presumably due to a certain brain-event, which is its productive condition. If this event can be traced to changes of blood-pressure in my eyes or to something happening in my liver, these would count as originative conditions. Whether all sensations have transmissive conditions is uncertain. It is certain that most of them have, and probably the difference between those which obviously do, and those which apparently do not, is a difference of degree rather than one of kind. It is perfectly obvious that an ordinary sensation of light or of sound has a long train of transmissive conditions, both external and internal. It is fairly clear that a sensation of itching in the finger, or of stomach-ache, has internal, though not external, transmissive conditions. But, if an auditory or visual experience were started by a change of blood-pressure in a part of the brain immediately adjacent to that in which the productive conditions of such experiences are localised, the transmissive process would be so short as to be evanescent. Still, we are probably justified in saying that the vast majority of sensations have originative, transmissive, and productive conditions.

We must next notice (a) that some kinds of sensations have *only* internal originative (and therefore internal transmissive) conditions. These are the sensations connected with our somatic sense-histories, such as feelings of headache, stomach-ache, etc., and kinæsthetic sensations. It is a well-known fact that the places of somatic sensa in their fields are not always a safe guide to the places of their originative conditions in physical space. A toothache occupies a certain sensible place in the total somatic field of the moment, and it may go on

occupying similar places in successive somatic fields. These somatic places will be correlated, through past experience, with certain places in the movement-continuum, which are optically occupied by the visual appearances of my tooth and physically occupied by certain scientific events which dentists profess to know about. As a general rule the part of my body which thus corresponds to a given sensible place in my somatic fields is the seat of those scientific events which originate the somatic sensum which occupies this sensible place. *E.g.*, if a feeling of toothache be located in a certain sensible place in my somatic field, my dentist will *generally* find something wrong with the particular tooth which I point out to him as occupying the physical place correlated with this sensible place. Sometimes, however, he will find that nothing relevant is happening in this tooth, but that the originative conditions of my toothache are located in a part of physical space which is correlated with a quite different part of my somatic field from that in which the feeling of toothache is located.

(β) Another important fact is that, although experiences of a certain kind may *generally* have external originative (and therefore partly external transmissive) conditions, yet experiences of the same general character may *sometimes* be originated by purely internal conditions. This is best illustrated by experiences of the visual type. Generally these are originated by some external luminous body, which starts waves that travel to the eye and there set up a disturbance which travels up the optic nerve to the brain. But in dreams we have perfectly distinct visual experiences, very much like those of waking life, although our eyes are shut and we may be in a perfectly dark room. Again, visual images are rather like visual sensa; and we can apprehend them best in the dark and with our eyes shut. Thus it is evident that the originative conditions for experiences of the visual type *need* not be external to

the body in every case. It is worth noticing that here presumably the internal originative conditions are extremely unlike the normal external originative conditions. The inside of the body is quite dark; so that, whatever be the internal conditions which originate the visual experiences of dreams, they must be extremely different from the luminous events which are the originative conditions of normal visual sensations.

I think that visual experiences provide the only perfectly clear case where very similar experiences are originated sometimes from without and sometimes from within, and where the two kinds of originative condition are extremely different in character. If we take auditory experiences, the facts are much less certain. It is quite true that I have auditory experiences in dreams, and that these are very much like those of waking life, which are originated by events outside my body. It is also true that many people can apprehend auditory images, and that these are a good deal like auditory sensa. So far, the facts about auditory experiences resemble those mentioned above about visual experiences. But now we have to notice two important differences: (i) It is much harder to be sure that the auditory experiences of dreams are not originated externally than to be sure that the visual experiences of dreams are not thus originated. Rooms are dark and our eyes are shut when we are asleep. But we cannot shut our ears, and few rooms are wholly free from those physical events which would suffice to originate auditory experiences in a waking man. It is therefore uncertain whether the auditory experiences of dreams be not originated externally.

(ii) As I have said above, our bodies are dark inside, *i.e.*, there are no physical events in them of a kind which would suffice to originate normal visual sensations in a waking man. But it cannot be said that our bodies are silent inside. All sorts of processes are going on in them, which would be quite capable of producing, in a mild

form, vibrations of the kind which strike a waking man's ears when he hears an externally originated sound. Moreover, our bones are capable of transmitting sound-waves just as well as air or any other material medium. Thus, even if there be auditory experiences which are originated internally, it cannot be confidently asserted that their originating conditions are different in kind from those of externally originated auditory sensations. *E.g.*, "head-noises" may quite well be noises of perfectly normal origin, which are heard by the sufferer and not by others, simply because his brain is nearer to and better connected with their originative conditions than the brain of anyone else can be. Thus we are reduced to the apprehension of auditory images, as the one clear example of auditory experiences whose originative conditions are almost certainly internal and almost certainly different in character from the external originative conditions of normal auditory sensations. I am indeed prepared to believe that some of the auditory experiences of dreams and disease probably do originate internally, and from events which are not like ordinary sound-vibrations; but I take this view, rather on the ground of analogy with visual experiences, than on account of any purely auditory phenomena known to me.

(γ) The question might be raised whether there be any type of sensible experience which is *always* originated by external conditions. I should not care to assert anything so sweeping; but I think it may be said that tactual experiences have a fair claim to this position. Tactual experiences are far less common in dreams than are visual or auditory experiences. Tactual images are extremely rare. If they exist at all, I certainly do not apprehend them myself, and I have not met anyone else who admitted doing so. Moreover, it is quite impossible to prove that such "hallucinatory" tactual experiences as there are, do not originate through actual contact between the skin and other bodies. For it is certain that throughout the whole of our waking and sleeping

life parts of our skin are in contact with other bodies. Again, there must always be contact between various parts of our internal organs ; and between some of these and the blood, undigested food, and so on. Thus, I think it would be very difficult to show even that any tactual experience was not originated by contact with *external* objects, and impossible to show that such experiences are ever originated except by contact of *some* kind, either internal or external. This is doubtless why most of us agree with the Apostle Thomas, who thought that *touch* was the best test for distinguishing normal from hallucinatory perceptions.

The theoretical importance of the points which we have just been raising will be seen in a later sub-section, where we shall consider how far we are justified in holding that certain brain-events are *sufficient* conditions of every sensation. Before ending the present sub-section we must discuss one point about originative and transmissive conditions. It is fairly obvious what part of the whole process is to be taken as the *productive* condition of a sensation. At least it is obvious where it *ends ;* for it ends where the sensation begins. Exactly how far back it stretches from this date is less determinate, and will need further discussion later on. But it is much less clear what stage in the long process, which ends up with a certain sensation, ought to be taken as the *originative* condition of that sensation. Let us return for a moment to the example of the striking bell. We took the stroke of the bell as the originative condition of the auditory sensation. But it might fairly be asked whether we should not have had just as good reasons for taking an earlier or a later stage in the total process as the originative condition. Whenever the process passes from one substance to another of a different kind, and changes sharply in character, there is an outstanding slice of it which might plausibly be taken as the originative condition. Now one such point is where and when the transmissive process of sound-

waves in the air ends and the transmissive process of nervous disturbance in the auditory nerve begins. Why should we not take a terminal slice of the external transmissive process as the originative condition of the sensation? Again, the process, of which one stage is the stroke of the bell, does not begin at that stage. Probably a man struck the bell; a contraction in his muscles caused the blow; a nervous current in a motor-nerve caused the contraction; and so on to infinity. Why should we not take one of the innumerable stages which precede the stroke as the originative condition of the sensation?

To these questions I answer (1) that we do recognise the last stage of the external transmissive process as important, and do mark it out by the special name of *stimulus*. For the physiologist and the physiological psychologist this is the earliest outstanding part of the total process which is of special importance. (2) The importance of the stage which immediately precedes the external transmissive process arises from its *common* relation to a number of *different* observers. If there be a number of observers listening to the same bell, there are as many different external and internal transmissive conditions, stimuli, and productive conditions, as there are observers. But all these different processes *diverge from* a common centre in Space-Time, and at this centre is located the physical event which is taken to be the common originative condition of all these very similar auditory sensations. (3) We can see how closely the notion of originative conditions is bound up with the fact of common optical and other centres for the corresponding sensa of different observers, by noting how difficult it becomes to apply this notion where the sensa of different observers are not correlated in this way. For instance, when we see a mirror-image we are doubtful what we ought to regard as the originative conditions of our visual sensations. The mirror-image is a partial optical object, and there is a certain place

behind the mirror which is optically occupied from many, though not from all, directions by sensa belonging to this object. A child or a cat might be inclined to suppose that this place is physically occupied by those events which are the common originative conditions of all the sensations whose sensa together make up the optical object. But the incompleteness of such optical objects prevents a grown man, even if he be ignorant of physics, from locating the originative conditions of his sensation in the optical place of these objects. We are left with the choice of events in the mirror or events in the reflected physical object, as the originative conditions of such sensations ; and, whichever choice we make, we have to admit that the place which is optically occupied by our visual sensa and the place which is physically occupied by the originative conditions of our sensations are widely separated. If we say that the events in the mirror are the originative conditions of our sensation, we must remember that they will not originate similar sensations in observers in *all* directions, as the normal originative conditions of visual sensa do. If we say that the events in the reflected physical object are the originative conditions of our sensation, we must remember that, unless mention be made of the mirror as well, we cannot account either for the peculiar optical place or for the peculiar "inversion" of the image-sensa.

(*b*) *Dependently and Independently Necessary Conditions.*—As I have said, it is commonly held that certain brain-events are the necessary and sufficient conditions of the occurrence of all our different sensations. We have now to clear up the notion of "necessary and sufficient conditions," and to see in what sense, if any, it is true that brain-events are the necessary and sufficient conditions of all our sensations. A number of conditions a, b, and c, are said to be severally necessary and jointly sufficient to produce an event x, if (1) whenever they are all present x happens, and (2)

whenever they are not all present *x* does not happen. It is obviously much easier to be sure that *a*, *b*, and *c* are severally necessary than that they are jointly sufficient to produce *x*. If we can omit in turn *a*, *b*, and *c*, and find that *x* does not happen, we can be sure that each of these conditions is necessary. But it is far from safe to assume that, because *abc* has always been followed in our experience by *x*, therefore these conditions are jointly sufficient to produce *x*. It is never really possible to get *abc* in complete isolation from the rest of the world, and there may have been some fourth factor *d*, which was, in fact, present in all the cases that fell under our notice and was necessary for the production of *x*. Statements that such and such conditions are jointly sufficient to produce a certain result should therefore always be viewed with suspicion.

If *abc* be sufficient to produce *x*, it follows that no other factor (unless it be simply a constituent of one of the factors *a*, *b*, or *c*, or a combination of them, such as *ab*), can strictly be necessary to produce *x*. For to say that *abc* is sufficient to produce *x*, is to say that *whenever abc* happens *x* follows. Hence both *abcd* and $abc\bar{d}$ will be followed by *x*, whatever *d* may be.* And if *x* follows in the absence of *d*, as it does in the case $abc\bar{d}$, *d* cannot be necessary for the occurrence of *x*. If then a certain brain-event be really *sufficient* to produce a certain sensation (say that of the sound characteristic of a certain bell), the existence of the bell and the air, and the occurrence of a stroke on the bell, and so on, cannot be strictly necessary to produce this sensation. Yet we should commonly say that the striking of the bell, and the other conditions which we have enumerated, *are* necessary, if that particular noise is to be sensed at that particular time. Our ground for this statement is that we believe that no such sensation would have happened then, if no bell had existed, and if it had not been struck shortly before.

* Here "$\bar{d}$" simply stands for "the absence of *d*."

It is clear from this that we use the word "necessary" in two different senses. In one of them, nothing can be necessary to produce an event unless it be contained in the smallest set of conditions which will jointly suffice to produce the event. In the other, many factors which are not contained in the smallest set of conditions which will jointly suffice to produce an event are yet said to be necessary for its production. We must, in fact, distinguish between *independently* and *dependently* necessary conditions. If a certain brain-event be really sufficient to produce the sensation of the sound of a certain bell, then the striking of the bell, the disturbance of the air, and so on, are only dependently necessary to the production of this sensation. That is, they are necessary to produce the sensation only in so far as they are necessary to produce the whole, or some part of, that brain-event which is sufficient to produce the sensation. We may say in general that a is a dependently necessary condition of the event x, if a be necessary to produce the whole, or some part of, the conditions which are independently necessary and jointly sufficient to produce x.

Now a very important question at once arises. Can a certain event a be *both* dependently and independently necessary to produce x? I think that this would commonly be denied; but we shall see in a moment that it can only be denied on the basis of certain assumptions about causation, which have very little plausibility when they are explicitly stated. What would it mean to say that a is both dependently and independently necessary to produce x? It would mean that a, b, and c (say) were all needed to produce x, and that they are all that is needed, but that a plays two parts. It *produces* b (say). And it *co-operates* with b and c to produce x. Supposing it to be possible that a should play both parts, and supposing it to be certain that a is *dependently* necessary, then it would always be impossible to know that a is not also *independently* necessary to

produce x. For, if a be dependently necessary to produce x, there is some factor b in the necessary and sufficient conditions of x, which cannot occur unless a has preceded. Since b never does occur without a preceding, we cannot possibly know whether b does not need the co-operation of a in order to produce x, unless we have some positive reason for holding that a dependently necessary condition of an event cannot also be an independently necessary condition of it.

Let us apply this abstract logical argument to the concrete case of the auditory sensation of the noise of a bell. If the brain-event which produces this sensation could not occur unless the bell had rung a little earlier, we cannot be sure that the brain-event is by itself a *sufficient* condition of this sensation, unless we are sure that a dependently necessary condition *cannot* also be an independently necessary condition of the same event. If the brain-event never happens without the bell-event preceding, we cannot possibly know that the brain-event, without the co-operation of the bell-event, would suffice to produce the auditory sensation, unless we have some *a priori* ground for this belief. For the only conclusive *empirical* ground for such a belief would be to get the brain-event without the bell-event, and to find that the sensation still followed. But, *ex hypothesi*, we cannot get just this kind of brain-event without a bell-event preceding, and therefore this empirical argument cannot be used. Conversely, of course, we cannot be sure that the bell-event *is* independently as well as dependently necessary for the production of the sensation.

Now, is there any *a priori* argument against the possibility of a certain condition a being at once dependently and independently necessary to produce a certain event x? I know of one and only one way in which such a possibility could be refuted. If it be held that all the independently necessary conditions of an event must be contemporary with each other, it will

follow that the same factor cannot be both independently and dependently necessary to produce a certain event. For the dependently necessary condition will precede that one of the independently necessary conditions which it produces. Consequently it could not itself be an independently necessary condition, if these have all to be simultaneous with each other.

But I cannot accept the premise of this argument. (1) It does not seem to me to have the slightest trace of self-evidence. I think there is something to be said for the proposition that cause and effect must be continuous with each other in time, and that the complete cause must itself be a continuous process in time. This, however, is quite compatible with a and b being successive, and yet both of them being independently necessary conditions of x. Suppose that the end of b is simultaneous with the beginning of x, and that the end of a is separated by a lapse of time from the beginning of b. Then the principle of the temporal continuity of causation would only show that the complete cause of x consists, *not merely* of a and b, but also of some process which bridges the gap between the .wo. It has no tendency to show that b is the complete cause of x, and that a is only dependently necessary.

(2) Apart from the lack of self-evidence in the principle that all the independently necessary conditions of an event must be simultaneous, there is a serious positive objection to it. We have seen that no two events are *intrinsically* simultaneous, unless they also have no spatial separation. Events which are separated in the timeless space of one permissible frame, and are simultaneous with respect to that frame, will be temporally separated with respect to any other frame which moves in the timeless space of the first. Thus the principle would presumably have to be stated in the much milder form that the independently necessary conditions of an event must not be *intrinsically* separated in time, *i.e.*, that there is at least one permissible frame

with respect to which they are all simultaneous. But, when it thus loses its original sweet simplicity, it seems to lose any trace of self-evidence which it may have had before.

(3) Lastly, it seems to me almost certain that the sufficient productive conditions of many sensations could not be momentary, and, therefore, must include non-simultaneous factors. I do not merely mean by this that "momentary" conditions are not existent facts and can only be defined by Extensive Abstraction. I mean that, if you tried to apply Extensive Abstraction to the conditions of many sensations you would find that these do not converge to a set of contemporary momentary states. It is practically certain, *e.g.*, that the external originative and transmissive conditions of sensations of light and sound are *periodic*, and it is reasonable to suppose that the subsequent internal processes in nerves and brain are periodic too. There is a very accurate correlation between the colour or pitch of the sensum and the period of the external originative and transmissive events. Now it is impossible that the characteristic periodicity of red light, or of a certain note on the piano, should be carried by a purely *momentary* brain-event. Presumably the brain-event, which is the productive condition of even the shortest sensation of red, must last, at least as long as one complete vibration of red light. Or, if we prefer to express ourselves more guardedly, we must, at least, hold that the productive conditions of the shortest possible sensations of (say) red and blue must *both* have characteristic finite durations, and that these durations must have to each other the same *ratio* as the periods of a complete vibration of red light, and a complete vibration of blue light. If the productive conditions have durations, they must have non-simultaneous parts. And, if the whole finite event be the least that is sufficient to produce the sensation, all its successive parts must be *independently* necessary to produce the sensation. If, further, the event in question

be transmissive in character (if, *e.g.*, it be the passage of some kind of disturbance through a finite tract of brain and nerve) the earlier parts of it will also be *dependently* necessary conditions of the sensation, since the later parts will not happen unless the earlier ones happen and produce them.

The upshot of this discussion seems to be that we cannot prove by any direct empirical argument that *any* condition which is dependently necessary to produce a sensation is not also an independently necessary condition of it. And we cannot prove *a priori* that dependently necessary conditions cannot also be independently necessary, except from a premise which is not self-evident, is of very uncertain meaning when the relativity of physical simultaneity is considered, and is almost certainly false as applied to the productive conditions of some of our most important sensations. It follows that it is rash in the extreme to expect to be able, even in theory, to isolate a momentary event at a definite place in the brain, and to say: "This is the necessary and sufficient condition of such and such a sensation." We cannot be absolutely certain that even such remote dependently necessary conditions as the stroke of the bell are not also independently necessary conditions of our sensation of the sound which is characteristic of the bell. And we can feel fairly confident that at least the later stages of the internal transmissive conditions of a sensation are independently as well as dependently necessary conditions of its occurrence. To put it shortly: The productive conditions of a sensation almost certainly include the later stages of its internal transmissive conditions; and, for all that we can certainly know, they might include the external transmissive and the originative conditions as independently necessary factors.

I think it is possible to produce a more or less plausible *indirect* empirical argument, which renders it *probable* that the independently necessary conditions of

some at least of our sensations do not extend so far back as the external transmissive or the originative conditions. But it is only an argument from analogy, and, as we shall see, the analogy is none too good. The argument would run as follows: Although the particular sensation s would not have arisen when it did, unless certain external originative and transmissive conditions had been fulfilled, there are sensible experiences s', very much like s, which happen (*e.g.*, in dreams) when there is good reason to believe that no such external originative or transmissive processes are operating. If so, internal conditions are sufficient to produce s'. And the analogy between s' and s may suggest that purely internal conditions are sufficient to produce s, though these cannot, in fact, arise unless certain external conditions be first fulfilled. If this be so, the external conditions are *only* dependently necessary for the production of s. To take a concrete example. Although I should not have sensed a certain flash at a certain moment unless someone had struck a match very shortly before in my neighbourhood, yet I do have visual experiences very much like this sensation in dreams. The latter must have been produced by purely internal conditions. Hence purely internal conditions are sufficient to produce experiences very much like this particular sensation. Therefore probably the sufficient conditions of *all* visual experiences are internal; and the external conditions, which are necessary for the production of many such sensations, are *only* dependently necessary. That is, the striking of the match is necessary *only* for producing the internal process which is the sufficient condition for the sensation of the flash; it is not also necessary as a condition which co-operates with the later stages of this process.

It is evident that such an argument could *never* establish more than a probability that external events are not independently necessary conditions of those sensations to which they are dependently necessary.

The strength of the argument in any particular case will depend on two factors, viz.: (1) the degree of analogy between the experiences s', which are alleged to be originated wholly from within the body, and the sensations s, which are externally originated; and (2) the degree of certainty with which it can be asserted that the experiences s' are originated altogether internally. When the experiences s' are apprehensions of so-called "mental" images I should not deem the analogy strong enough to bear any great weight of argument. For, although visual and auditory images are a good deal like visual and auditory sensa respectively, yet there are such marked differences between them that we hardly ever mistake one for the other in normal waking life. I should be inclined to say that only the experiences of dreams, and other forms of hallucination, bear enough likeness to auditory and visual sensations to support an argument such as I have outlined above. Now, in the last sub-section we saw that it is by no means certain that auditory experiences (other than images) are ever originated save by external physical events or by internal events of precisely the same character. It is therefore doubtful whether there be any facts about auditory experiences which the present argument could use as premises. With tactual experiences, as we saw, the position is still less favourable. In fact, it is only with visual experiences that there is really good evidence that something very much like normal sensations can be originated by events which are wholly internal and are quite unlike the external originative conditions of the normal sensations. Thus we can argue with a fairly high degree of probability that the sufficient conditions of visual sensations are internal, and that the external originative and transmissive conditions are *only* dependently necessary; but, for auditory and tactual sensations, a similar argument leads to only a weak probability.

It must be remembered, on the other hand, that it

is equally impossible to prove (what the naïver Realists would like to believe) that the external originative conditions of our sensations *are* independently, as well as dependently, necessary conditions for the occurrence of these sensations. Thus, so far as I can see, empirical facts and *a priori* principles about causation justify little more than complete agnosticism on this subject. There is, therefore, an almost open field for different hypotheses, each carrying the independently necessary conditions backwards in Time and Space by different amounts. Each will lead to a somewhat different theory as to what is involved in the perception of external physical objects and events, and the hypothesis which leads to the theory of perception which best unifies all the known facts is the one to be preferred.

Within the body I know of no means of setting even probable limits to the distance backwards in Space and Time to which the independently necessary conditions of a sensation may stretch. It may be that the events in the brain are sufficient, and that the process in the sensory nerve is *merely* transmissive. On the other hand, it is equally likely, so far as I can see, that the process in the nerve is an independently necessary, as well as a transmissive condition, for the occurrence of the sensation. The former alternative appears to be unhesitatingly taken by physiologists, and accepted, on their authority, by the general public. But this conviction rests on no stronger basis than a failure to draw certain distinctions among "necessary conditions," and a simple faith in certain dogmas about causation which will not bear the light of common day.

I will end this sub-section by considering a rather confused semi-popular argument, which tries to raise doubts about the existence of external objects and events, on the ground of physiological theories about the conditions of our sensations. I will call this position *Physiological Scepticism*. The argument would run somewhat as follows. "My only ground for believing in

the existence of external physical objects is the occurrence of certain sensations which I ascribe to them. But physiology proves that states of my body are *sufficient* conditions of all my sensations. Hence I have no right to conclude from the occurrence of sensations to the existence of external physical objects and processes, as their originative conditions." To this we may answer: (1) That, even if internal processes be sufficient conditions of our sensations, we do not know and have no reason to believe, that these internal processes would take place unless certain external events were happening and affecting our bodies. Thus we may still argue to the existence of such external objects, as, at least, the *dependently* necessary conditions of many of our sensations. Moreover, the resemblance between many of the sensa which I sense and those which are sensed by other observers, the fact that visual sensa from different observers' sense-histories are in the same optical place, and the somewhat similar facts about auditory sensa, suggest strongly that there is often a remote external physical event, which is located in this place, and is a common dependently necessary condition of all these correlated sensations. (2) We have seen that it is impossible to be sure that these dependently necessary external conditions are not also independently necessary. It is, therefore, quite uncertain whether internal processes *are* sufficient conditions of all my sensations. If this be held at all, it can only validly be held as a probability based on certain partial analogies. (3) It is perhaps worth while to point out that Physiological Scepticism cannot consistently stop at the stage of doubting the existence of *external* physical objects. If such arguments be valid at all, they must finally be applied to one's own body and its supposed internal structure. All that anyone knows about the physiology and internal anatomy of his own body he has learnt by studying and dissecting other organised bodies. Now, for each observer, these are simply external physical

objects, of whose existence and inner structure he learns by sensations of sight and touch. If then he is forced to be wholly sceptical about external physical objects, he ought, if he wants to be consistent, to be equally sceptical about all statements which imply the existence of a permanent inner structure and variable states of his own body. The conclusion of Physiological Scepticism blows up its own premises, and the only consistent result is complete scepticism about *all* physical objects and processes, including those with which physiology professes to deal. Physiologists with a tendency to philosophical speculation are liable to combine Naïve Realism about the purely hypothetical states of their brains with Subjective Idealism about all other physical objects, including those which they have had to study in order to learn about their own brains. To parody Mr Gibbon's remark about the Jews: "In contradiction to every known principle of the human mind this singular people seems to have yielded a stronger and more ready assent to" the hypothetical entities of their science "than to the evidence of their own senses."

(*c*) *Occurrent and Continuant Conditions.*—In the last sub-section I brought forward certain abstract logical considerations to show that it is impossible to tell how far the series of independently necessary conditions of a sensation must be carried in Space and Time. But, quite apart from these considerations, it is practically certain that no event in the brain is a *completely* sufficient condition for the occurrence of any sensation. Every event depends on two kinds of conditions, which we may call *occurrent* and *continuant*, borrowing two useful names from Mr W. E. Johnson. We are always very liable to notice the occurrent and to ignore the continuant conditions, and then to think that the former are sufficient to produce the event. It would commonly be said that the stroke of a bell is a necessary and sufficient condition of the occurrence of certain vibrations in the surrounding medium. So it

is, *provided* that there is a material medium in contact with the bell, and that it is capable of being set in vibration by a disturbance of this particular period. It is evident that the latter condition is as necessary for the setting up of vibrations as the former. But the striking of the bell is a short outstanding event in that long and fairly uniform strand of history which is the bell ; whilst the medium and its structure existed before the bell was struck, and will exist with very little change for long afterwards. Moreover, in our experience, bells are much more often than not surrounded with such a medium. The medium is thus such an unexciting and such a usual piece of physical history that we hardly think it worth mentioning. Now I should call the striking of the bell an *occurrent condition*, and the existence of a surrounding medium of suitable structure a *continuant condition*, of the setting up of the vibrations. Both are necessary, and neither by itself is sufficient. Together they are sufficient. We can, if we like, call the striking of the bell the *necessary and sufficient occurrent condition* of the vibrations, but we must on no account call it the necessary and sufficient condition without qualification.

I do not pretend that an absolutely hard and fast line can be drawn between occurrent and continuant conditions. An occurrent condition is a short outstanding slice in some long strand of physical history, which is fairly uniform up to this slice and again shows uniformity, often of the same kind as before, after the slice. A continuant condition is a long and practically uniform strand, which stretches out with little variation before, during, and after the occurrent condition. Obviously terms like "short," "outstanding," "uniform," etc., are relative. But, for our purpose, all that we need to notice is that some of the conditions of an event are always of the continuant type, and that the more a condition is of the continuant type the more likely it is to be overlooked.

Let us now apply these general considerations to

the necessary and sufficient conditions of our sensations. When a stimulus, which normally produces a certain kind of sensation, acts on a sense-organ, such as the eye or ear, no sensation will be produced unless the nerve be intact and the general structure of the brain be not disintegrated beyond a certain very small degree. Again, the structure of the sense-organ, sensory nerve, and brain may (so far as we know) be intact, and yet no sensation will be produced if the man be dead. If he be alive, but asleep or in a swoon or under the influence of a drug, the stimulus may also fail to produce a sensation in his mind. Again, there are such phenomena as "psychic" blindness, deafness, etc., which happen spontaneously in hysteria, and can be induced artificially by hypnosis. Here there is no reason whatever to suppose that there is any defect in the structure of sense-organs, nerves, or brain—indeed there is evidence to the contrary—and yet the external stimulus is not followed by any correlated sensation in the conscious mind of the patient. Lastly, we have seen in an earlier chapter that similar external stimuli will often produce in different observers sensations whose sensa are partly different in quality, and that these differences can be correlated with differences in the past histories of the observers.

It is evident then that one general continuant condition for the production of sensations is that the sense-organ and the nerve which are specially concerned, and at least a considerable part of the brain, shall be structurally intact. Given this condition, it is also necessary that the body shall be "alive." This is probably a distinct condition from the one just mentioned. Although the structure of the brain and nervous system does not remain intact for very long after the death of the body, it would be rash to say that it disintegrates profoundly *immediately* after death. Motor nerves can certainly be kept alive for some considerable time after the death of the body. I should suppose that

"being alive" involves at least the maintenance of a certain moving equilibrium among bodily changes. We might therefore call it the *general somatic occurrent condition* of sensations. I suppose that "being awake" or "being conscious" involves at least a certain moving equilibrium among processes in the brain. This might therefore be called the *general cerebral occurrent condition* of sensations. Since a man can be alive without being awake, though he cannot be awake without being alive, there is a partial dependence and partial independence between these two sets of conditions.

The *bodily* conditions on which psychic blindness or deafness depend, if such there be, are quite unknown to us. It seems to me theoretically possible that the conditions of such phenomena are wholly psychic, and have no bodily correlates at all. Whatever view we may take on this point, we can at least say that they are *special*, and not simply *general* conditions, such as we have so far been describing. A patient is not, as a rule, psychically blind to all lights or psychically deaf to all noises. Most usually he is blind or deaf only to those which have some special association for him, or to those about which suitable suggestions have been made to him by himself or by others. We may reasonably suppose that psychic blindness or deafness, if it have a bodily correlate at all, depends on certain *disconnexions* between the particular nervous process which would normally give rise to the sensation, and the rest of the brain. Thus the condition that we shall *not* be psychically blind or deaf when a certain stimulus acts on us may be called a *special connective condition* for the occurrence of the sensation. As it is a condition which usually holds, unless there be special causes to disturb it, it should presumably be counted as continuant rather than occurrent. Lastly, when the quality of the sensum partly depends on the past experiences of the observer, we may say (borrowing a useful expression from Mr Russell) that the sensation has *mnemic conditions*. (By

using this phrase I do not imply either the acceptance or the rejection of that peculiar kind of causation which Mr Russell calls "mnemic causation.") On the ordinary view that past experiences leave traces which persist, and that it is these which condition our present sensations, I suppose that mnemic conditions would be partly continuant and partly occurrent. The trace, having become part of the permanent structure of the nervous system, would be a continuant condition. The connexions between this trace and other parts of the brain, which have been formed by association, will also be continuant connective conditions. But the excitement of this particular trace, when a certain part of the brain is excited by some external stimulus, is a special occurrent condition.

All the conditions which I have just been enumerating must be fulfilled if a certain stimulus is to be followed by a characteristic sensation at a given moment. The mnemic conditions may, in a sense, be called "less necessary" than the others, since (α) there are probably some sensations in whose production they play little if any part; and (β) even if they be necessary to produce a certain sensation at a certain moment, it is probable that a *rather* similar sensation would be produced without them, provided that all the other conditions were fulfilled. On the other hand, if any of the other conditions be not fulfilled, no sensation at all will be produced in the conscious mind * of the observer.

The question can now be raised as to which of these conditions are only dependently necessary, and which are also independently necessary, for the production of a sensation. The structural integrity of a special nerve,

* I use the expression "conscious mind" here, because I think that it is theoretically possible that sensations may be produced in connexion with a certain brain and nervous system, which do not form parts of that mind which normally manifests itself through this organism. Such sensations (if they exist at all) might not form parts of anything that deserves to be called a mind; or again, they might form parts of a mind which seldom or never manifests itself.

and its "being alive," are presumably dependently necessary conditions; since, unless they be fulfilled, no disturbance will be produced in the brain. Whether they be or be not also independently necessary it seems impossible to tell, for the reasons given in the last sub-section. But I should suppose that, on any view, the substantial structural integrity of the brain as a whole, in addition to that of the particular part that is immediately connected with a special sensory nerve, is an independently necessary condition for the production of a sensation. In addition to this, I should suppose that the general balance of cerebral processes, which is involved in the statement that the observer is "awake," is an independently necessary condition. The special connective conditions, which are needed for the absence of psychic blindness or deafness, are also independently necessary. And, if the sensation has mnemic conditions, these are independently necessary for the production of *just this* sensation, though a sensation a good deal like it might be produced in their absence.

We see now how loose it is to talk of a certain brain-event, very definitely localised in time and place, as the sufficient condition for the occurrence of a sensation. Apart altogether from the fact, elicited in the last sub-section, that we do not know how many of the dependently necessary conditions are also independently necessary, we see that such assertions ignore many conditions, some occurrent and some continuant, which are independently necessary. At the utmost we can call a certain brain-event, fairly definitely localised in Time and Space, the necessary and sufficient *special non-mnemic occurrent condition* of a sensation. In addition to this, every sensation needs at least the following conditions: (1) the general continuant cerebral condition of structural integrity of the brain as a whole; (2) the general occurrent cerebral condition of "wakefulness"; and (3) a special continuant connective condition to prevent psychic blindness, deafness, etc. Moreover, many

sensations require further (4) mnemic conditions, which are partly occurrent and partly continuant; (5) and all sensations require, as at least dependently necessary conditions, that the body as a whole, and especially the sensory nerve, shall be structurally intact (a continuant condition), and that the body shall be "alive" (a general occurrent condition). Beside all these, there may well be *purely* psychic conditions, having no bodily correlates, which must also be fulfilled if sensations are to arise in the mind. I am going to assume, for the sake of simplicity, in this book that there is such a complete parallelism between mind and body that it is enough to mention bodily conditions, because every psychic condition has its bodily correlate. I am very far from believing that this is *true*, and am not even sure that it has any very definite *meaning* which would survive analysis; so I assume it here simply as an excuse for avoiding additional complications which are hardly relevant to our present purpose.

Sensations, Sensa and Acts of Sensing.—For reasons given at the beginning of this chapter we have so far spoken of physiological and physical conditions as producing *sensations*. We have now to ask whether this involves the production of sensa, or of acts of sensing, or of both. Before we can hope to answer this, we must try to clear up the notion of a sensation a little more fully than we have yet had occasion to do.

(*a*) *The General Process of Sensing*.—A sensation, on our view, is a complex in which an objective factor (the sensum) and a subjective factor (the act of sensing) can be distinguished. Whether either of these can exist without the other is a matter which has so far been left in decent obscurity. It is obviously *logically* possible, and indeed quite plausible, that there might be unsensed sensa. It is very much harder to believe that there could be acts of sensing which did not sense anything, because an act of sensing would seem to involve a

special relation between a sensum (which is thereby sensed) and something else. Let us begin by asking whether every different sensation involves a different act of sensing.

It seems clear to me that we distinguish different sensations by means of the different sensa which are their objects. If two sensa be in different fields of the same sense-history we should say that the observer had two different sensations. If two sensa were in the same field, and completely overlapped in time, we should say that the observer had two sensations, provided the two sensa were separated spatially in the field by a background which differed qualitatively from both of them. I think it would be reasonable to say that sensa in successive fields are sensed by different acts, which are themselves successive. But I see no reason to postulate different acts of sensing for different sensa in the same field. When we remember that sensa do not exist in isolation, but are simply outstanding features in sense-fields, any such view seems far from plausible. It seems more reasonable to suppose that the same act of sensing grasps a whole sense-field. We can then distinguish as many sensations as there are outstanding sensa in the field ; but there seems no need whatever to assume a special act of sensing for each of these sensa. To say : "I have two contemporary sensations, one of x and the other of y," would seem to mean simply : "I sense a field f, in which x and y are two outstanding parts, which may overlap in time but are separated in space." Thus, although every sensation involves an act of sensing, it does not follow that the production of every sensation involves the production of a special act of sensing.

So far, we have been considering sensa which are in the same special field, *e.g.*, in some one visual field. But my general sense-history consists of a number of parallel special sense-histories, *e.g.*, visual, tactual, auditory, etc. My general sense-history goes on

throughout the whole of my waking life at any rate, though there may be gaps in any one of my special sense-histories. Now I do not see any reason to suppose that there are as many contemporary acts of sensing as there are contemporary special sense-fields. The various special fields are joined up with each other by sensible temporal relations to give a general sense-field. If I am aware at once of a visual and a tactual field, I see no more ground for postulating two acts of sensing, one visual and the other tactual, than for postulating two acts of sensing for grasping a red patch and a blue patch in the same visual field. I would rather say that there is a single general act of sensing, which happens to be supplied with both a visual and a tactual field for its objects. Certainly a tactual sensation is very different from a visual sensation. But so, too, is a sensation of a round red patch from a sensation of a square blue patch. The difference in the objects seems to be enough to account for the difference between the sensations in both cases, and it is needlessly multiplying entities to postulate different acts of sensing as well, unless there be some special positive reason for doing so.

I am therefore inclined to think that at any moment in our lives, while we are awake at any rate, there is a general act of sensing; and that these successive general acts join up to give a single general *process* of sensing, forming the subjective correlate to our general sense-history which is its object. Some slices of this general object consist of more, and some of fewer, special sense-fields. Consequently, we have sometimes more, and sometimes fewer, kinds of sensations. Again, one field of some special sense-history may be more differentiated into outstanding sensa than another. Consequently, we have sometimes more, and sometimes fewer, sensations of the same kind. But, if I am right, this makes no difference to the number of our acts of sensing. I do not deny for a moment that there may

be, from time to time, special mental acts directed on to special sensa. Sometimes one sensum particularly interests me, either because of its intrinsic character or because of its associations. If so, I may specially attend to it. In so far as this involves more than merely adjusting my body, so that I sense a new field in whose centre there is a larger and more distinct sensum correlated with the old one that first attracted my attention, it no doubt involves the directing of a special mental act on to a certain sensum. But specially to attend to a sensum is something more than merely to sense it, and therefore the fact just admitted is quite consistent with our earlier statement that there is no need to assume a distinct act of sensing for each distinct sensation.

(*b*) *Conditions of Sensing and Conditions of Sensa.*— Let us now apply some of the conclusions which we reached in the last section about the various conditions which are necessary for the production of sensations. We have just seen that not every special sensation involves a special act of sensing, though every sensation does involve *an* act of sensing. In the last section we distinguished between the special occurrent conditions of a sensation and certain equally and independently necessary general conditions, some occurrent and some continuant. Now it seems to me probable that the general process of sensing is kept up by the continuant and occurrent general cerebral conditions, which are involved in being "awake" and conscious. And it seems to me that the function of the special occurrent conditions is, not to produce acts of sensing, but to produce outstanding sensa in our special sense-histories, and thus to supply the general process of sensing with various objects. If the special occurrent conditions be fulfilled without the general cerebral conditions, it is conceivable that sensa may still be produced, but it is certain that they will not be sensed. And we know, from such facts as psychic blindness and deafness,

that, even when both sets of conditions are fulfilled, no sensum will be consciously sensed by the observer unless certain special continuant connective conditions be also fulfilled. In such cases it seems still more likely that sensa may be produced without being sensed. But these abstract possibilities of the production of unsensed sensa cannot be properly estimated until we have cleared up the notion of "production," which we shall try to do in the next section.

Now it might be said: "If you think it possible that the special occurrent conditions might produce unsensed sensa in the absence of the general cerebral conditions, do you think that the general cerebral conditions might produce a general process of sensing, with nothing to sense, in the absence of special occurrent conditions?" To this I answer: (α) Probably not; because I find it difficult to know what, if anything, would be meant by a process of sensing with no objects to sense, and am therefore doubtful whether anything of the kind be possible at all. I do not feel any similar difficulty about the possibility of unsensed sensa. And (β) in any case the question cannot be tested empirically, for the following reason. The cerebral conditions which keep up the general process of sensing are themselves dependent on more general somatic conditions. We cannot be conscious without being alive; though, if there be ever completely dreamless sleep or complete anæsthesia through drugs or disease or accident, we may sometimes be alive without being conscious. Thus, whenever the cerebral conditions for sensing are fulfilled, there is a rough balance of physiological processes in the body as a whole. These somatic conditions supply the general process of sensing with a continual series of internal sensa as objects. Thus, in practice, the general process of sensing never could lack at least a somatic sense-field to sense, for the dependently necessary conditions of the former are the originative conditions of the latter. Once the general

process of sensing is started and supplied with a somatic sense-history to sense, external stimuli acting on the organs will supply the process with sense-fields of other kinds, such as the visual and the auditory. The one process of sensing, which is permanently provided with a somatic sense-history for the reasons given above, grasps the other kinds of sense-field in its stride, as they are supplied to it from time to time by special occurrent conditions.

Here we might perhaps leave the matter; but there is a further speculation on this subject which it seems worth while to mention. I do not wish to stake too much on it, but it does seem to me to be hopeful, and not without plausibility. My suggestion is as follows: We have never attempted, so far, to analyse what is meant by an act of sensing. We have assumed that, when a sensum is sensed, it stands in some special relation to something else, and that it would not stand in precisely this relation to this something if it were not being sensed. But we have never attempted to state what this something is, nor to describe the relation. Now one result, which seems relevant for the present purpose, did emerge from our discussions in Chapter VIII on the question whether sensa are in any way mental. We saw there that the need of distinguishing between the sensum and the act of sensing was most obvious in the case of visual and auditory sensations, and that it was least evident for bodily sensations. In fact, we suggested that it was possible that bodily "sensations" are not true *sensations* at all, but are of the nature of presentations. This would mean that they are unitary experiences, in which there really is no possibility of distinguishing act and object. We have also just seen that, even if the distinction between act and object is to be drawn for bodily "sensations," the general cerebral conditions of the process of sensing cannot, in fact, arise apart from those general somatic conditions which supply this process with somatic sensa

as objects. If we combine the latter result with the suggestion that bodily "sensations" are really not distinguishable into act of sensing and sensum, we reach the following tentative conclusion: The general cerebral and the general somatic conditions co-operate to give a continuous series of unitary bodily feelings, in which no distinction between act of sensing and sensum can be drawn. This constitutes the somatic sense-history; and it is broken during life only, if at all, in dreamless sleep and other states of complete unconsciousness. Granted that these general conditions are in operation, suitable stimuli on the special organs of sense cause special sensa, visual, auditory, etc., to unite with the somatic sense-history and thus to form the general sense-history. Now I suggest, very tentatively, that "getting sensed" may just mean "coming into such relations with the somatic sense-history as to form with it a general sense-history." On this view a sensation of a red patch would be a red sensum, so related to a *somatic* field that they form together a *general* field in a certain sense-history. A contemporary auditory sensation would consist of a noise-sensum, related in the same kind of way to the same somatic field. The somatic field itself would consist of feelings or presentations, which are not objects of acts of sensing, but are unanalysable mental states. It will thus form the subjective factor in all true sensations. If we ask: "What is the relation which a special sensum must have to a somatic field in order to be sensed?" the answer seems to be that the sensum must stand in the relation of sensible simultaneity to some part of the somatic field, *i.e.*, that the two must fall into a single Specious Present. For this is certainly the only known relation which binds various special sense-fields together into a single general sense-field. Of course, it may well be that something further than this is needed, but at any rate this seems to be the most noticeable feature in the relation. If this sug-

gestion be right, what we have formerly called the "general process of sensing" is just the somatic sense-history, and what we have called "getting sensed by the general process of sensing" is just coming into the relation of sensible simultaneity with some part of the somatic sense-history.

What is meant by the "Production" of Sensa.—We have agreed that, in some meaning of the word, sensa are "produced." The production of a sensation consists in supplying the general process of sensing with a certain sensum at a certain time as an object. And, if the suggestion made at the end of the last section be accepted, this means causing a certain sensum to be sensibly simultaneous with a certain part of the somatic sense-history. Even so, the notion of "production" remains highly ambiguous, and we must start by clearing up its various possible meanings.

(*a*) *Selection and Generation.*—Dr Johnson is reported to have described his one meeting with Mr David Hume in the following terms: "On the sole occasion, Sir, on which I entered into the intimacy of a familiar conversation with that notorious Sceptic, *his* contribution to the mutual conviviality was to *produce* a drawing, so unutterably gross in its conception as to merit a murmur of disapprobation even within the walls of a brothel!" Now Dr Johnson's statement leaves us in doubt as to exactly what happened at this memorable meeting, and the doubt is due to a characteristic ambiguity in the word "produce." Did Mr Hume *select* for Dr Johnson's inspection one of a number of objectionable pictures which (like too many of his countrymen) he was carrying in his pocket? Or did he take a pencil and pollute a previously virginal sheet of paper by *generating* such a picture upon it? We may compare Dr Johnson to the general process of sensing, Mr Hume to the productive conditions of a sensation, and the picture to the sensum itself. And we may raise the

question whether, when a sensation is produced, the special occurrent conditions simply pick out a certain sensum from a mass of already existing sensa, and connect it up with the general process of sensing; or whether they have to generate the sensum which is sensed. Of course, it may well be that sensa are subject to both kinds of production. Even if the production of a sensation only needs the selection of a certain sensum from a mass of already existing sensa, it is hardly likely that these sensa have existed for ever. If they have not, they must at some time have been generated. Conversely, if the production of a sensation involves the generation of its sensum, it does not follow that this is *sufficient* to produce the sensation. No sensation will be produced unless the sensum which is generated gets properly connected with a general process of sensing; and it is not obvious that a sensum could not be generated without *ipso facto* becoming connected with a general process of sensing.

We may say then, in general, that *production* must be differentiated into *selection* and *generation*. Now selection may be either *positive* or *negative*. We may select a card from a mass of other cards, either by picking it up and leaving the rest on the table, or by leaving it on the table and sweeping all the others on to the floor. I should call the first process *positive*, and the second *negative*, selection. In general, to select x from a group g implies the following facts: (1) All the members of g originally stand in like relations to the selector s. (2) A particular member, x, of the group g is made to stand in a different relation from all the rest to s. This result can be reached either by leaving the rest of the group in their old relations to s and changing the relation of x, or by leaving x in its old relation to s, and changing the relations of all the other members of the group to s. The former is positive and the latter is negative selection.

Both forms of selection imply that a mass of sensa

already exists for us to select from. It will first be necessary to see what precisely this means. A sensum, which I sense, is an event with a certain short duration. If I say that *it* existed before I began to sense it, and that *it* will exist after I cease to sense it, I cannot literally mean that precisely and numerically the same event as that which I sensed exists before and after my sensing of it. What I must mean is that this sensum, which I sense, is a short slice of a longer strand which stretches out before the beginning and after the end of my sensum. This strand must be qualitatively alike in all its sections if it is to be true, even in a Pickwickian manner, that my sensum "existed before and after I sensed it." The strand, as a whole, is not contained in my sense-history; but I can understand what is meant by such a strand, since there are plenty of sense-objects which *are* contained in my sense-history. The physiological and other conditions must be supposed to pick out a short slice of such a strand, and to connect it up with my general process of sensing, so that it becomes one of my sensa. So the selective theory would seem to imply that *all* sensa are short slices of longer and practically uniform strands, even when these strands are not, as wholes, sensed by us, and therefore are not sense-objects in our histories.

On such a view I take it that the selective process would have two different parts to play. (1) It would select one or more out of a much larger number of such strands; and (2) out of each selected strand it would further choose the particular slice, long or short, which is to be connected with my general process of sensing. Suppose, *e.g.*, that a certain source were to send out a flash of red light and a flash of ultra-violet light. On the present view these would both be sense-objects. The former would consist of a successive series of very similar red sensa. The latter would consist of a successive series of sensa with a different sensible quality from the former. The structure of our eyes, or optic

nerves, or brains, would completely prevent us from sensing any part of the latter sense-object. This would be an example of negative selection. Again, we should not be able to sense more than a short slice of the former sense-object. The position of my body and the relevant events in my brain and nervous system presumably select this particular short slice out of the whole red sense-object to be a sensum in my history.

Now, of course, there is no doubt that our bodies do act selectively. If we turn in one direction, we automatically cut out the appearances of objects in many other directions. Again, it is presumably the structure of our bodies which determines the comparatively small range of ethereal vibrations to which sensations of colour correspond, and so on. But the question is: Do our bodies select *sensa*, and are they *only* selective in their action? Or are they also generative? I take it that the ordinary view of educated common-sense is that they do not select *sensa*, and that they do *generate* sensa. The ordinary view would be that our special sense-organs and sensory nerves select vibrations of certain wave-lengths, and transmit corresponding disturbances to the brain; magnetic vibrations, light-waves of too high or too low frequency, and so on, are automatically cut out, and fail to disturb the brain. The selection, so far, is made out of a number of *physical vibrations*, not out of a number of different *sense-objects*. Again, it is commonly supposed that if, and only if, a disturbance reaches the brain, a sensum is *generated*.

Now I do not think that there is any direct way of deciding between purely selective and generative theories. All that we can do at present is to point out the main merits and defects of theories of the selective type. On the face of it their chief merit is that they make the ontological status of sensa in the world easier to understand than do generative theories. With the latter there is a sharp distinction between scientific

objects and events, on the one hand, and the sensa, which, under certain peculiar circumstances, they generate, on the other. The very notion of generation is not easy to understand, whilst that of selection is fairly intelligible. And the status of sensa, when generated, in a world which consists almost wholly of scientific events and objects, is certainly most peculiar. Finally, we are directly acquainted with many sensa, and therefore do know *that* there are such things and *what kind* of things they are. Now the natural complement of a selective theory of the production of sensa is a theory that physical events and objects are composed of sensa, some few of which are sensed and the great majority of which are unsensed. It might reasonably be said that the hypothetical entities of such a theory are less hypothetical than those of the generative theory, which makes physical events and objects to differ in *kind* from sensa and sense-objects. On the view of physical objects and events which corresponds to the selective theory of the production of sensa, all that we need to postulate is unsensed sensa and unsensed sense-objects. That is, we only need to assume more entities of the same kind as we meet with in our sense-histories.

Thus we may fairly say that, *if* a purely selective theory can be made to work, and *if* it can be accompanied by a satisfactory theory of physical objects as composed wholly of sensa, it will have the double merit of avoiding the difficult notion of generation and of giving sensa a less ambiguous status in the universe than any generative theory is likely to do. I will now point out certain difficulties in theories of the selective type, and in the view of the nature and status of sensa which generally accompanies such theories.

(1) It is difficult to work a purely selective theory without postulating a perfectly enormous number of unsensed sensa. I am not now alluding to the sensa which have to be put in places where there are no

observers. After all, *any* theory has to put *something* (*e.g.*, light-waves, etc.) into such places and times; so that the selective theory is here no worse off than the generative theory. For similar reasons I do not make it an objection that there will have to be many kinds of sensa (*e.g.*, magnetic, ultra-violet, and so on) which no one ever senses. What I am thinking of is the following fact. At a place, where the physicist would say that a single physical process is going on, it is possible for all sorts of qualitatively different sensa to be sensed by putting in different observers or by altering the internal states of a single observer. If physiological processes be purely selective, we shall have to postulate as many different kinds of sensa co-existing at a given place and time as any observer, however abnormal his bodily condition, can sense if put there at that time. I say *co-existing*, although we cannot literally have the same observer in two different states at once, or two different observers in the same place at once. For we do find characteristic changes in the sensa which are sensed from a place *whenever* we suitably alter the internal state of the observer there or introduce a suitably abnormal observer into his place. If you hold that the internal states of the observers' bodies are causally independent of the sensa which they sense, and that they act merely selectively, you must conclude, in accordance with the argument of Chapter XI, that sensa *like* all those which the various observers sense *co-exist*, although the sensa which are *actually sensed* are successive. (*Cf.* pp. 422 to 429.)

I will take one very simple example to illustrate my meaning. An observer stands in a certain place and senses a certain sense-object. He pushes his eye aside with his finger, and begins to sense two similar sense-objects which are sensibly separated. This happens whenever he chooses to push his eye aside. If bodily conditions be purely selective, there must have been two separate and similar sense-objects all the time,

one of which remains unsensed except when he pushes his eye aside. I find this very difficult to swallow; and a supporter of a purely selective theory will have to swallow a large number of equally unpalatable doses. If the sensa which an abnormal observer, or a normal observer in a temporarily abnormal state, senses from a certain place were *absolutely unlike* those which normal observers sense from that place, a purely selective theory would be more plausible. The difficulty is that the abnormal sensa are a great deal like the normal ones, and yet distinctly different. It is very difficult, under these conditions, to resist the conviction that both the abnormal and the normal sensa are *generated* by two sets of conditions, one common to both, and one varying from observer to observer. The former accounts for the likeness, and the latter for the difference, between the sensa.

The only purely selective theories that I know of are M. Bergson's in *Matter and Memory* and Prof. Alexander's in *Space, Time, and Deity*. M. Bergson holds, so far as I can understand, that physiological conditions are purely selective, and that the selection is negative. Our minds would normally be in similar cognitive relations to every event in Nature, and the whole function of our bodies in perception and memory is to *shut out* the vast majority of these events from our cognisance. Unfortunately, M. Bergson does not condescend to enter into detail, and the only possible way to decide for or against selective theories is to work them out in detail and to see whether they can be made to fit the known facts. Prof. Alexander is not open to this objection. He has made the most heroic efforts to work out a purely selective theory, and he accompanies it with a definite and extremely interesting view as to the nature of sensa and their status in the universe. He takes physical objects to be four-dimensional strands of history; and here he is undoubtedly right. He then supposes sensa to be "sections" across

such strands. Sensa are thus "contained in" physical objects, as the various sections which could be got by slicing an ordinary cylinder in various directions are "contained in" the cylinder. The position of the observer's body selects the particular physical objects, and the particular sections of each of these, which his mind can "contemplate" there and then. The function of the physiological processes in brain and nervous system is to keep up that process of "enjoyment" which is the contemplating of such sections. Such a theory has many advantages, if it could be made to work. It accords with common-sense in making sensa fragmentary and dependent, as compared with physical objects. And yet it makes all sensa, whether sensed or not, exist as "parts" of physical objects, in a perfectly definite and intelligible way. They exist in physical objects, as the various possible sections of a geometrical solid figure exist in it. Some are momentary, and may be compared to the various circular sections of a cylinder, if we compare the axis of an ordinary cylinder to the time-direction of a strand of physical history. Others consist of a set of momentary events of various dates, all falling within a certain short duration ; these might be compared to oblique sections of an ordinary cylinder.

Unfortunately, it seems very difficult to uphold such a theory in face of all the facts. If we never dreamed, and if we always saw objects through a perfectly homogeneous medium, without mirrors, lenses, etc., and if people and things never moved about, it would be more plausible. I cannot, of course, attempt any adequate criticism of it here, but I will raise one point: When I see an image of a pin in a mirror, of what physical object precisely are my visual sensa sections? If they be sections of the pin's history, why are they optically present at a place quite remote from that which is occupied by the pin? And how can the image-sensa and those which I sense when I look directly

at the pin be sections of the same strand of physical history? If the image-sensa be not sections of the history of the pin, are they sections of some strand of physical history which is located at their optical place? Surely not; for it is well known that no relevant physical process is going on there. Are they then sections of some strand of physical history located at the surface of the mirror? If so, why is their optical place at some distance *behind* the surface of the mirror instead of upon it? Prof. Alexander has tried his hardest to deal with such difficulties, and in the course of his discussion much of value has emerged; but he has provided no answer which I can fully understand or accept.

(2) If, in face of difficulties of this kind, we add the smallest trace of generation to a purely selective theory, the latter at once loses many of its advantages. I will take Mr Russell's theory, as expounded in his *Lowell Lectures* and his *Analysis of Mind*, as an example of a predominantly selective theory with a small trace of generation in it. He regards a physical object as a group of connected sensa, with members in all parts of physical Space-Time. The vast majority of these are unsensed. If the body of a living observer be at a certain place at a certain time, he will sense one sensum from each such group, and one only; though he will, of course, be sensing sensa from many different groups at once. So far the theory is purely selective. But I understand Mr Russell to hold that those sensa, belonging to a given physical object, which occupy regions of physical Space-Time where there is no living organised body, are systematically different in quality from sensa of the same group which occupy regions of Space-Time where such a body is present. This would seem to suggest that the observer's body and its internal processes are generative, as well as selective, in their action, and that they at least modify qualitatively those sensa of any group which are in their

neighbourhood. Mr Russell seems generally to regard organised bodies as analogous to distorting media, like coloured glass. I take it that Mr Russell's theory, in its present form, is admittedly transitional; it is only a first step in the direction which he wishes to follow. This makes it a very delightful "Aunt Sally" for the numerous philosophers who are more anxious to score neat verbal hits than to help in unravelling the complexities of Nature. I propose to state some of the main difficulties which strike me in the theory, as presented; without imagining for a moment that they are fatal objections to this type of theory, or that Mr Russell is not quite as well aware of them as I am.

(i) A purely selective theory, if it could be worked out, would have two advantages, one ontological, and the other epistemological. The ontological advantage is that sensa would be given a definite and intelligible status, as, in some sense, "parts" of physical objects; whereas, in theories of the generative type, it is hard to see how they exist side by side with the physical events and objects which generate them. The epistemological advantage is that the hypothetical entities, which every theory needs in order to fill the gaps between our sensations, are here of the same kind as the sensa which we sense. We are therefore only postulating more entities of a kind which we already know to exist.

Now it does seem to me that a theory like Russell's, however successful it might be on the ontological side, sacrifices most of the epistemological advantages of a purely selective theory. If our brains and nervous systems be a kind of "medium," they are media from which even the "Free Man" cannot get free. And it is admitted that they "colour" to an unknown extent all the sensa with which we can possibly become acquainted. We therefore do not really know that sensa can exist at all apart from brains and nervous systems. And, even if we decide to postulate sensa of

some kind in places and times where there are no brains and nervous systems, we cannot have the slightest idea what intrinsic sensible qualities such sensa will have. We really know just as much and just as little about them as we do about the hypothetical scientific events and objects of the Critical Scientific Theory. To call them *sensa*, under these circumstances, seems rather misleading; for it is liable to disguise the purely hypothetical character of these events, and to suggest that we know a good deal about their intrinsic qualities. Really we know nothing about the events which happen at intermediate times and places between the opening of a shutter and our sensing of a flash, except that they obey Maxwell's Equations.

(ii) In Chapters IX and X I pointed out that perceptual physical objects are *composita*, made up of various correlated constituent objects, optical, tactual, etc. Now, Mr Russell's theory seems to have been built up wholly by considering the *optical* constituents of perceptual physical objects. It is a theory of *complete optical objects*, and, so far, of nothing else. It cannot even be said that he has yet dealt with *partial optical objects*, like mirror-images, or with the still worse complications of non-homogeneous transmitting media. When Mr Russell tells us that he can easily deal with Nature by regarding it as a six-dimensional spatial whole, in which all sensa have their places, and by regarding physical objects as groups of sensa which form three-dimensional spatial wholes, I cannot help suspecting that he is thinking only of visual sensa and of complete optical objects. At least, I can understand more or less what he means, on this interpretation, but not at all if he expects to work all kinds of sensa and all the various components of perceptual physical objects into such a scheme.

(iii) Closely connected with this is the fact that Mr Russell has not yet treated the observer's body in terms of his general theory of physical objects. The

body is a physical object; and, regarded as a perceptual object, it has all the components which an ordinary piece of matter has, together with a special component, viz., the somatic history. If Mr Russell's general theory be right, my body must consist of a set of correlated groups, each composed of correlated sensa of a certain kind; and it must be this composite set which selects and "colours" the sensa of the other physical groups which we sense. I am not sure that his theory does not at present owe some of its plausibility to the fact that, while we read his exposition, we think of our own bodies (and perhaps of other media, like mirrors and coloured glass) as physical objects in the non-Russellian sense, and of all other pieces of matter as physical objects in the Russellian sense.

(iv) It might, perhaps, be objected that Russell's theory makes sensa too substantial and self-subsistent, whilst it makes physical objects too ghostly. Certainly Alexander's theory is, in this respect, more in accordance with common-sense. But I am not inclined to attach much weight to this objection myself. After all, on Russell's theory, unsensed sensa do not as a rule exist in isolation. They are members of physical groups, connected together by qualitative similarity and regular rules of spatio-temporal correlation. And the alleged substantiality of physical objects, as compared with sensa, may well rest on nothing but our special practical interest in those groups of sensa which happen to be pretty stable, and our practical ignoring of isolated sensa, or of abnormal and less permanent groups, such as mirror-images.

The upshot of the discussion seems to be that selective theories are at present rather in the position of democratic government. There is no *positive* argument for them; the only arguments *for* them are the objections *against* their alternatives. And the analogy may be carried further, in so far as there are serious positive objections to all selective theories that have yet been

suggested. If, to avoid these, we introduce a certain amount of generation, we may keep many of the ontological advantages of selective theories, but we lose most of their epistemological benefits and we introduce the new and difficult conception of generation.

(*b*) *Causation and Creation.*—It remains to consider the form of production which we have called *generation.* This is itself an ambiguous term ; and generation must be distinguished into *causation* and *creation.* We shall see that the distinction between creative and causal theories does not rest on an absolute difference of kind ; still it is important, and it must be firmly grasped before we can criticise generative theories of the production of sensa.

When I say that the friction of two bodies "generates" heat, I am using "generation" in the causal, and not in the creative, sense of the word. I mean that a certain process in two pre-existing bodies (*e.g.*, the rubbing together of a drill and a piece of iron) is followed by a change of quality (or rather, by a change of intensity in an already existing quality) in both of them. All ordinary generation is of this type. It presupposes one or more already existing substances, as continuant conditions ; and it asserts that one specific kind of change in their qualities or relations is followed, according to a general rule, by another specific kind of change in their qualities or relations. *Creation*, on the other hand, would mean that certain occurrent conditions in a pre-existing substance or substances are followed by the springing into existence of a *new substance* of some specific kind. The difference may be stated shortly, in terms of occurrent and continuant conditions. Both causation and creation involve these two kinds of condition. In ordinary causation, the event which is determined by them *joins up with* one or other of the continuant conditions, and becomes a part of *its* history. In creation, the event which is determined does not join up with any of its continuant

conditions to form a further stage in their history; it either remains isolated or is the beginning of an altogether new strand of history.

Now, in real life, there are no examples of pure creation. However isolated an event may be when it is generated, it has some place and date in Nature, and thus joins up with and continues the history of Nature *as a whole*, if not the history of sóme particular *pre-existing object* in Nature. Moreover, if it be determined by events in pre-existing substances, its place, date, and specific qualities will be fixed by those of its determining conditions. So it is, at least, joined on by *causal* connexions to one or more special pre-existing parts of Nature; although it lacks that qualitative similarity and spatial continuity with any of these parts, which would be needed before we could say that it actually joins up with and continues the history of some particular pre-existing substance. Thus, we may speak of one generative process as being "more of the creative type," and of another as being "more of the causal type"; but we can hardly speak of any process as "purely creative." In proportion as a generative process is more of the creative type, it is less intelligible to us; and one difficulty about generative theories of the production of sensa is that, at first sight at any rate, the generation of sensa by physical and physiological processes seems to be predominantly of the creative type. Let us see how far this is true.

If processes in our own bodies be sufficient conditions for generating sensa, it cannot be said, as a rule, that the sensa which they generate join up with and continue the history of the conditions which generate them. If a change in my optic nerve or my brain generates a red sensum, there is no obvious way in which this sensum can be said to join up with and continue the history of my brain or optic nerve. If sensa and sense-objects differ in kind from scientific events and objects, it is clear that there cannot be

much literal continuity of quality or position between a sensum and its generative conditions. The only continuity is temporal and causal. Even if we suppose that physical objects, including our brains and nerves, are groups of sensa, some of which are sensed and most of which are not, there is still very little continuity between most of our special sensa and their somatic conditions. For, on such a view, my body is presumably a large group of somatic sensa, out of which I sense a certain small selection which forms my somatic sense-history. The physiological conditions which generate other sensa would therefore be somewhere in this mass of somatic sensa. Now, visual and auditory sensa are not in the least like somatic sensa; they fall into different special sense-histories, and not into the somatic sense-history. Hence, even if our brains and nervous systems be simply groups of somatic sensa, it cannot be said that the visual and auditory sensa, of which they are the continuant generative conditions, join up with them and continue their history in any plain and straightforward way. (Of course, these remarks do not apply to the generation of somatic sensa themselves; for they do join up with the somatic sense-history, and the latter simply is a selection out of that whole mass of somatic sensa which would constitute my body on the hypothesis under discussion.) Thus we may say that, on no view of the nature of physical events and objects, can visual and auditory sensa be said to join up with and continue the history of their generative conditions, if the latter be processes in our brains and nervous systems. Thus, if such sensa be generated at all by physiological processes, it must be admitted that the generation is rather of the creative than of the causal type.

On the other hand, we must not exaggerate the isolation of visual and auditory sensa. (1) All those that we sense are at any rate events in our *general* sense-history, and are thus related at least by sensible

temporal relations to parts of our somatic sense-history. (2) Again, it is very rare for a visual sensum to occur apart from other visual sensa. This does happen indeed if we sense a single flash on a dark night. But usually a visual sensum is an outstanding part of a much larger visual field, and this visual field is itself a slice of a visual sense-history, which stretches out before and after it. So, in the vast majority of cases, visual sensa when they occur, do join up with a special pre-existing continuant, viz., the observer's visual sense-history. This is less frequently true of auditory sensa, though it is often true of them too. (3) Often a visual sensum does not merely continue the visual sense-history in general, but continues the history of some particular sense-object within it. This is true of most of the outstanding sensa in our visual fields, if we look steadily in any one direction. (4) Even when a sensum is a quite isolated event in *my* general sense-history, and not part of any sense-object in one of my special sense-histories (*e.g.*, when it is a single flash sensed on a dark night), it may have specially close correlations with sensa in the histories of other observers. It may be a member of a group of very similar sensa, which constitutes a complete or partial optical object and has members in various observers' histories. And the sensum in another observer's history, which is thus correlated with an isolated sensum in mine, may not itself be isolated. It may be a slice of a long sense-object. For instance, another man may be gazing at a lighted candle, and between it and my body there may be an opaque object with a shutter. If this shutter be suddenly opened and immediately afterwards closed again, I shall sense an isolated visual sensum. But it will be correlated with a very similar sensum in the other man's history, and this other sensum will be a short slice of a long sense-object. So that, indirectly, my isolated sensum will be correlated with a certain special sense-object, although this sense-object is not in *my* history.

Thus it is far from being true in general that sensa are perfectly isolated occurrents, and that they do not join up with the history of pre-existing continuants. What we must say is that *sometimes* they seem to be extremely isolated ; that *often* their connexion with pre-existing continuants is rather remote and indirect ; and that apparently they *never* join up with the history of that particular continuant (viz., the brain) which is the seat of their most immediate special occurrent conditions. These facts show that the generation of sensa by physical and physiological processes must be considerably different from the causation of a change in one physical object by a change in another. But they do not suggest that the generation of sensa, if it take place at all, is a perfectly unintelligible process of creation.

(*c*) *Physical Causation and Causation of Sensa.*—We have seen that there is no radical distinction between causation and creation, but that the generation of physical events is more of the causal type, and that of sensa more of the creative type. We ought therefore to be able to give a definition of generation, which shall cover both cases, and then to point out what distinguishes the generation of sensa from that of physical events.

In order to do this, we must enter a little more deeply into the nature of events. An event is a particular existent, and therefore the generation of any event is the generation of a new particular existent. By this I simply mean that precisely and numerically the same event cannot possibly recur, although, of course, qualitatively similar events can occur at many different times and places. Next, we must distinguish between *determinateness* and *particularity*. A perfectly definite shade of red is determinate, but is not particular. The difference between determinateness and particularity will best be seen by an example. Let us take (1) redness in general, (2) a perfectly definite shade of red, and (3) a certain sensum which has this shade of red. The

relation of (3) to (2) is quite different from that of (2) to (1), though this is often disguised by the statement that (2) is an *instance* of (1) and (3) is an *instance* of (2). The difference is that the sensum cannot recur, though other sensa of exactly the same shade may occur at other times and places. On the other hand, the definite shade of red is still a universal; since any number of sensa may have precisely this shade of red. It is therefore best to say that the definite shade of red is *a lowest determinate under the determinable of redness* (to adopt Mr W. E. Johnson's phraseology), and that the sensum is *a particular instance* of this determinate. The analogies and differences between being a determinate under a determinable, and being an instance of a determinate, are the following: (1) Determinables have a plurality of determinates, and determinates have a plurality of instances. But (2) the number of determinates under a given determinable is a necessary consequence of the nature of the determinable, whilst the number of instances of a given determinate is purely contingent. It is of the nature of redness that there should be just such and such shades of red, but the number of instances of any shade of red depends on the make-up of the existent world. And (3) the instances of determinates are always particulars, whilst the determinates under determinables are always universals.

Now an event is fully described *i.e.*, is marked off from all other events, if we know (1) its place and date in some Space-Time; (2) its extension and duration; and (3) the determinates of which it is an instance. For example, a certain visual sensum is completely described if we know where and when it occurs in an observer's sense-history, what shape it has, how long it lasts, and what precise shade of what precise colour it has. Thus, the occurrence of any event consists in the "occupation" of a certain definite region of some Space-Time by one or more determinates under one or more determinables. Now the nature of the

"filling" of one or more regions may fix, according to general rules, the nature of the "filling" of a certain other region. If so, we say that the events which consist in the former regions being "filled" with such and such determinates *generate* the event which consists in the latter region being "filled" with such and such other determinates.

We can now give a definition of *generation* in general. The widest form of causal law would seem to be of the following kind: If any determinate c of the determinable C inheres in a region r of the Space-Time S, then a certain correlated determinate γ of a certain correlated determinable Γ inheres in a certain correlated region ρ of a certain correlated Space-time Σ. (Of course, the antecedent may involve more than one determinable, and more than one region; but there is no need to complicate matters further for our present purpose.)

Now I take it that ordinary physical causation is distinguished by a very great simplification of this most general type of law. (1) All the events under consideration are in the same Space-Time (viz., physical Space-Time) so that $S = \Sigma$. This is true, in spite of the fact that physical Space-Time can be split up in many different ways into time-axes and timeless spaces. (2) Very often in physical causation we have only to deal with a single determinable, *e.g.*, physical motion. This would be true if, *e.g.*, we were considering how the motion of one billiard-ball generates that of another. In such cases $C = \Gamma$. (3) The determinables are generally such that their determinates can be fixed by giving a particular numerical value to some quantitative variable. If so, c and γ will be connected by a mathematical formula, such as $\gamma = \phi(c)$. Lastly (4), since we are dealing here with a single Space-Time, we may be able to assign a single system of co-ordinates to the whole of it. The regions r and ρ will then have co-ordinates in the same frame, and the correlation between them will be expres-

sible in an equation or set of equations of the form $\rho = \psi (r)$.

Now the peculiarity of the causation of sensa may be that these special simplifying conditions are not fulfilled here. Take, *e.g.*, the production of a red sensum by processes in the optic nerve and brain, supposing that these are sufficient occurrent conditions. (1) The brain-events consist in the filling of a certain region of physical Space-Time with certain physical determinates. The sensum consists of the filling of a region in the observer's visual Space-Time with a determinate shade of red. Thus two different Space-Times are involved. (2) In consequence of this, the correlation between r and ρ will be of a much more complicated type than it would be if r and ρ were just two regions in the same Space-Time. (3) We are here concerned with two quite different determinables, viz., physical motion (say) and redness. Thus we cannot put $C = \Gamma$. (4) The determinates under redness, *i.e.*, the definite shades of red, cannot be expressed simply by different values of the same numerical variable, since they differ qualitatively. Thus we cannot put $\gamma = \phi (c)$, where this is an ordinary algebraic equation or set of equations.

All this complication is doubtless troublesome, but it does not really render the causation of sensa different in *kind* from the causation of one physical event by another. The scientist has simply banished nearly all *qualitative* differences from his world, and has contented himself with the residuum. But the whole mass of sensible appearances, from the most impressive to the most trivial, and from the most normal to the most outlandish, forms part of the total content of the existent world. We have no right then to feel surprised if the structure and laws of the existent world as a whole fail to show that sweet simplicity which distinguishes the particular part of it to which natural scientists have confined themselves. Science has been able to make the great strides which it has made by deliberately

ignoring one side of reality. The end has justified the means, for the world is so complex that it can only be understood bit by bit. Moreover, the success of this abstraction does show that reality as a whole has less unity than certain departments of it. The physical part of reality and the sensible part do not indeed form water-tight compartments, but it does seem as if there were characteristic forms of unity in each which do not stretch across from one to the other. From the philosophic point of view, the procedure of natural science has rather resembled that of those diplomatic Conferences which have done so much to brighten European life since the Allies inaugurated the New Jerusalem in 1918. The most edifying unity has been secured on each occasion by turning a blind eye to all the less convenient facts, and referring them to a future Conference for further discussion. In philosophy, as in economics, facts do not cease to be real by being ignored; and the philosopher becomes the residuary legatee of all those aspects of reality which the physicist (quite rightly, for his own purpose) has decided to leave out of account. The analogy only breaks down when we contrast the relative success of the scientists and of the politicians in their respective fields.

The difficulty which we feel about the ontological status of sensa may be put as follows: We feel that anything which can successfully claim to be "real," must be some*where* and some*when*. And we are so much accustomed to physical Space-Time, and to the way in which physical events and objects occupy regions in it, that we think that an event cannot be "real" unless it occupies some region of physical Space-Time in the way in which a physical event does so. Now, it seems clear that either (1) sensible determinates (such as some particular shade of red) do not inhere in regions of physical Space-Time, but in regions of some other Space-Time; or (2) that, if they do inhere in regions of physical Space-Time, they must inhere in the latter

in some different way from that in which physical determinates (like physical motion) do so. Either there is one sense of "inherence" and many different Space-Times, or there is one Space-Time and many different senses of "inherence." On either alternative the world as a whole is less simple than we should like to believe; and, if we have come to think that there is only one possible Space-Time and only one possible kind of inherence, we shall be inclined to suppose that sensa are nowhere and nowhen, and therefore are mere fictions. Since this is plainly contrary to fact, unless the whole way of treating sensible appearance which is developed in this book be wrong, we must accept one of the two alternatives just mentioned.

Now, it seems to me that these two alternatives are not mutually exclusive, but are complementary. We have long ago dropped the notion that a Space-Time is a kind of empty warehouse, with various cellars ready to receive different materials; although it remains convenient to talk as if this were so. Our view is that a Space-Time is a characteristic form of relational unity which pervades a whole set of entities, and binds them together into a peculiar kind of complex whole, whose fundamental structure is summed up in the geo-chronometry of the Space-Time in question. When we say that a determinate "inheres in a certain region of a certain Space-Time," we only mean that an instance of it enters into certain relations with other instances of the same and of other determinates, and that the relations which it has to them are of the same type as those which they have to each other. I think that my view of the structure of Nature as a whole, with its peculiar mixture of unity and disunity, can be more clearly explained by a familiar analogy than by a great deal of formal exposition.

Let us compare a Space-Time to a family of brothers and sisters. Then, coming to occupy a region of this Space-Time will be like being born into this

family. Let us take such a family, and suppose that all its members are children of the same husband and wife. This fundamental family F_1 shall be taken as analogous to the physical world, and the simple relation of brother or sister within it shall be analogous to the structure of physical Space-Time. Now we can suppose that some of the members of F_1 have children, and that others do not. Those who do may be compared to organised bodies, and those who do not to unorganised bodies. I am going to take the children as analogous to sensa. Now consider the families of two members of F_1. Let these two members be A and B, and let us call their families respectively F_A and F_B. Then we notice the following facts: (1) Each of these families forms a group analogous to F_1. This corresponds to the fact that the sensa of each individual (provided they are of the same sort) form a spatio-temporal whole. (2) F_A and F_B do not together form one family, in the sense defined. This corresponds to the fact that the sense-histories of different observers form different Space-Times. (3) Neither F_A nor F_B forms with F_1 a single family, in the sense defined. This corresponds to the fact that sensa are not literally in physical Space-Time, and that physical events are not literally in any sensible Space-Time. (4) In spite of this, there are relations between members of F_A and members of F_B, viz., the relation of cousinship. Similarly, there are relations between members of F_A or F_B and those of F_1, viz., the relations of child-and-parent or of nephew-and-uncle. Thus, although the whole set of individuals of the two generations does not constitute one family, in the sense of one set of brothers and sisters, yet it does constitute a set of interrelated terms, which may be called a "family" in a wider sense. In precisely the same way, I take it, the physical world and the various sense-histories form one interrelated whole, although the relations which stretch across from one sense-history to another or from a sense-history to the physical

world are more complex than those which interconnect physical events or interconnect sensa in the same sense-history. (5) Lastly, we might suppose that some of the members of F_1 had married twice in succession, and had thus had two families. Or, again, some of them might have embraced Mormonism and a plurality of contemporary wives. We should thus get a peculiar relation, viz., that of half-brother, to which there is nothing exactly analogous in the family F_1. The whole family of M, the Mormon member of F_1, would split up into two or more families. The relation between a member of one of these families and a member of another of them would be more intimate than that of cousinship and less intimate than that of complete brotherhood. This is analogous to the fact that the general sense-history of an observer splits up into a number of special sense-histories, such that sensible temporal relations do, and sensible spatial relations do not, stretch across from one to the other.

Now, if we had taken the original family F_1 as fundamental, and had "placed" all the members of the second generation by stating their various relations, such as child, nephew, etc., to various members of F_1, this would be analogous to taking physical Space-Time as fundamental and saying that sensible determinates of different kinds inhere in different ways in regions of this one Space-Time. If, on the other hand, we take the notion of families, in the strict sense, as fundamental, this will be analogous to saying that there is a plurality of different, though correlated, Space-Times, and that sensible determinates inhere in their own Space-Times in the same way as physical determinates inhere in physical Space-Time. It is obvious that these are only two different ways of treating the same set of interrelated facts. Logically the two methods are equivalent to each other.

I have taken this elementary example to illustrate in rough outline how we can combine sensa and physical

events into one universe, in spite of their many important differences. The exact details of this must be left to the symbolic logician ; but the complexities which arise even in the simple example of family relationships will show the reader that the complication of Nature as a whole is compatible with the ultimate relations between its elements being comparatively few and simple. The mistake is to try to force Nature *as a whole* into the mould which fits one important *part* of it ; and then to suppose that, because this attempt breaks down, Nature as a whole has no structure at all, but falls into completely isolated and incoherent fragments. There are, I believe, two different levels of "simplicity," and between them there is a region of "complexity." There is the lower kind of simplicity, which we find when we isolate one fragment of Nature from the rest, and ignore all the awkward facts that refuse to fit into the scheme which applies to this fragment. There is, or there well may be, a higher kind of simplicity, where we have recognised the fundamental structure of Nature as a whole, and have seen how the structure of special regions of Nature is just a special case of these fundamental relations. But, in order to pass from the lower to the higher kind of simplicity, we must traverse an intermediate stage of confusion and complexity, in which we confront the lower simplicity with all the awkward facts which it has ignored. This is a task in which we can all help, if we keep our heads clear and refuse to be put off with cheap and easy explanations. The final stage, that of finding the simple plan on which all this complexity is constructed, can only be accomplished by men who combine the insight of genius with technical mathematical ability of the highest order. To this combination of gifts few of us can lay claim, and the present writer is certainly not one of those who can. In our day one man, Einstein, has shown what such a combination can accomplish within the region of physics. We still await the man who

will show us in detail how the world of physics and the world of sensible appearance are united into the one whole of Nature. The utmost that we can claim to have done here is to have stated some of the facts which he will have to take into account and to unify.

The following additional works may be consulted with advantage:

A. N. WHITEHEAD, *The Principles of Natural Knowledge*, Parts II. and IV.

——— *The Concept of Nature*, Chaps. I., II. and VII.

——— *The Principle of Relativity*, Chaps. II. and IV.

B. A. W. RUSSELL, *Our Knowledge of the External World*, Lects. III. and IV.

——— *The Analysis of Mind*, Lects. V. and VII.

S. ALEXANDER, *Space, Time, and Deity*, Bk. III.

H. BERGSON, *Matter and Memory*.

G. E. MOORE, *Philosophical Studies*.

G. F. STOUT, *Mind*, Vol. XXXI. No. 124.

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"How index-learning turns no student pale
Yet holds the eel of Science by the tail."
(Pope, *The Dunciad.*)

PRINTED IN GREAT BRITAIN BY
THE EDINBURGH PRESS, 9 AND 11 YOUNG STREET, EDINBURGH